

# DECOUPES GEOMETRIQUES (Geometric Dissections)

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« Les matières de Géométrie sont si sérieuses d'elles-mêmes, qu'il est avantageux qu'il s'offre quelque occasion pour les rendre un peu divertissantes » Blaise PASCAL

## INTRODUCTION

Il s'agit de découper une figure géométrique polygonale (convexe ou pas) pour qu'avec les morceaux (ou « pièces ») obtenu on puisse en reconstituer une autre (donc de même aire).

Deux polygones de même aire étant donnés un théorème nous assure qu'il existe toujours une découpe du premier en un nombre fini de pièces permettant de reconstituer le second.  
Théorème énoncé par M. LOWRY en 1814, redécouvert par W. WALLACE, puis par F. BOLYAI, G. GERWIEN en 1833.

Notons que HILBERT, dans l'un de ses célèbres 23 problèmes qu'il jugeait difficiles, avait posé la même question pour les polyèdres. Il fut alors rapidement montré que ce n'est pas possible pour tous les polyèdres...

## FIGURES A DECOUPER

Les plus fréquentes sont les suivantes :

CARRE (Sq) ; TRIANGLES (équilatéral, isocèle, scalène).

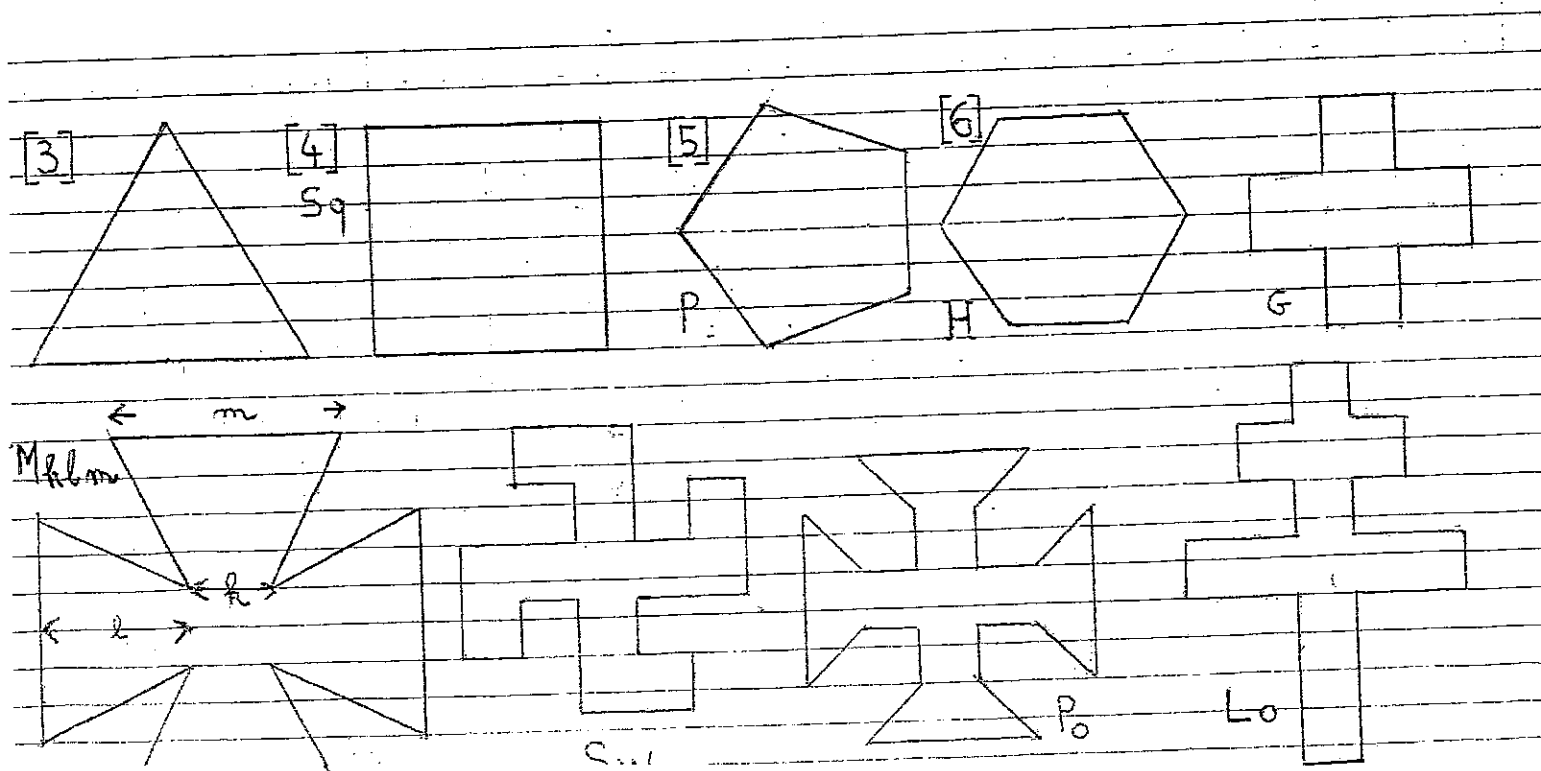
PENTAGONES, HEXAGONES, OCTOGONES, DODÉCAGONES (réguliers ou pas).

CROIX « GRECQUE » : (G) (celle de la Croix Rouge), CROIX de MALTE : M k,l,m :

selon l'ouverture des bras ; CROIX DE LORRAINE : (Lo), d'aire a = 13 ; CROIX

ORTHODOXE ; SWASTIKAS (d'aires 13,17,25, ...) ; CROIX PORTUGAISES (Po)

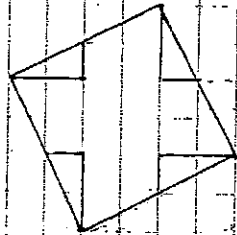
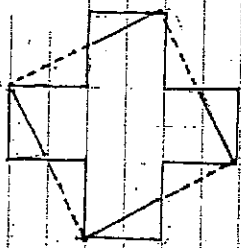
ETOILES : à 5 branches (Pentagramme : 5/2), à 6 branches (Hexagramme : Hx ou 6/2), etc



1er exemple

I

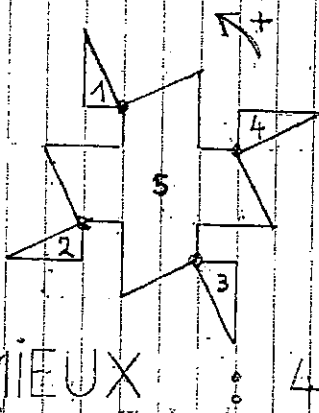
(2)



$G \leftrightarrow Sq$

5 p.  
mais  
mieux!

\* Articulable :



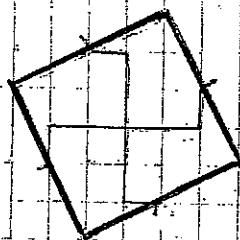
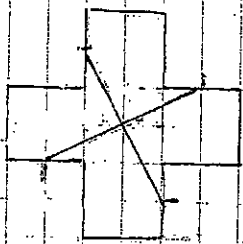
si on fait tourner les 4  
pièces de :

$\frac{\pi}{2}$   $\Rightarrow Sq$   
 $\frac{\pi}{2}$   $\Rightarrow G$

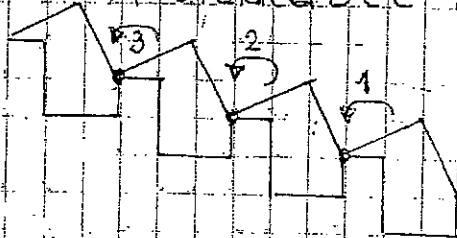
\* MIEUX

4 p.

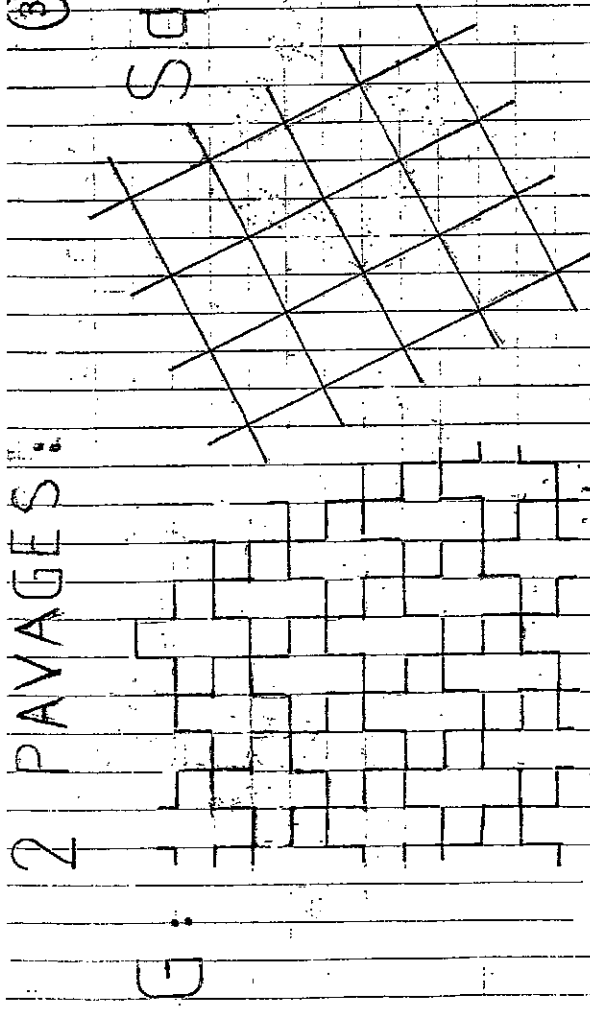
raison!



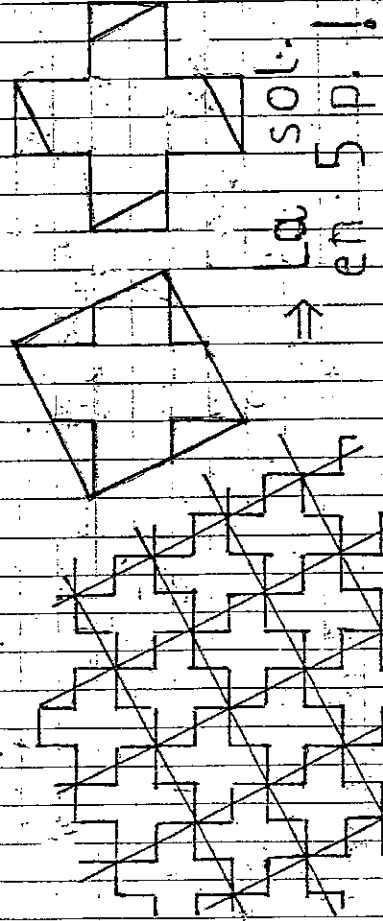
Articulable :



2 PAVAGES :

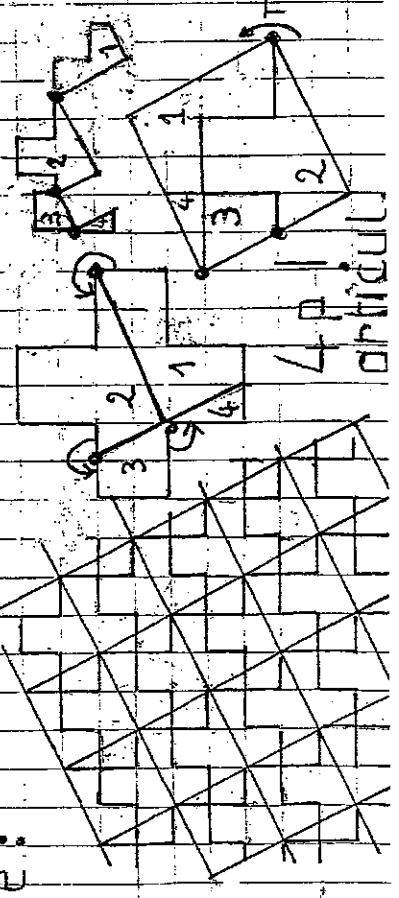


superposition :



⇒ la sol. en 5 p. !

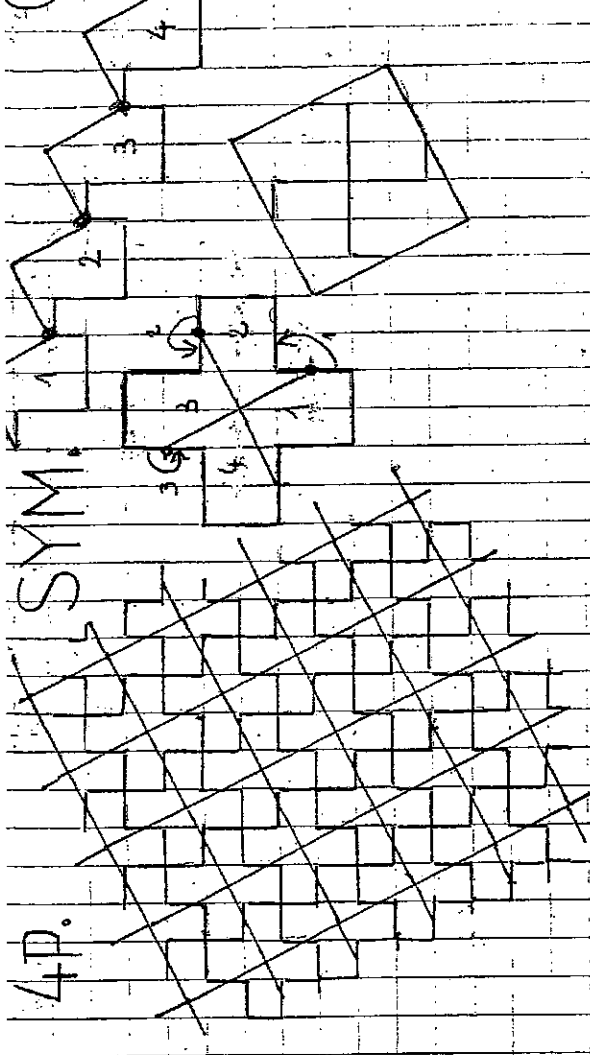
Autre :



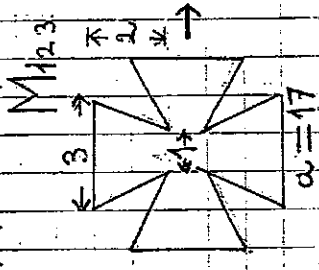
4 p. articulé

4 p.

SYM.

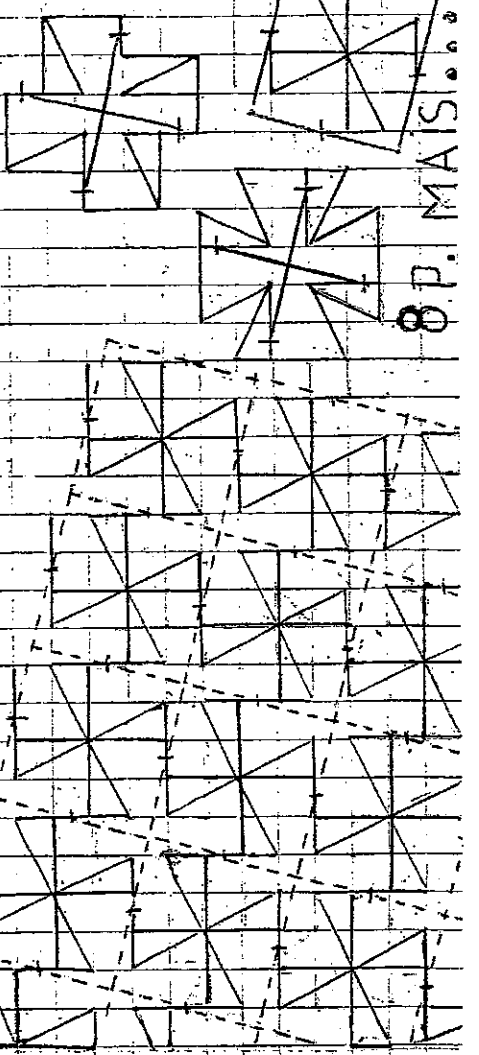


Autre appli. de PAVAGES SUPERPOS  
M<sub>123</sub> : CROIX de MALTE



4 TRIANGLES  
à retourner

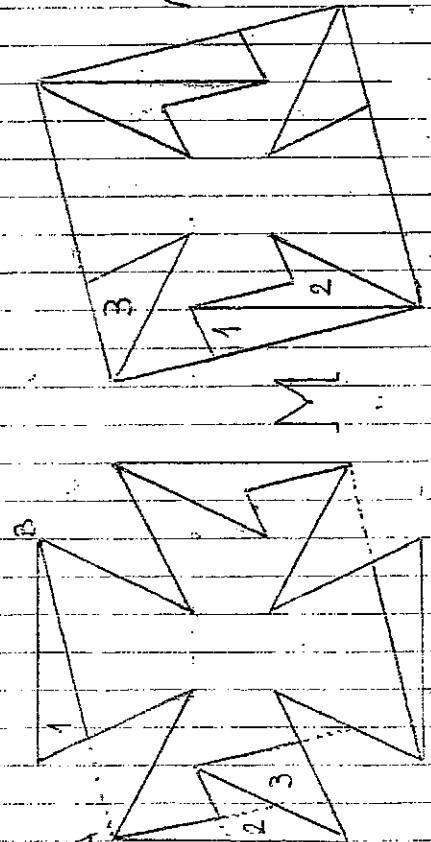
MOTIF de  
PAVAGE



8 p. MAIS...

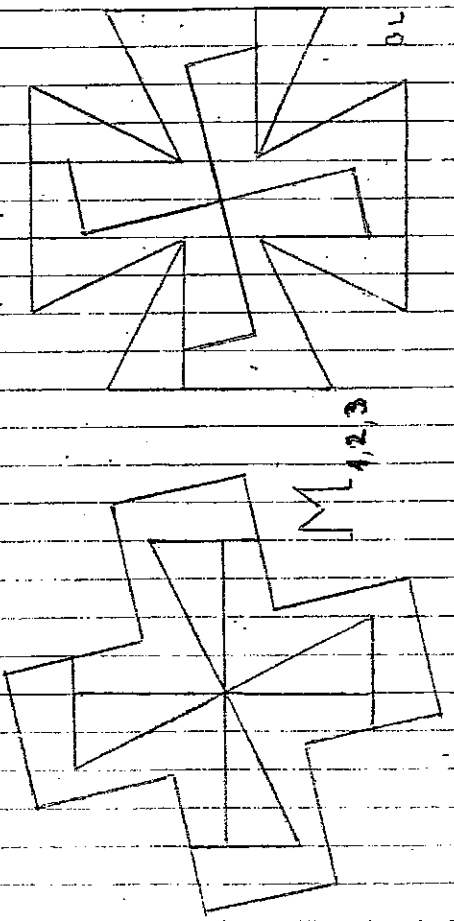
... E MIEUX !

7p. seulement :



A. E. HILL  
 E. H. DUDENEY  
 ca 1900

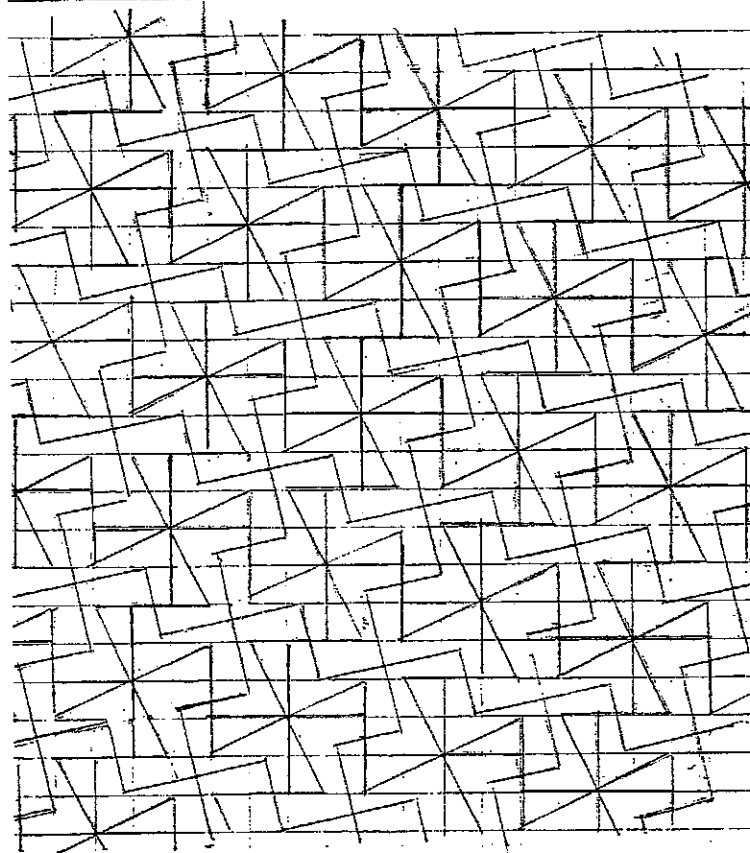
MAIS LE PAVAGE PRÉCÉDANT M'A PERMIS DE  
 TROUVER :  $G \leftrightarrow M$  EN 8p. (B.P. : H. LINDGREN)



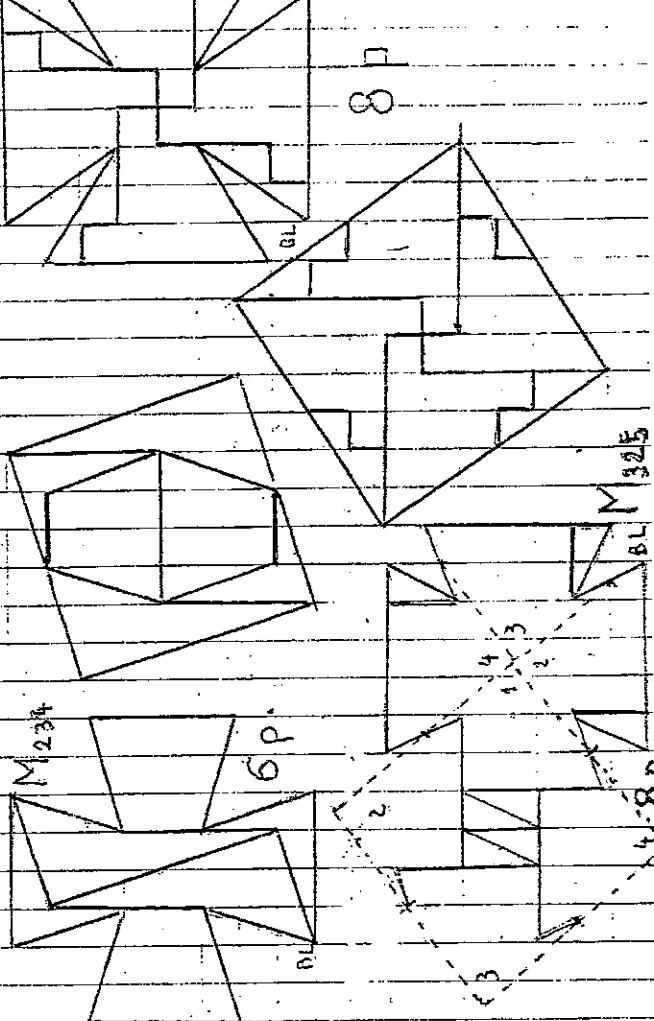
1973 : B.L. → MARTIN GARDNER → G.F.

(6)

SUPERPOSITION  
 DE 2 PAVAGES  
 • celui dérivé  
 de M par déplacement  
 de 4/5  
 • celui de G



3 AUTRES CROIX DE MALTE :

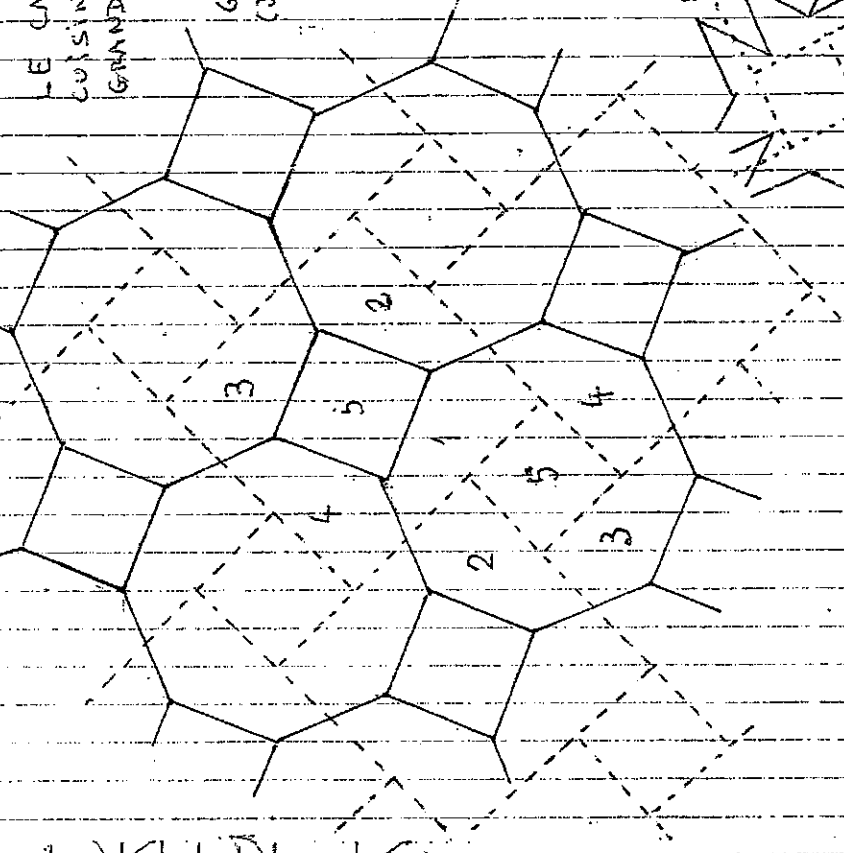


MAVALES A LOUUBLO MUIIT

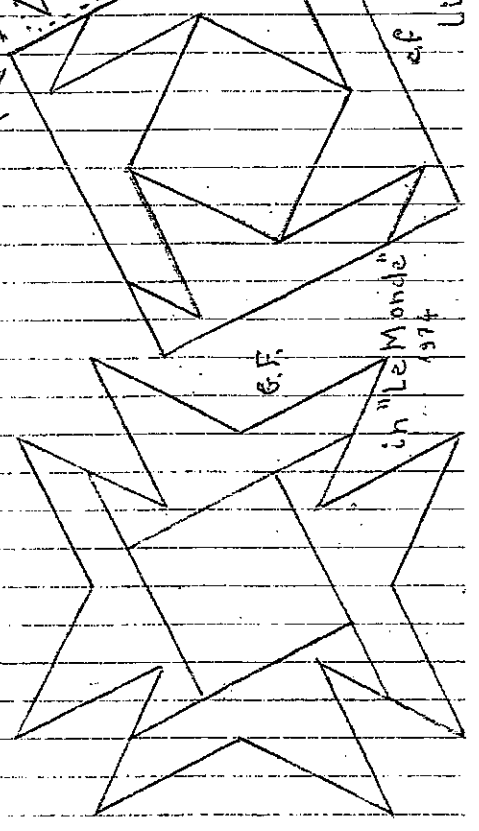
LE CARRELAGÉ DE  
CUISES DE NOS  
GRAND(S) - MÈRES

G. BENETT 192  
(J. Travers 19)

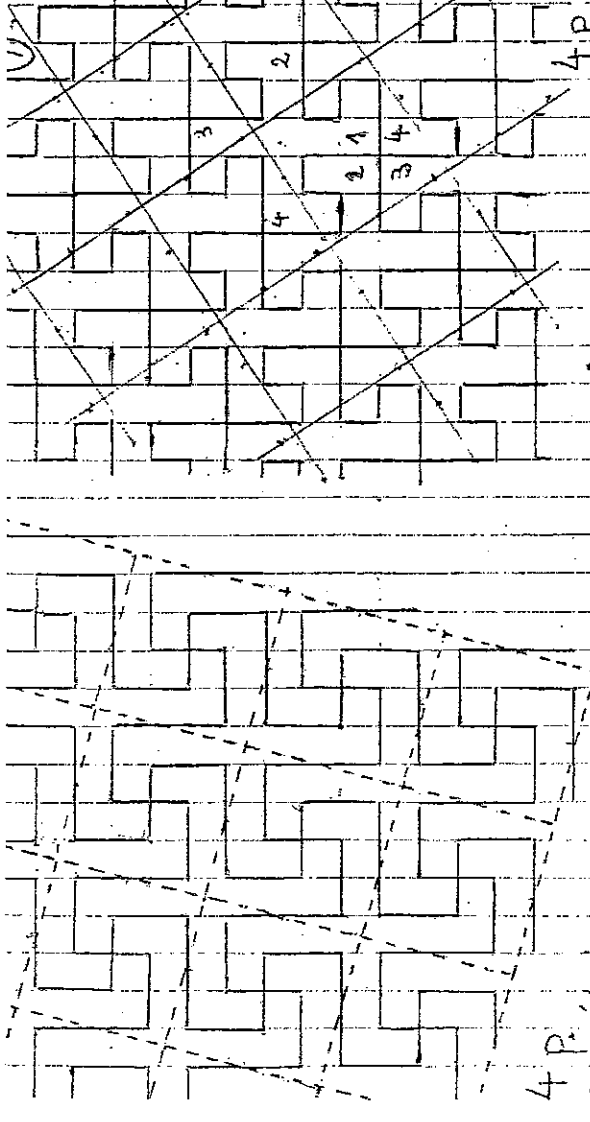
5 p.



DÉCORATION:  
GRAND - CROIX

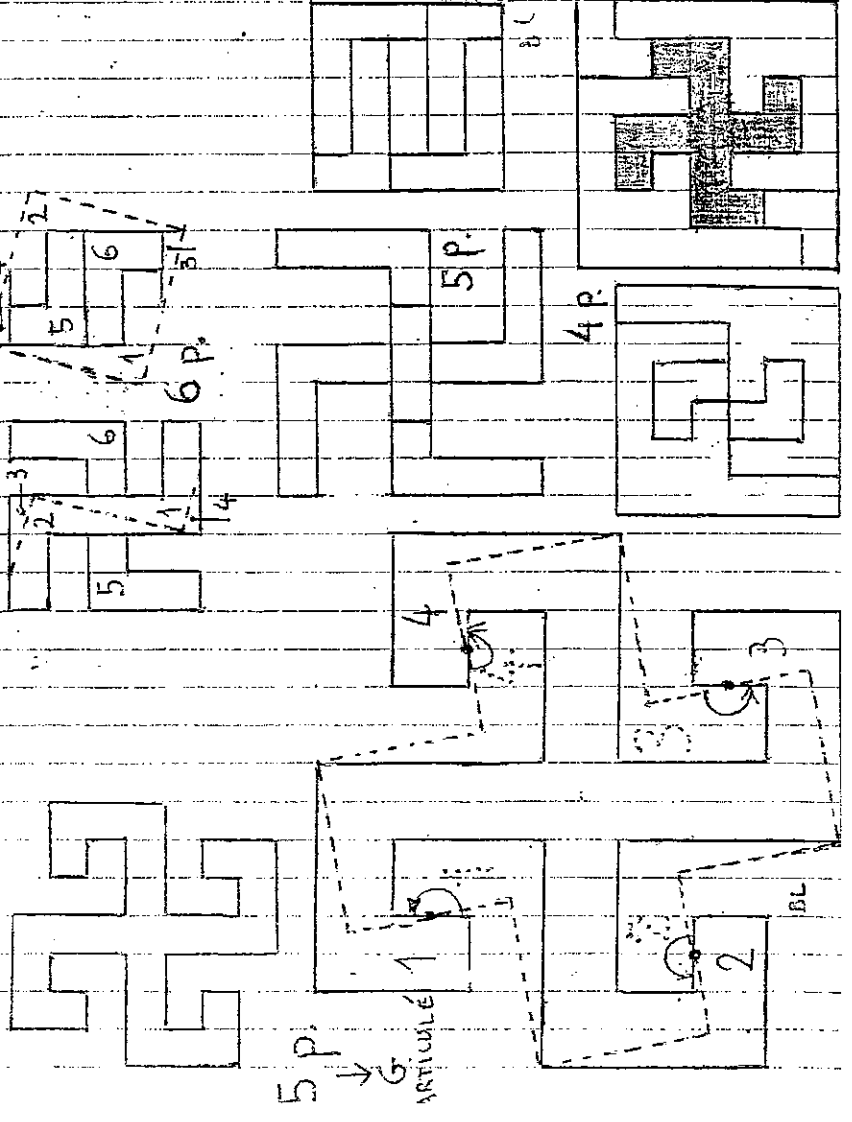


et jaquette de  
livre de G.F.



SWASTIKAS

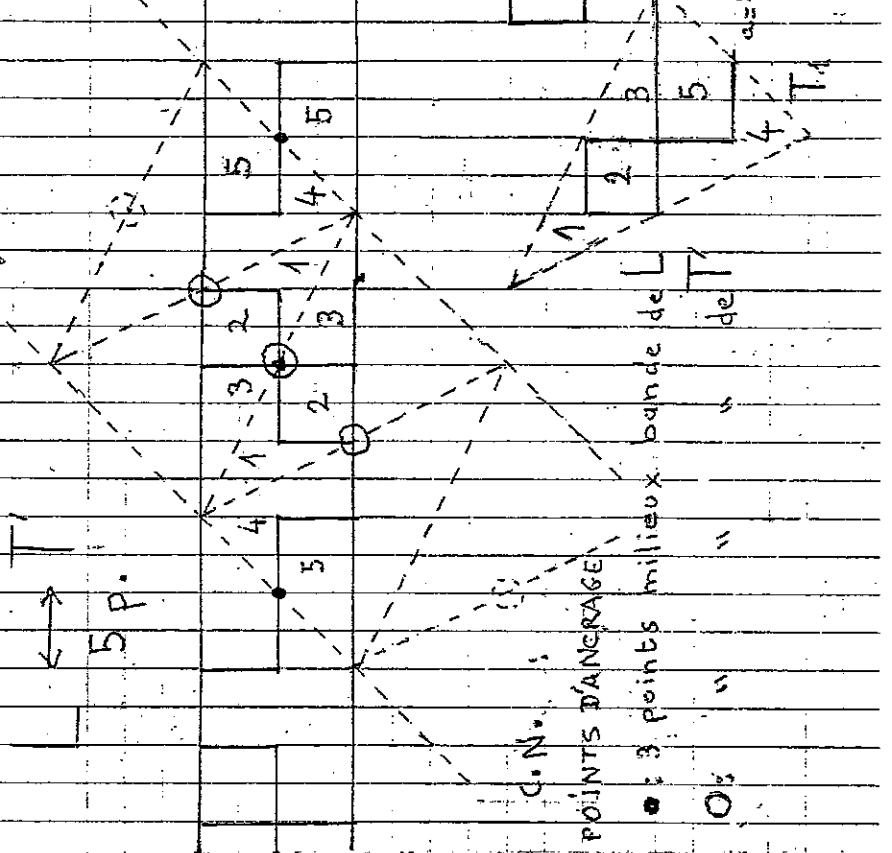
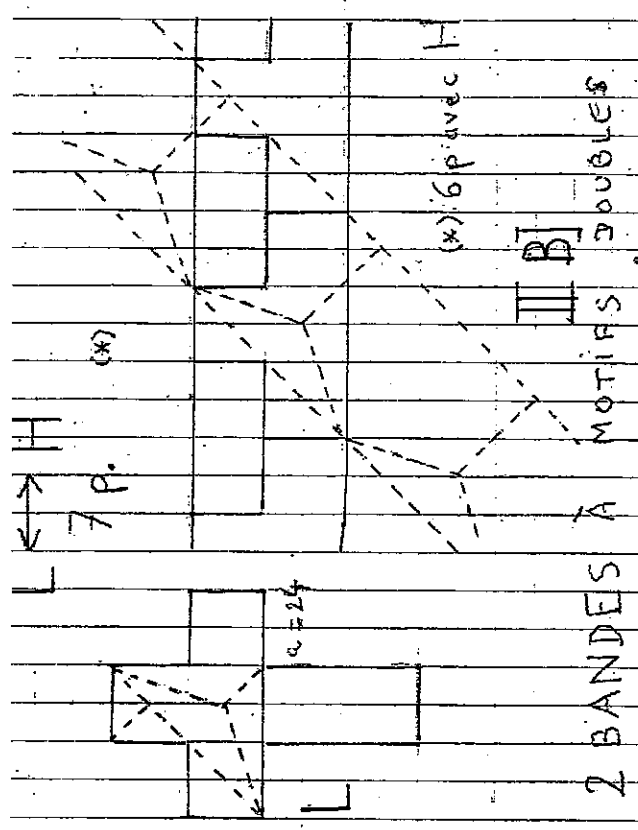
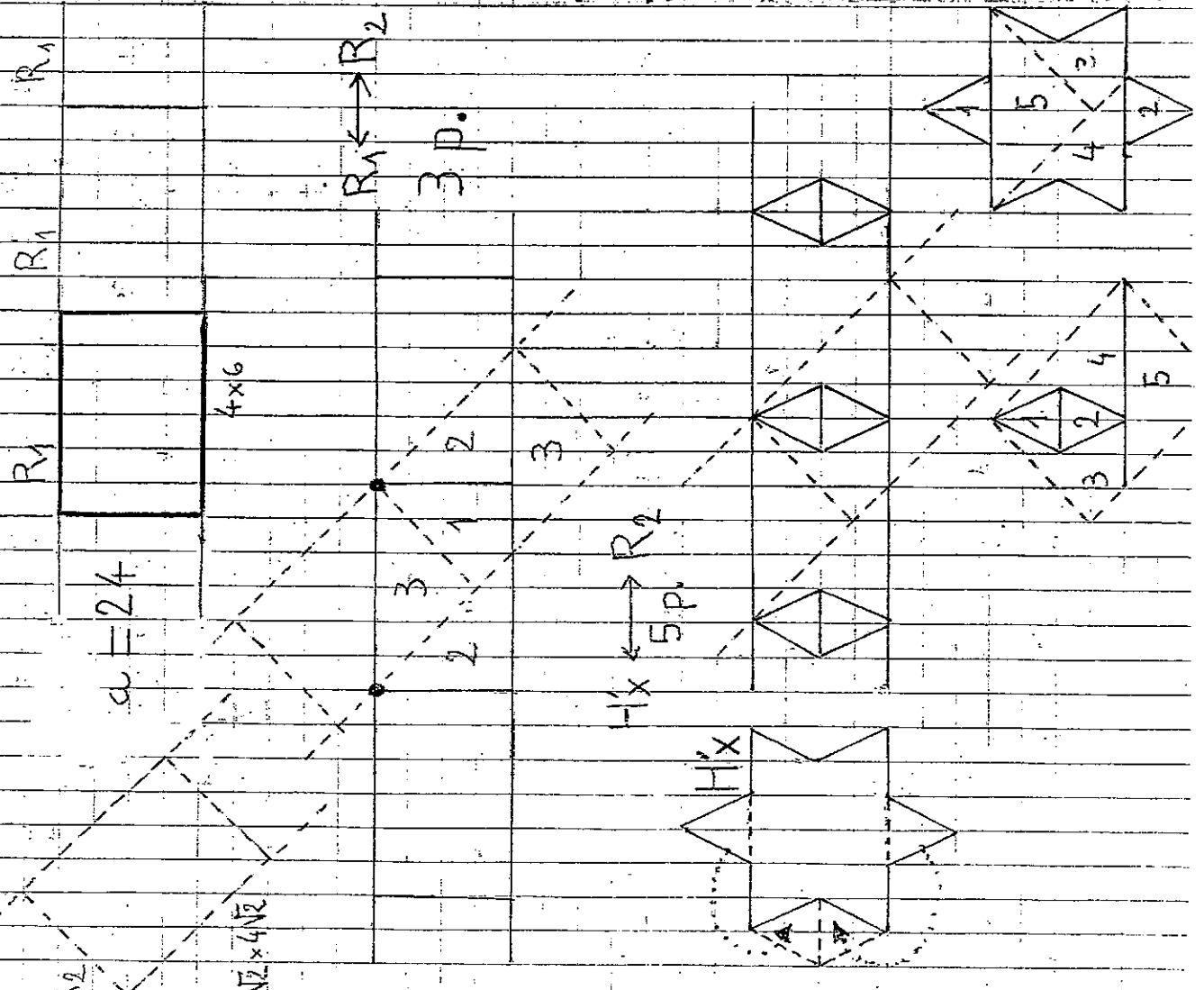
4 p.  
non sym.

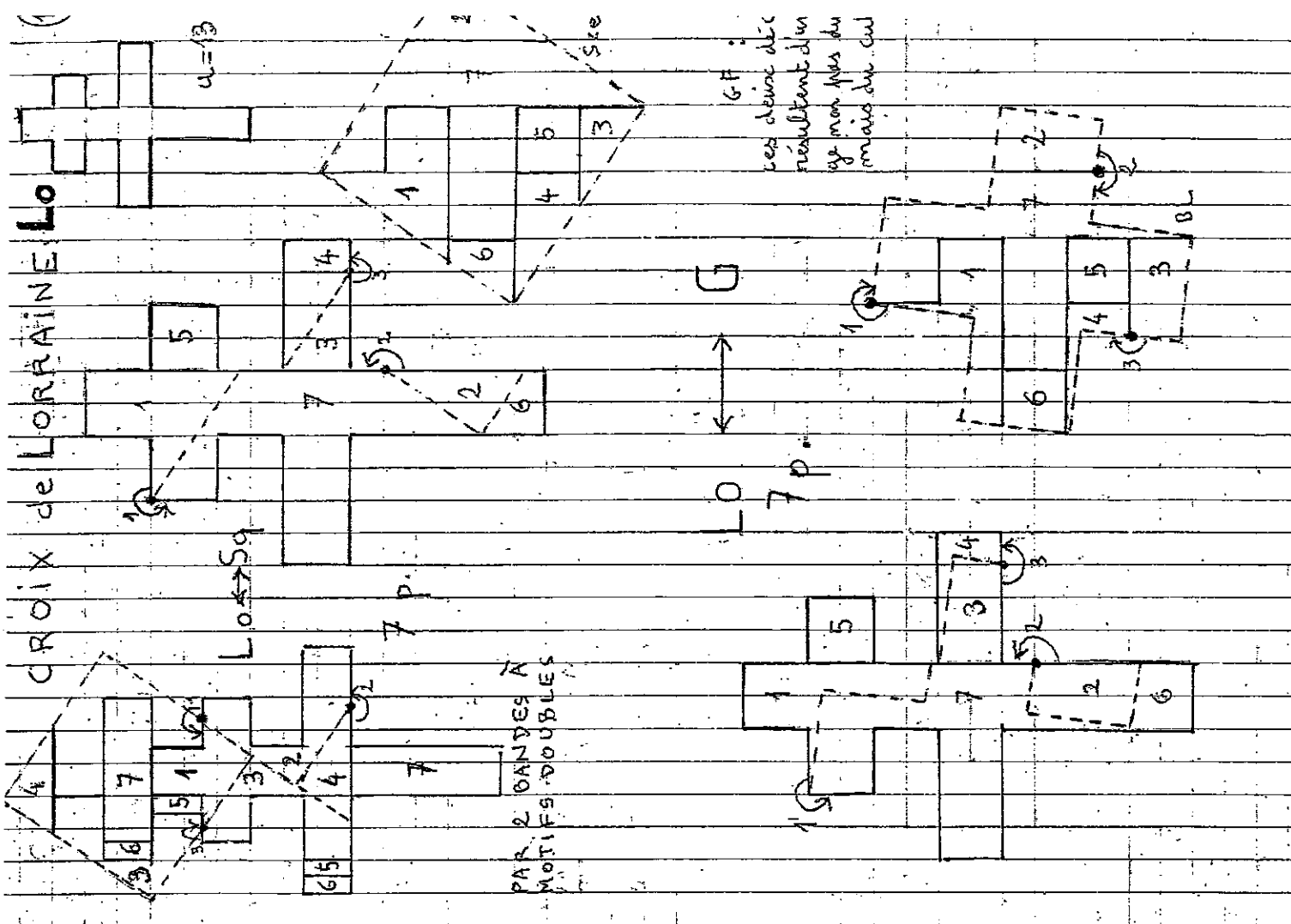
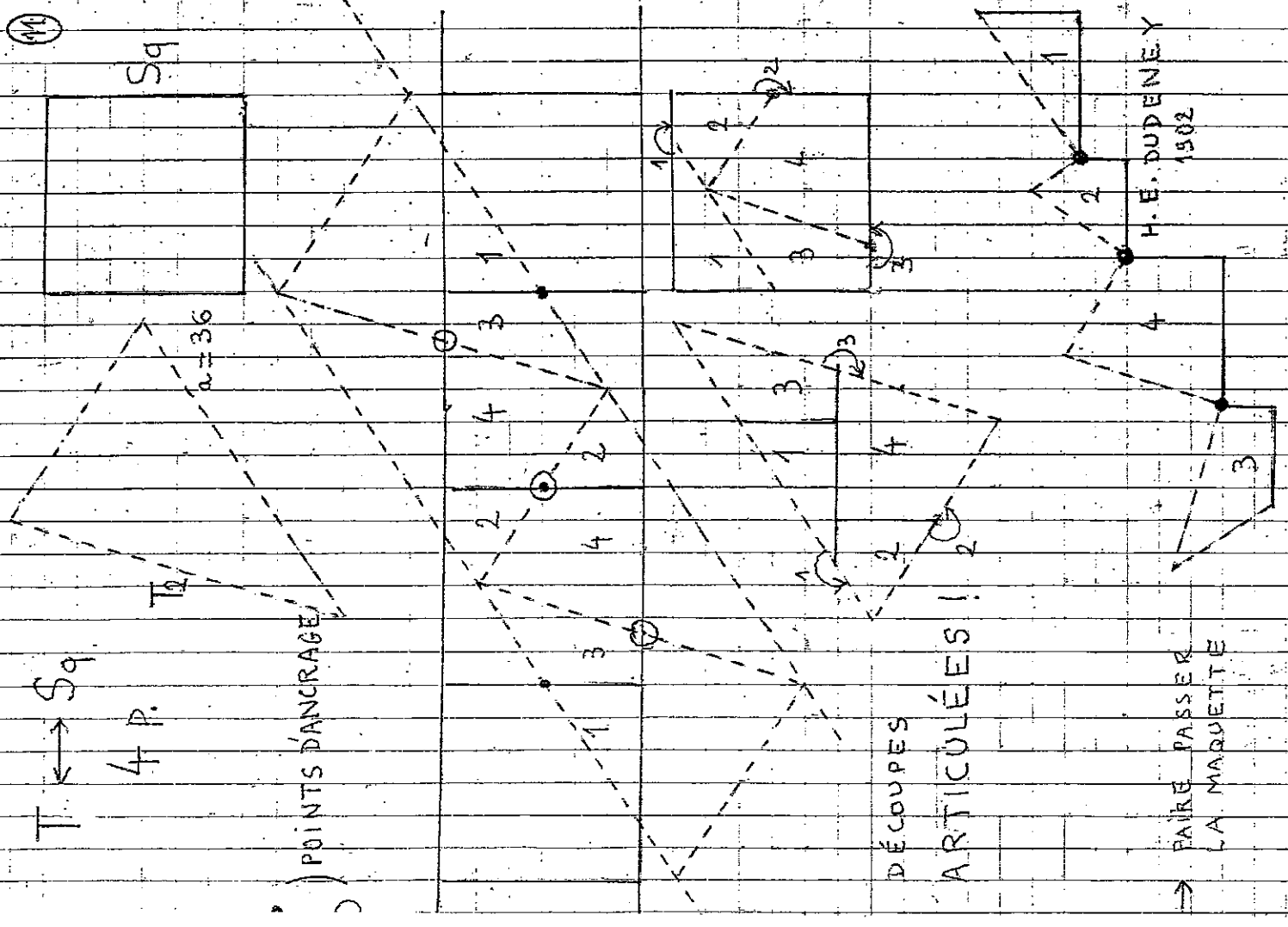


5 p. →  
ARTICULÉ

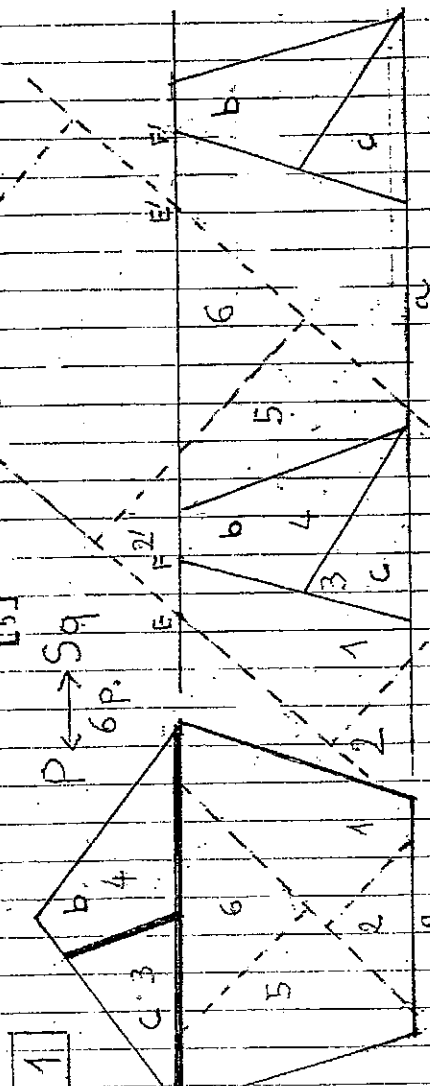
4 p.  
symétrique

# DÉCOUPES "strips" À 2 BANDES



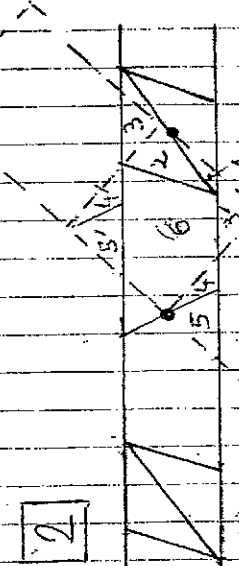


LAB] P  
3 BANDES POUR [51]

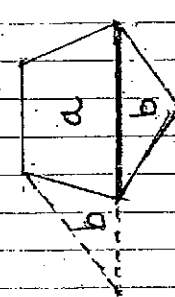


1

$P \leftrightarrow Sg$   
6P.

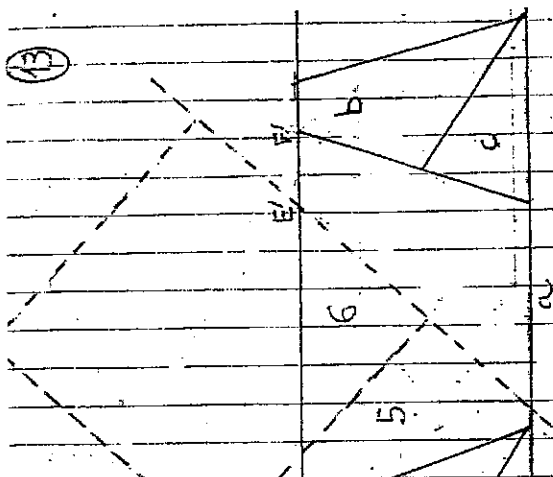


2

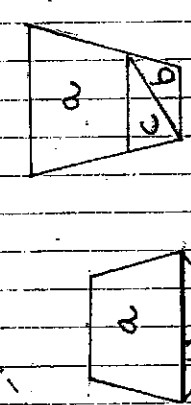
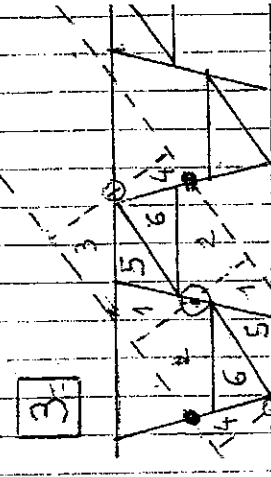


POINTS - MILIEUX

13



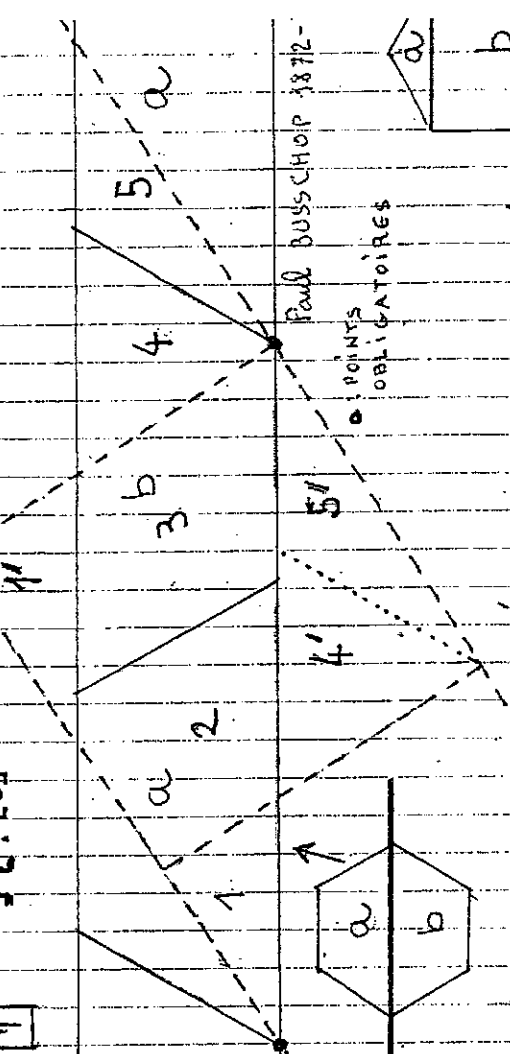
3



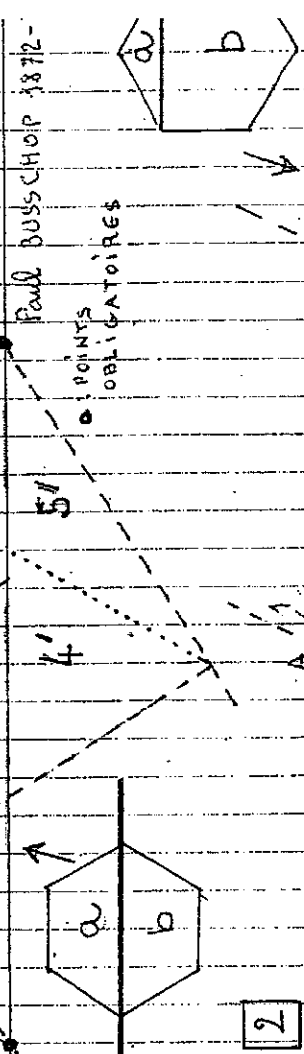
BANDES A MOTIFS DOUBLE

POINTS D'ANCRAGE

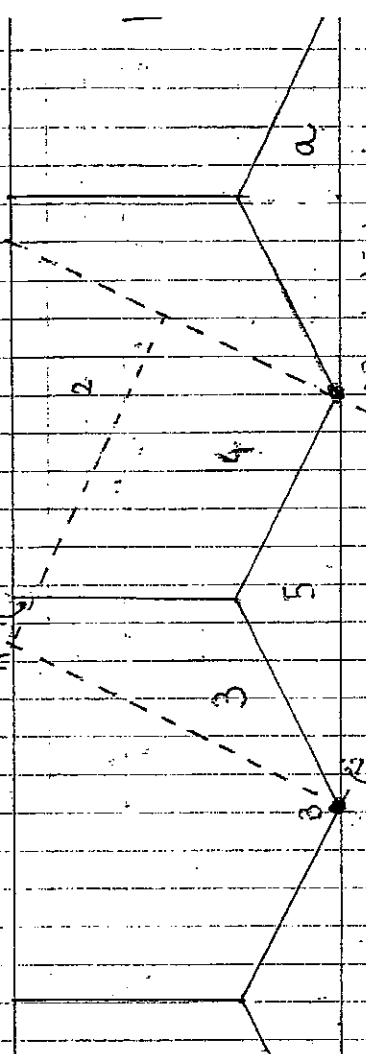
3 BANDES POUR  
H: [6]



1



2



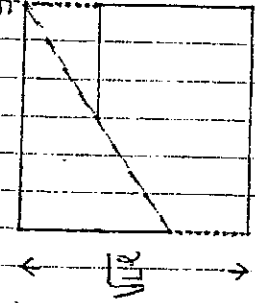
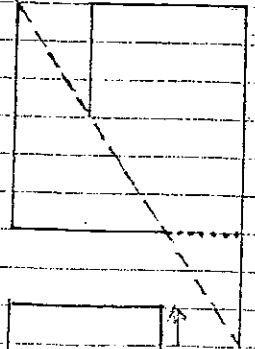
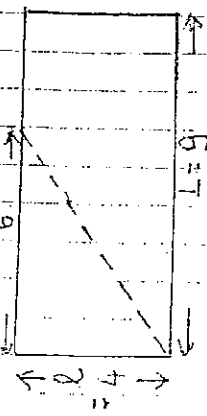
3

Paul BUSSCHOIP 1872-



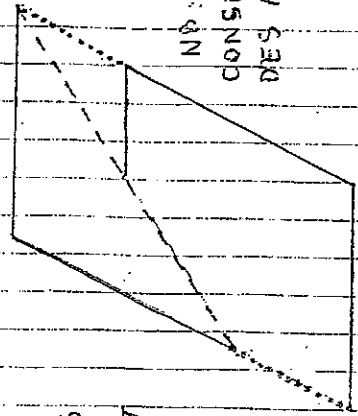
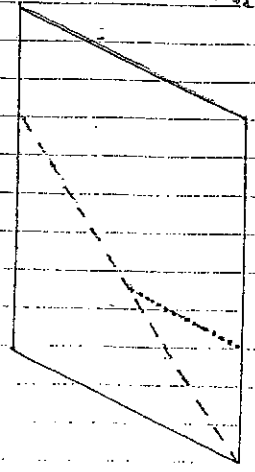
III DÉCOUPE PAR GLISSEMENT (slide)

RECTANGLE (4x6) ↔ CARRÉ (6x6)



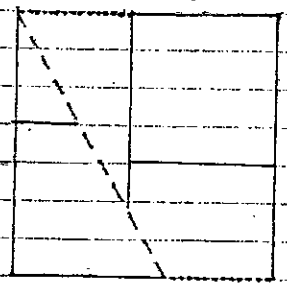
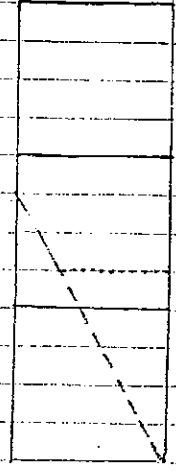
En 3 p.  
si L < 4L  
(si L = 4L → 2P)

PARALLÉLOGRAMMES



NP: CONSERVATION DES ANGLES

3 CARRÉS

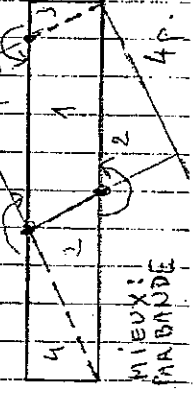
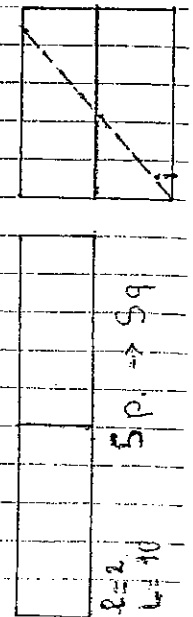


CARRÉ (5q)

ABOUL-WAFA ≈ 1960 → 9q  
PAUL BUSSCHOP 1875 → 7P.

Ex pour 4L > L

MIE PUDENEY 1900

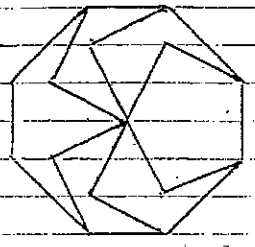
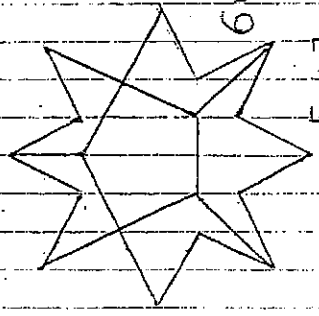


2=2  
L=10  
5P. → 9q

MIEUX: PARBANDE

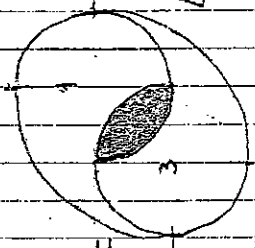
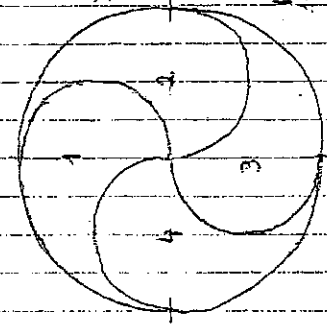
4P.

IV POUR ALLER PLUS LOIN: DANS LES ÉTOILES

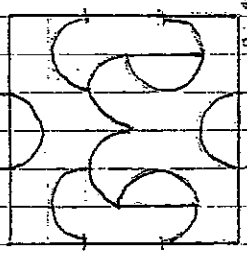
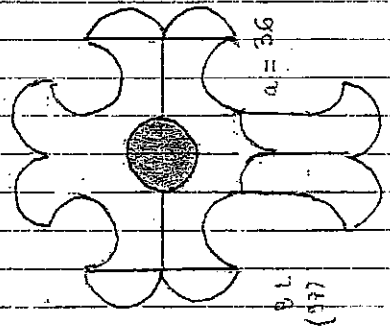


[5/3] [8]

LIGNES COURBES



Sam LOYD 1902

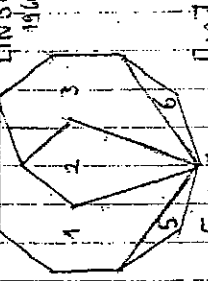
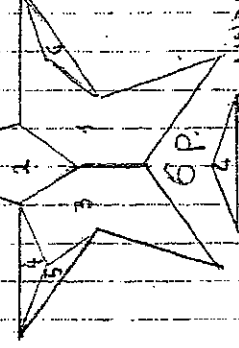


BL (177)

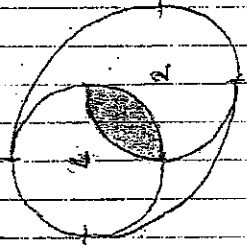
BL (177)

40 DÉCOUPES EN 3D...

IV POUR ALLER PLUS LOIN: DANS LES ÉTOILES

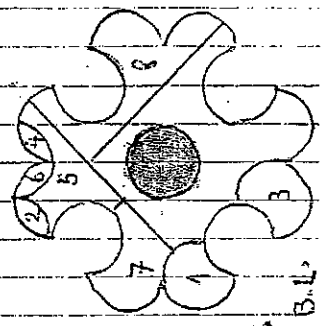


[5/2] [10]



CONTRAINTES

- Longueurs parties CONVE
- " " CONCAV
- Mêmes RAYONS de COURB



B.T., B.L.

## BIBLIOGRAPHIE

Zerschneidung jeder beliebigen Anzahl von gleichen gerad-linien  
Figuren in dieselben Stücke . Journal für die reine und angewandte  
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Recreational Problems in GEOMETRIC DISSECTIONS  
Revised and enlarged by Greg FREDERICKSON, 184 pages  
Dover Publications, New York 1972

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Greg N.FREDERICKSON, 2002, 287 pages  
Cambridge University Press

PIANO – HINGED DISSECTIONS : Time to fold !  
Greg N.FREDERICKSON, 2006  
A.K PETERS Ltd

## WEBOGRAPHIE

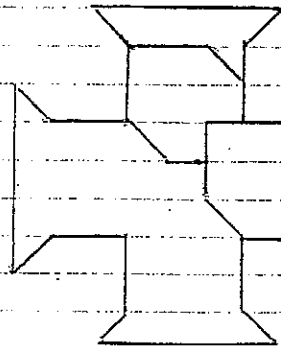
**Site de Greg N. Frederickson**  
Complements et mises à jour de ses livres  
[https : www.cs.purdue.edu/homes/gnf/book/Booknews/](https://www.cs.purdue.edu/homes/gnf/book/Booknews/)

**Site de Gavin THEOBALD : « GEOMETRIC DISSECTIONS »**  
Gavin a crée de nombreuses bandes (strips) complexes et  
amélioré de très nombreuses découpes, principalement de  
polygones réguliers  
[home.btconnect.com/GavinTheobald/index.html](http://home.btconnect.com/GavinTheobald/index.html)

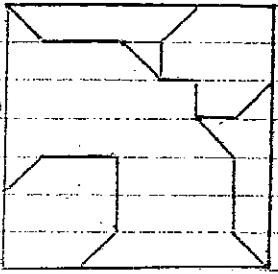
“Hinged dissections exist” : Proceedings of the 24 th Symposium  
on Computational Geometry, 2008, pp. 110-119 ;  
<http://arsav.org/abs/0712.2094> T.ABBOTT, E.DEMAINÉ and  
M.DEMAINÉ (M.I.T); Z.ABEL, S.KOMINERS (Harvard);  
D. CHARLTON (Boston) : ont étendu le théorème de LOWRY  
et al. aux découpes articulées .

# COMPLÉMENTS

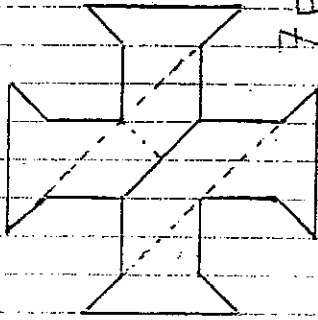
## Croix Portugaises



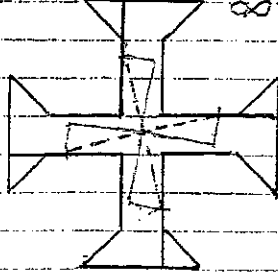
5P. SEULEMENT!



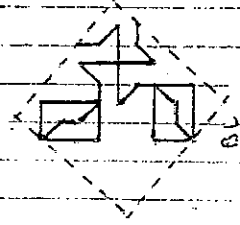
BL LISBONNE 2002L



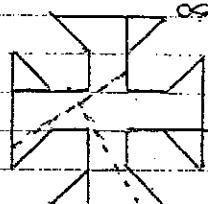
7P.



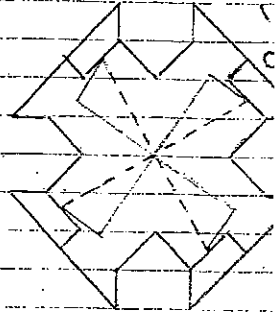
8P. G



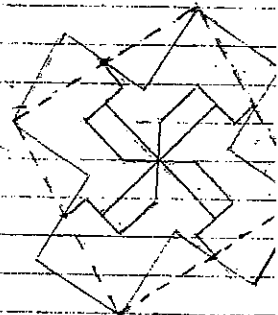
8P. BL



8P. BL

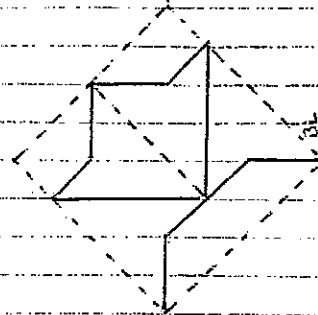


Sq

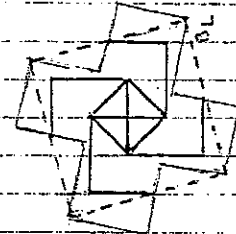


Sq

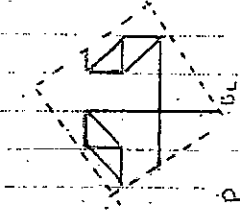
48



BL

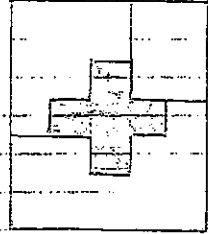


BL

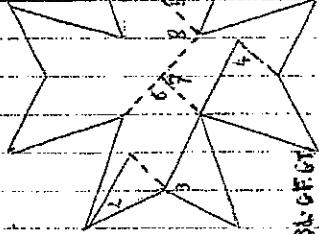


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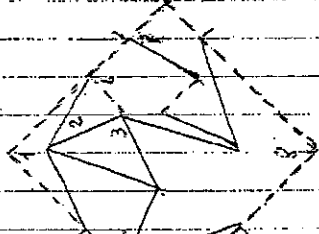
# FLORILEGE DE PERLES!



DRAPÉAU SUISSE ÉVIDÉ 3P.

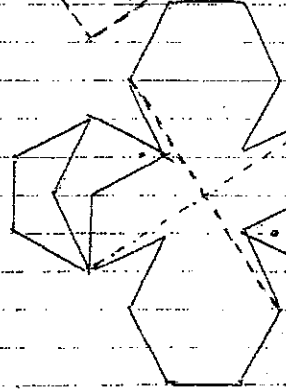


8P. BL

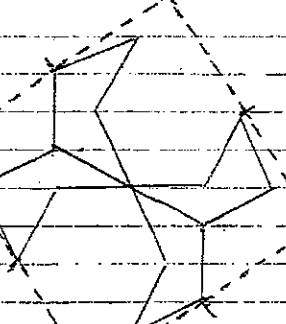


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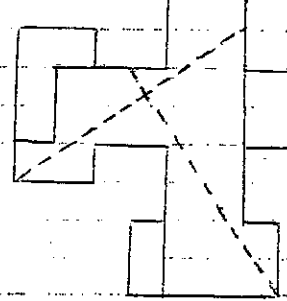
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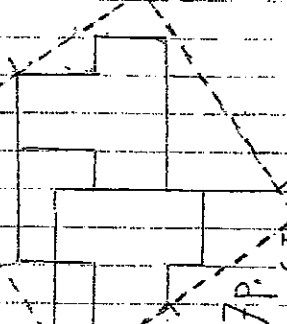
8P. BL



8P. BL

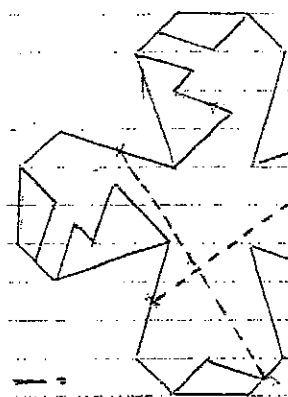


7P. 6TA

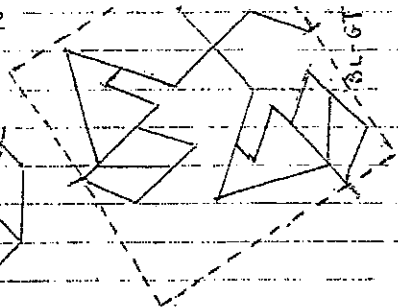


7P. 6TA

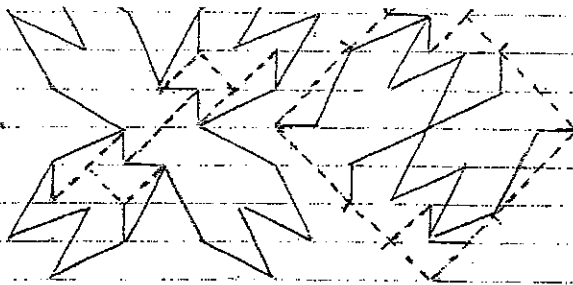
CROIX POTENCÉE



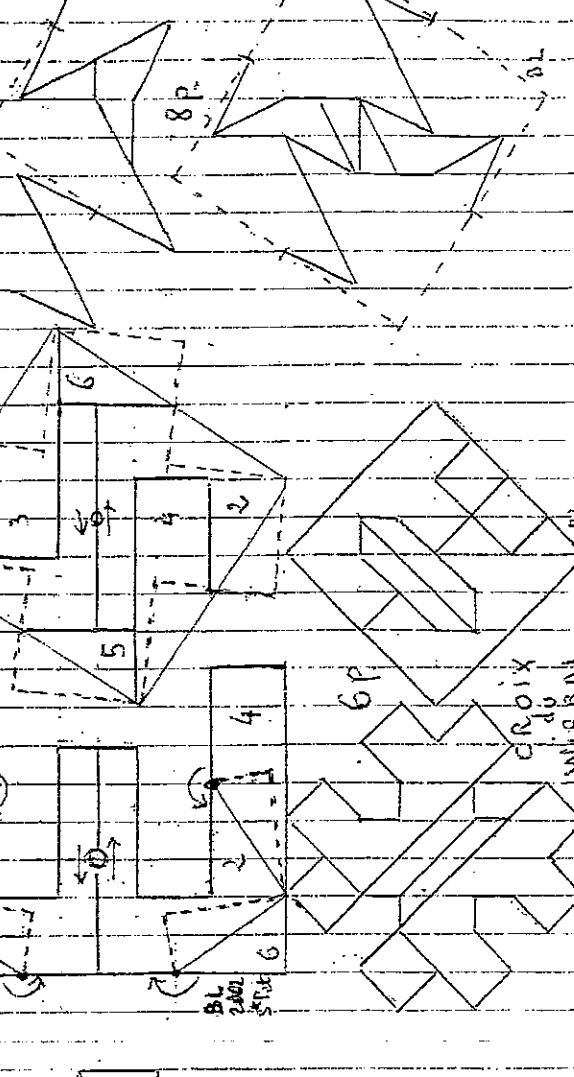
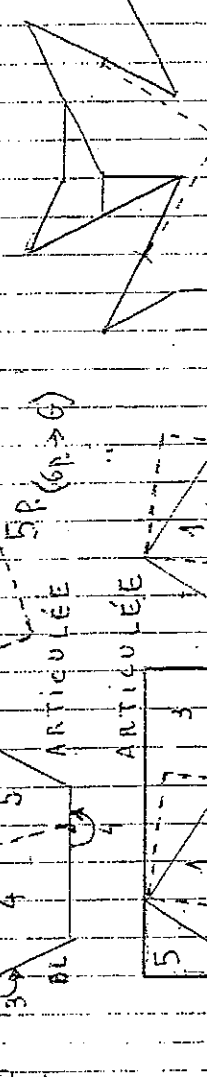
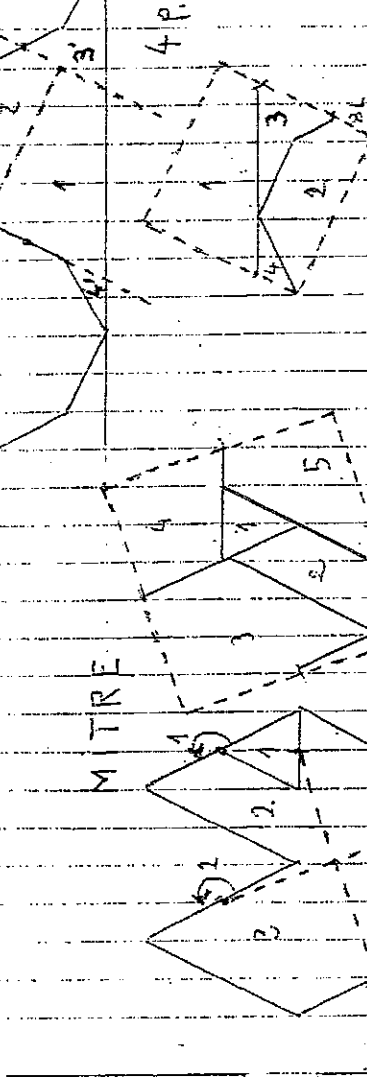
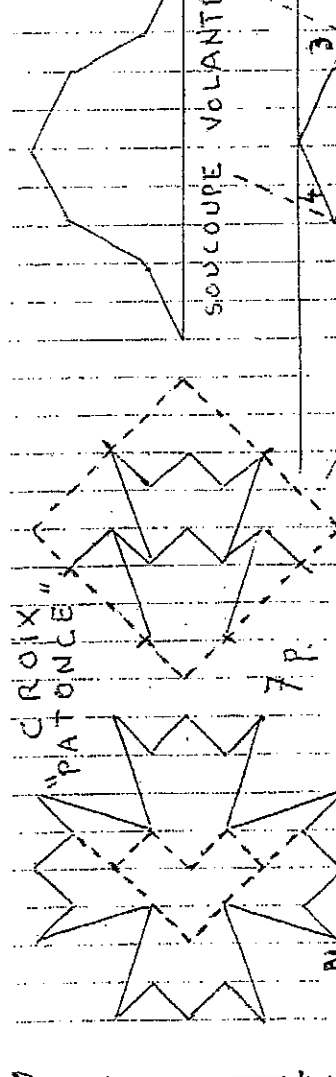
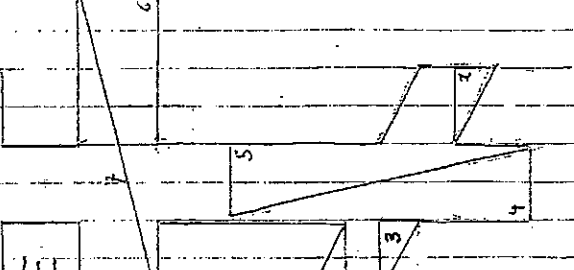
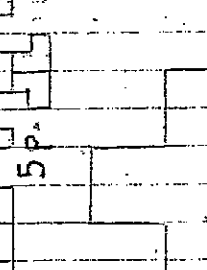
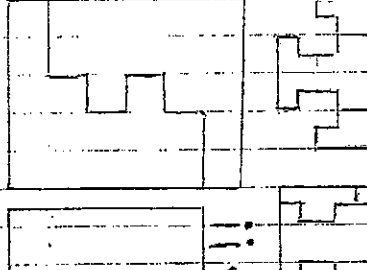
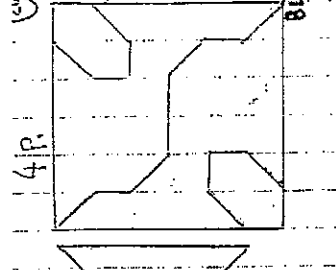
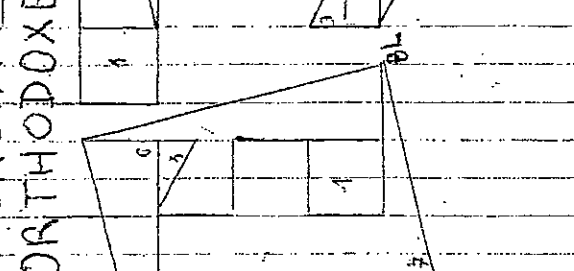
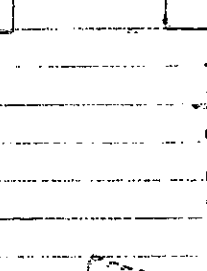
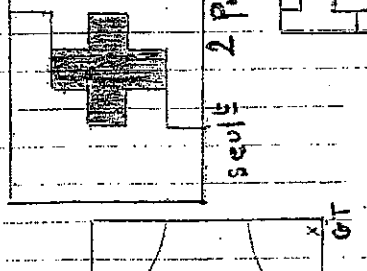
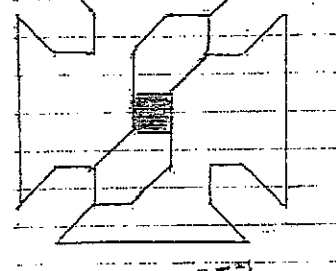
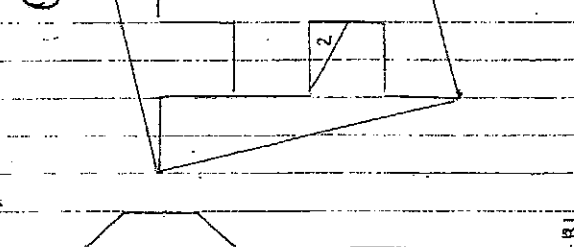
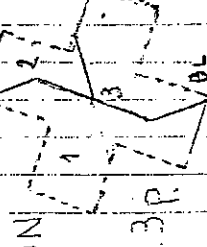
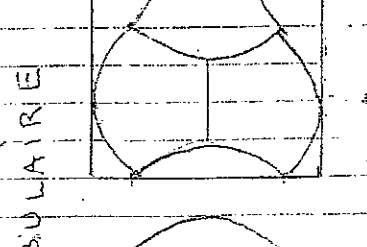
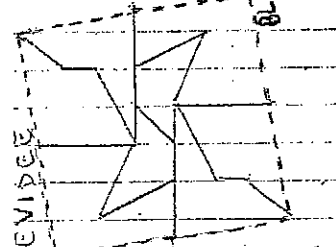
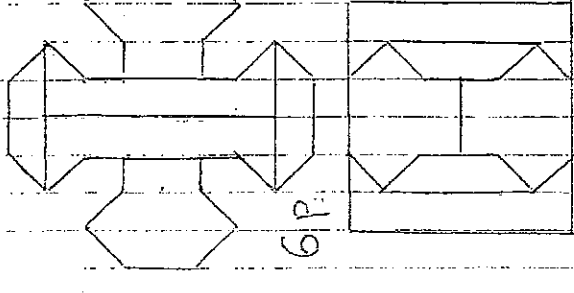
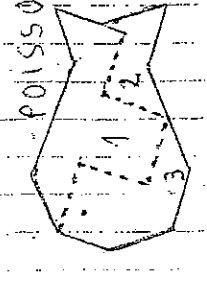
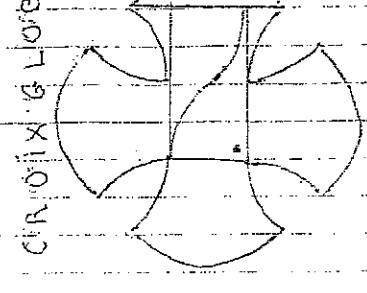
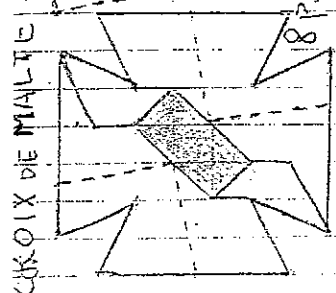
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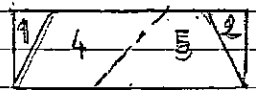
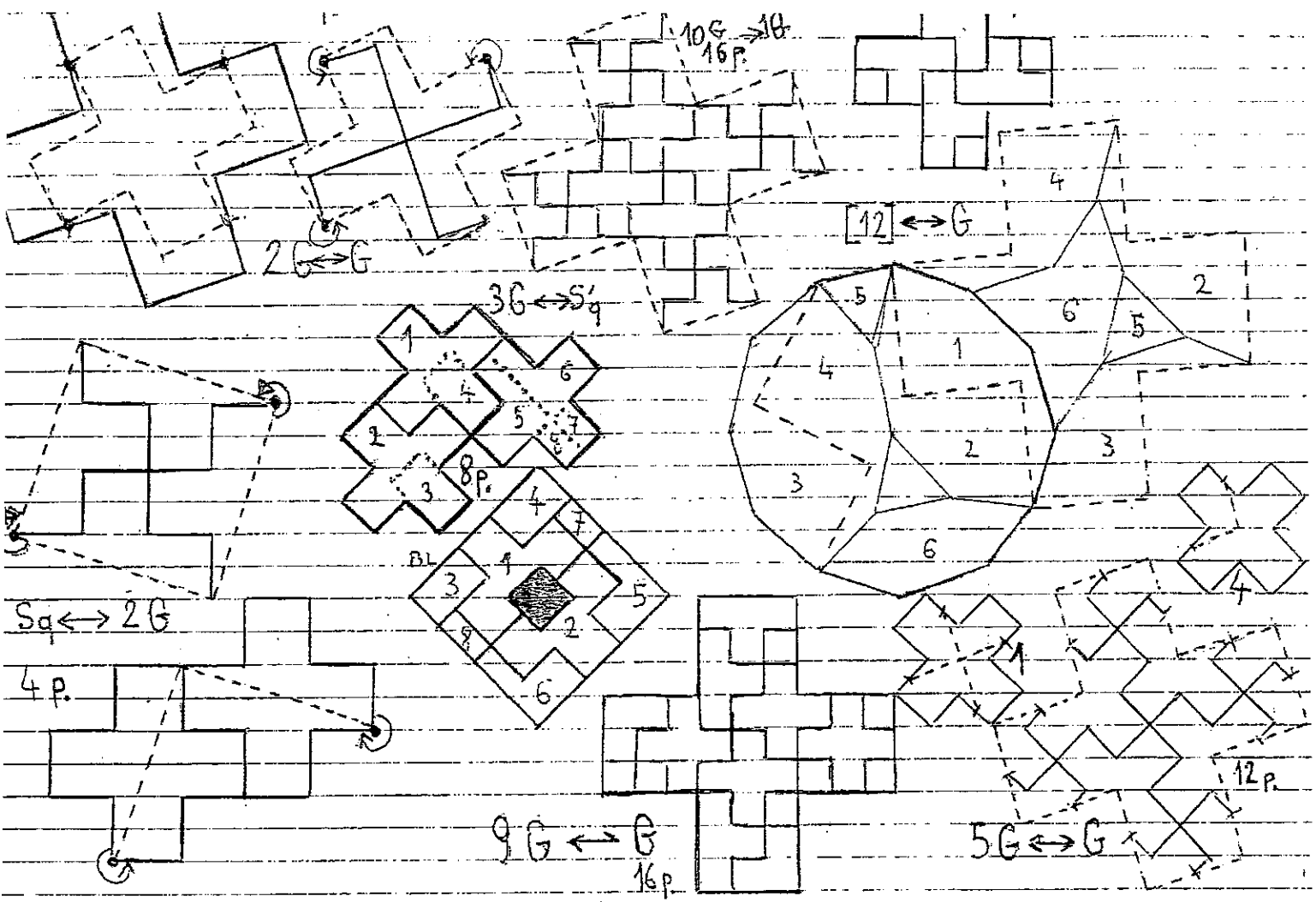


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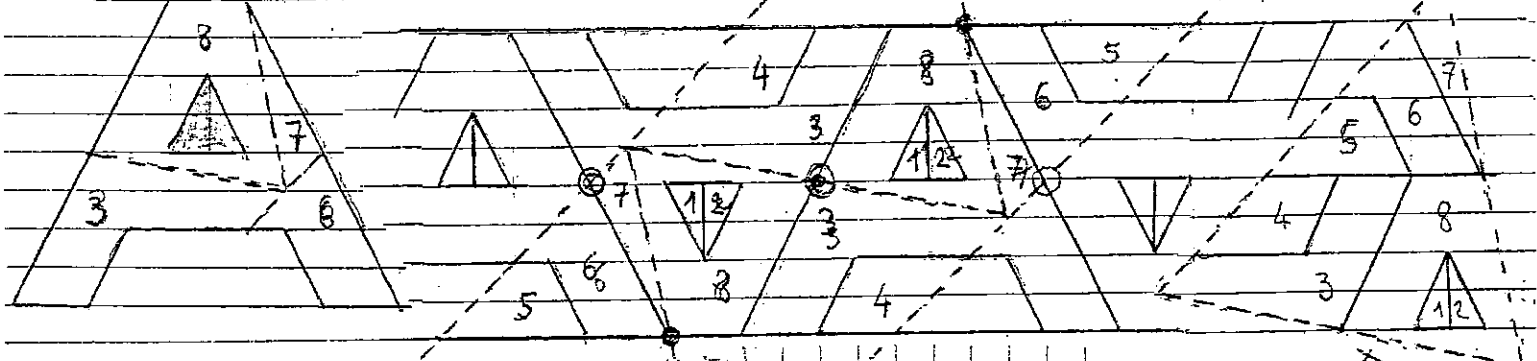
BL





A BASQUE ↔ TRIANGLE : 8P.

○ POINTS D'ANCRAGE



3 CARRÉS ↔ 1 CARRÉ

ABDŪL-WAFĀ  
≈ 960

ÉTOILE À 8 BRANCHES

