www.kafemath.fr

THÉORÈME 1. – *On a l'inégalité*

$$\sum_{\sigma\in G}\frac{|K_\sigma|(|K_\sigma|-1)}{K^2}\log\left(\frac{|K_\sigma|-1}{K\,c\,\Lambda_\sigma}\right)+\frac{K-1}{K^2}\sum_{\sigma\in G}\sum_{k\in K_\sigma}\|\log|\sigma\,a_k\|$$
$$\leq\left(1-\frac{1}{K}\right)\frac{2D}{K}\sum_{k=1}^{K}h(a_k)+\frac{D}{K}\left(1+\frac{|G|}{2D}+\log\frac{K}{2}\right)$$

"Café mathématique"

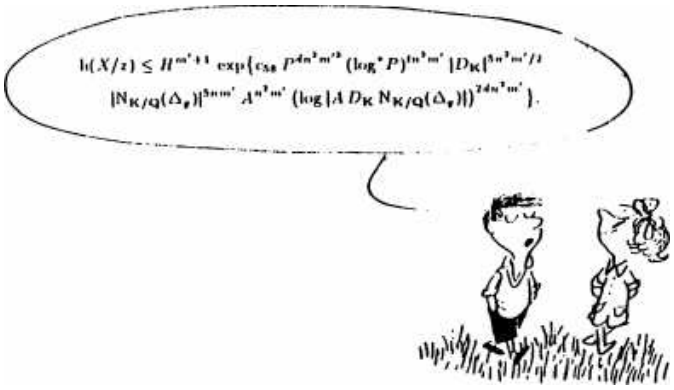
lundi 22 mars 2010 à 20 heures 30

"Polyèdres au coeur des arbres"

par Jean-Louis Loday

Mathématicien, Directeur de recherche au CNRS

à "La Commune Libre d'Aligre"



Séance suivante en avril...

25 janvier 2010.

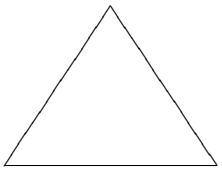
Café associatif "la Commune", 3 rue d'Aligre, 75012 Paris, métro Ledru Rollin.

Extraits du "Petit Nicolas en thèse" ; dessins de J.J. Sempé, formules de Y. Bugeaud, M. Mignotte, F. Normandin, texte de G. Tavio.

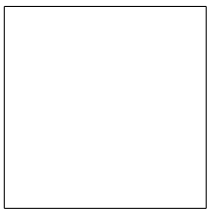
Polygones, polyédres, polytopes

En dimension 2 on a les polygones:

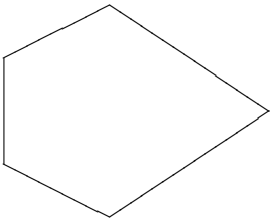
Triangle:



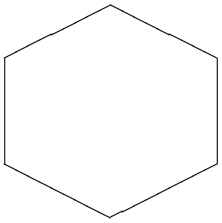
Carré:



Pentagone:

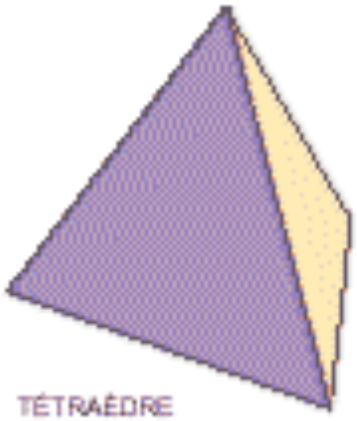


Hexagone:

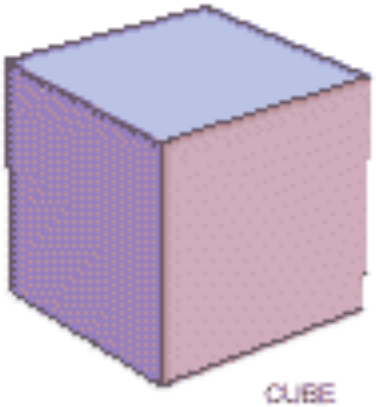


En dimension 3 on a des polyèdres. Les plus connus sont les polyèdres platoniciens:

Les Solides de Platon

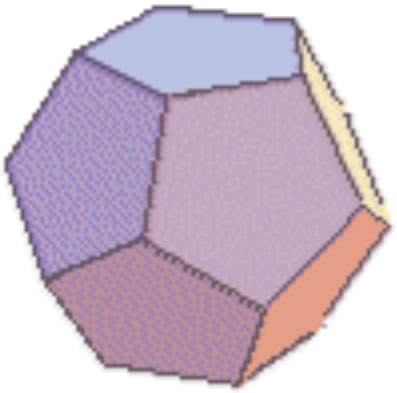


TÉTRAÈDRE



CUBE

OCTAÈDRE

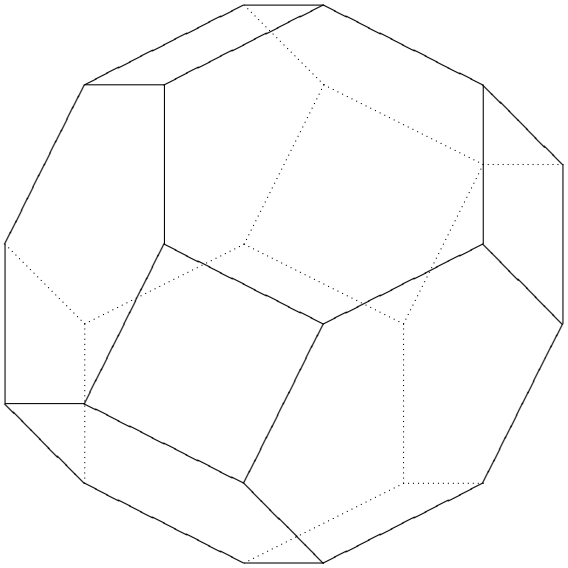


DODÉCAÈDRE

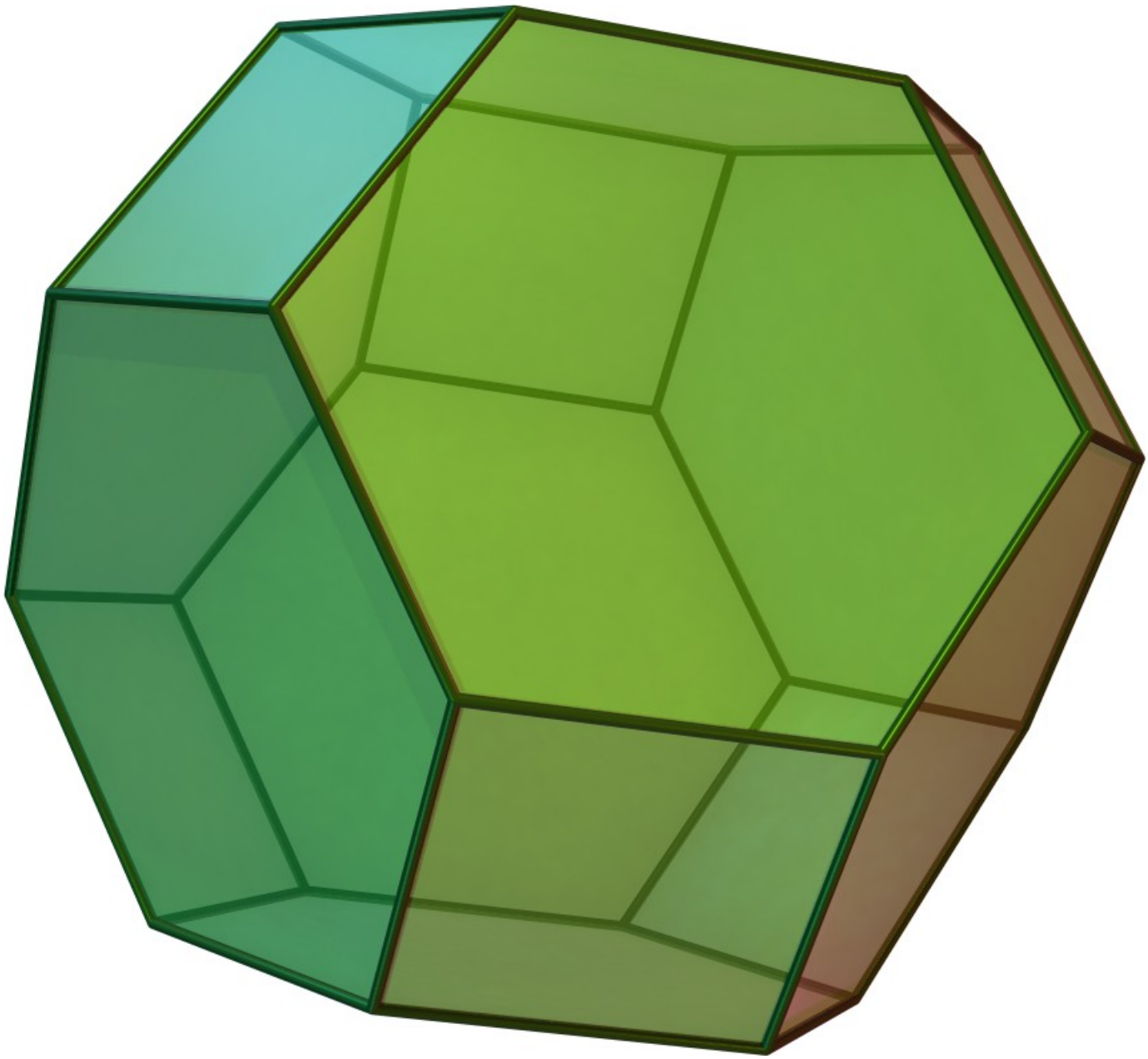
ICOSAÈDRE

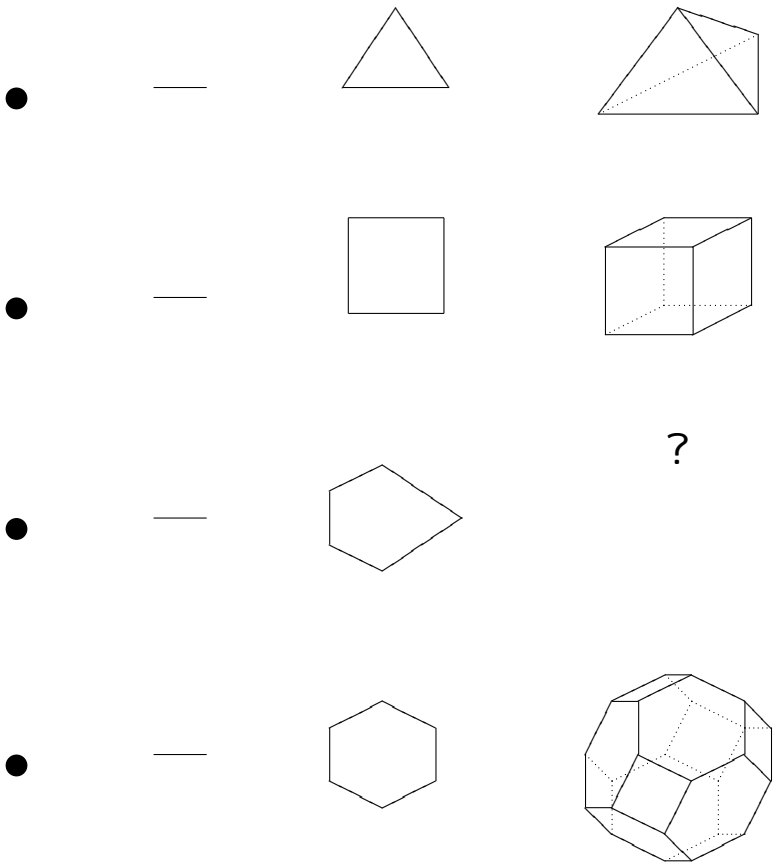
Pas de polyèdre avec que des hexagones (pavage hexagonal),

mais il existe un joli polyèdre avec hexagones et carrés:



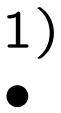
le permutoèdre



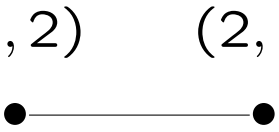


Coordonnées du permutoèdre:

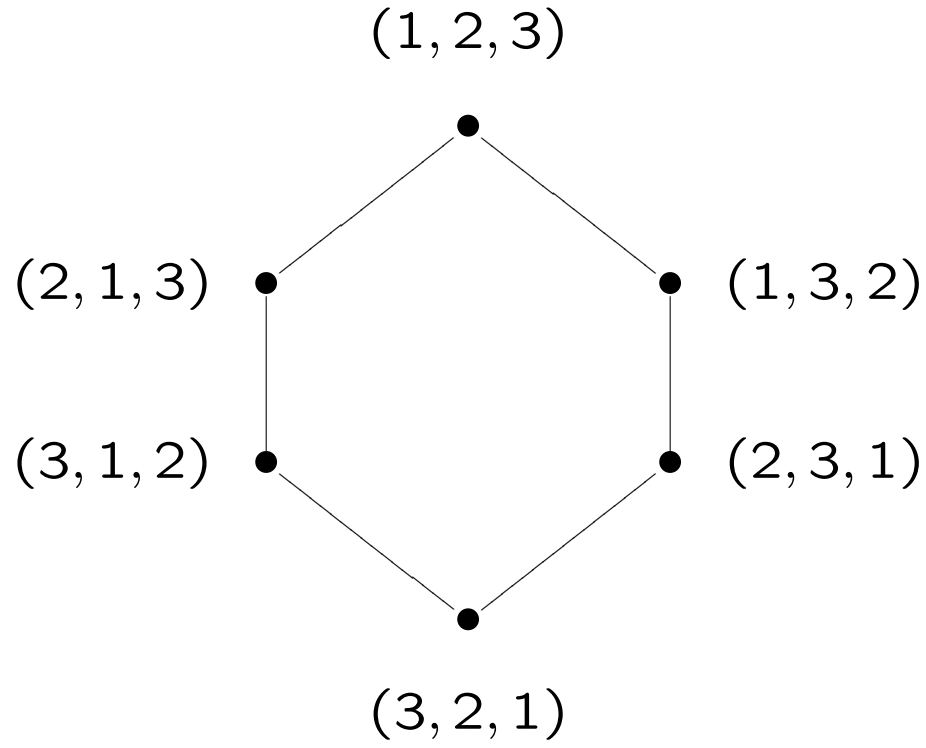
$\mathcal{P}^0 \subset \mathbb{R}^1$

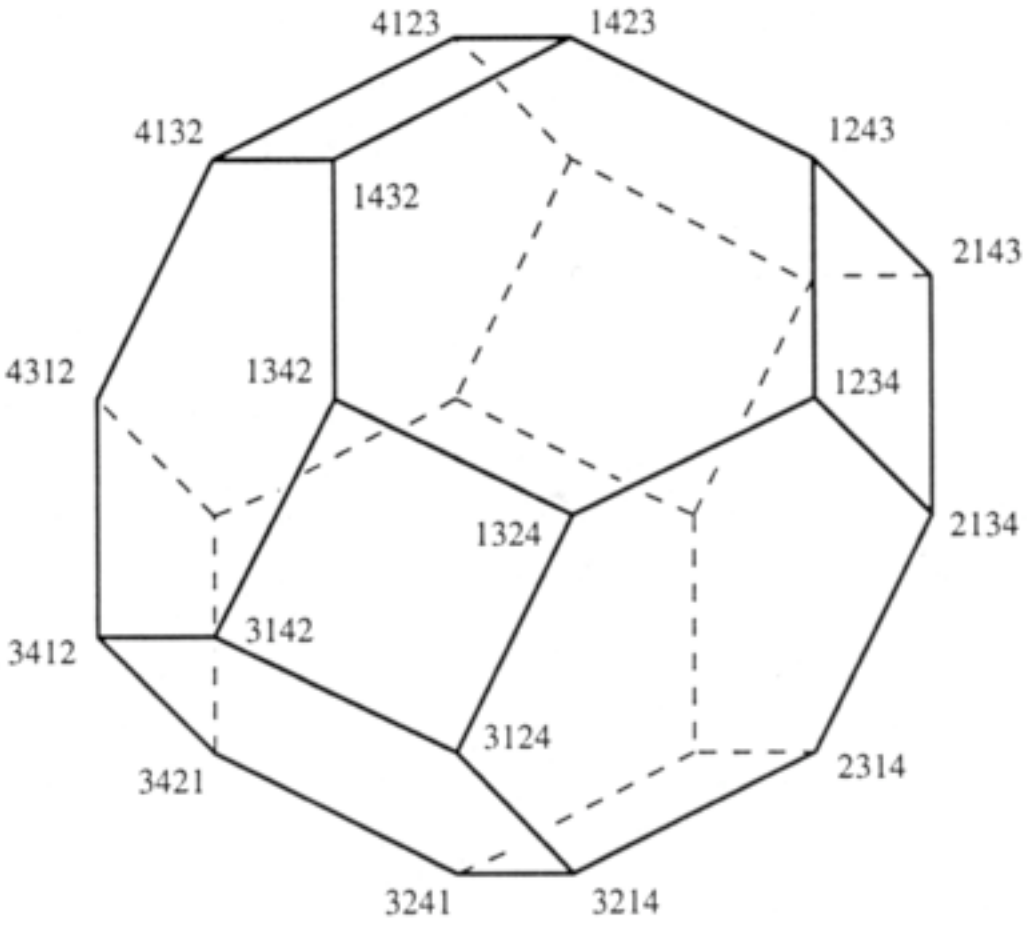


$\mathcal{P}^1 \subset \mathbb{R}^2$



$\mathcal{P}^2 \subset \mathbb{R}^3$




4123
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4132
1243
1432
2143
4312
1342
1234
1324
2134
3142
3412
3124
2314
3421
3241
3214

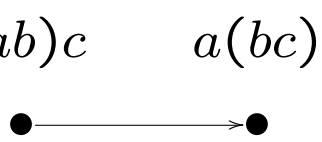
Parenthésages

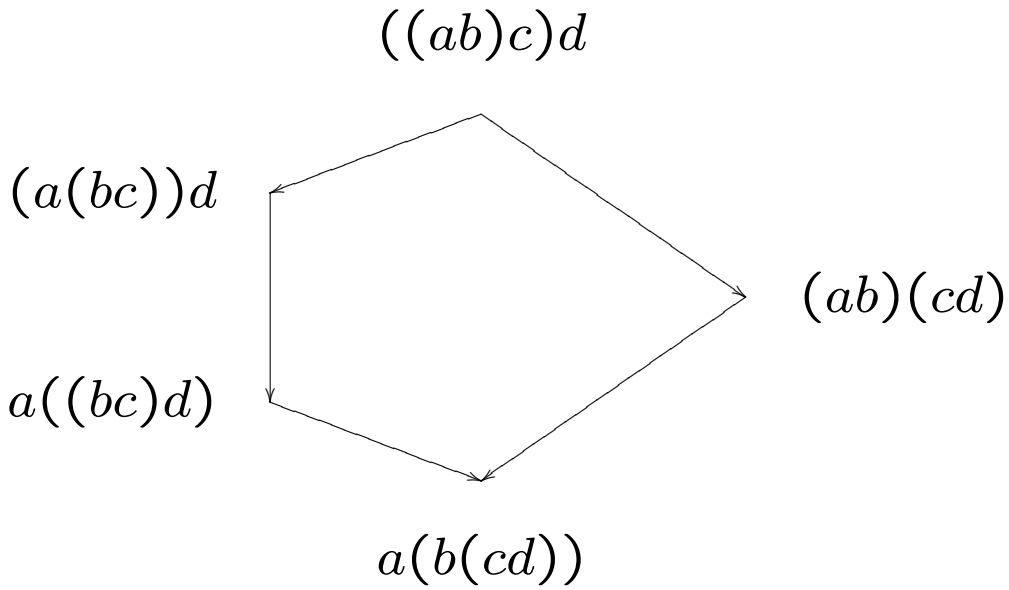
$$ab$$

$$(ab)c \quad , \quad a(bc)$$

$$((ab)c)d, (a(bc))d, (ab)(cd), a((bc)d), a(b(cd))$$

et polytopes





Théorème (Jim Stasheff 1966) Il existe un polytope de dimension n dont les sommets correspondent aux parenthésages d'un produit de $n+2$ variables.

Problème : Trouver les coordonnées à partir des arbres (comme dans le permutoèdre).

Solution (JLL 2004) :

1. Remplacer parenthésages par arbres

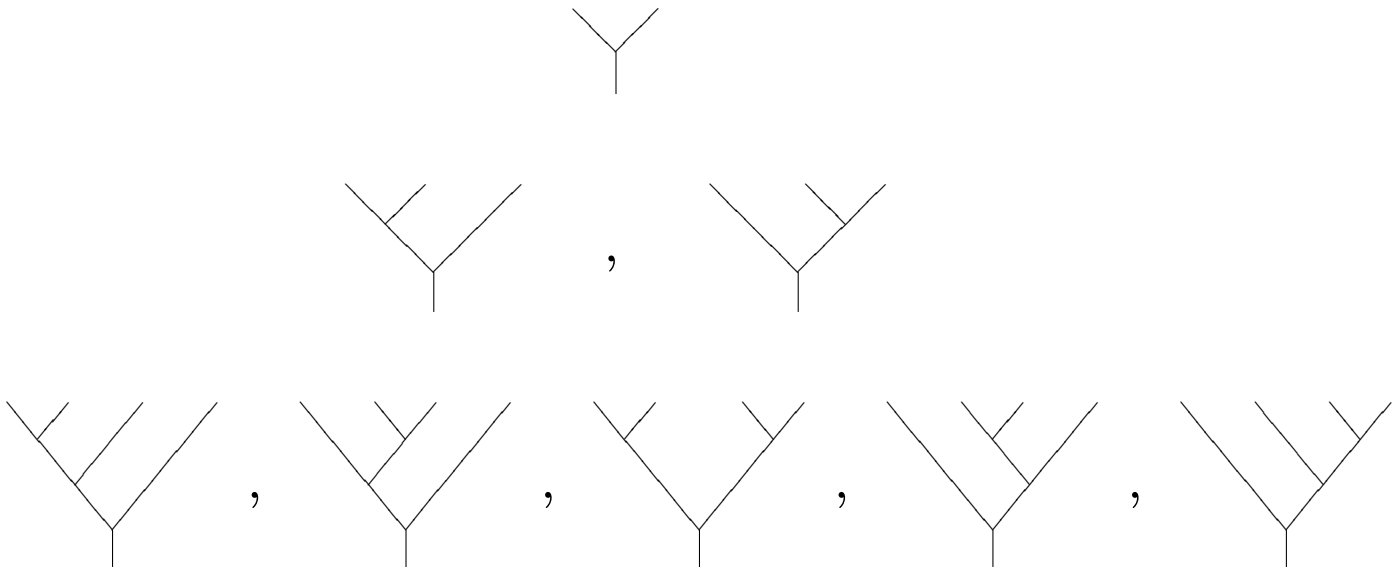
Parenthésages

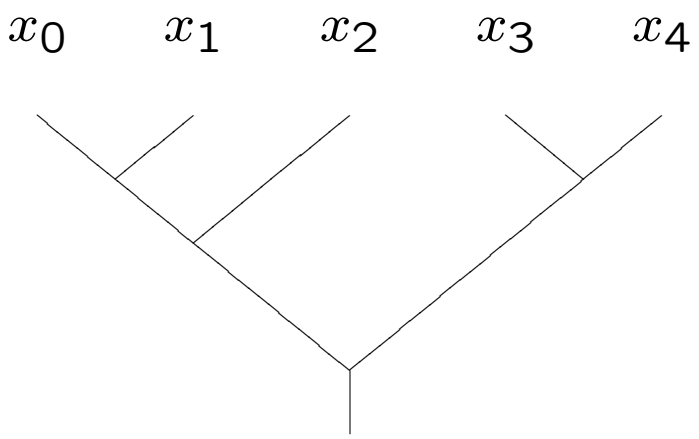
$$ab$$

$$(ab)c \quad , \quad a(bc)$$

$$((ab)c)d, (a(bc))d, (ab)(cd), a((bc)d), a(b(cd))$$

et arbres





$((x_0x_1)x_2)(x_3x_4)$

Numérotation des sommets:

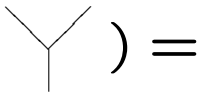
si on laisse tomber une bille entre les feuilles x_{i-1} et x_i on aboutit au sommet numéro i

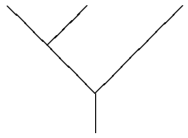
2. A chaque arbre t à $n+2$ feuilles on associe un point $M(t)$ à $n+1$ coordonnées

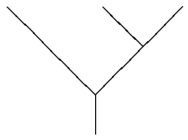
$$M(t) := (a_1 b_1, \ldots, a_i b_i, \ldots, a_n b_n) \in \mathbb{R}^n \ .$$

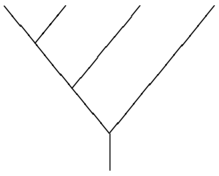
a_i := nombre de feuilles sur la gauche du sommet numéro i

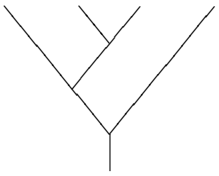
b_i := nombre de feuilles sur la droite du sommet numéro i

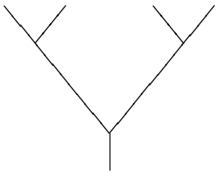
$M($  $) = (1 \times 1) = (1)$

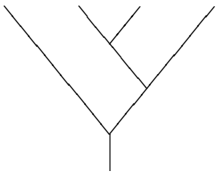
$M\Big($  $\Big) = (1 \times 1, 2 \times 1) = (1, 2)$

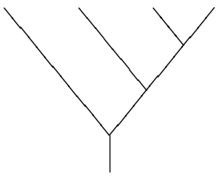
$M\Big($  $\Big) = (1 \times 2, 1 \times 1) = (2, 1)$

$M($  $) = (1, 2, 3)$

$M($  $) = (2, 1, 3)$

$M($  $) = (1, 4, 1)$

$M($  $) = (3, 1, 2)$

$M($  $) = (3, 2, 1)$

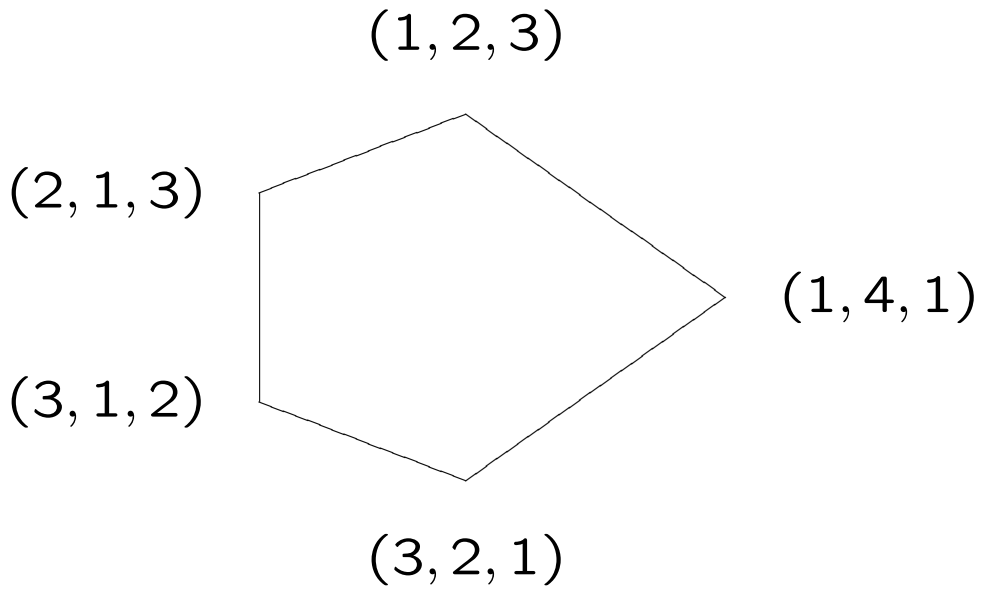
Pour l'arbre t correspondant à $((x_0x_1)x_2)(x_3x_4)$

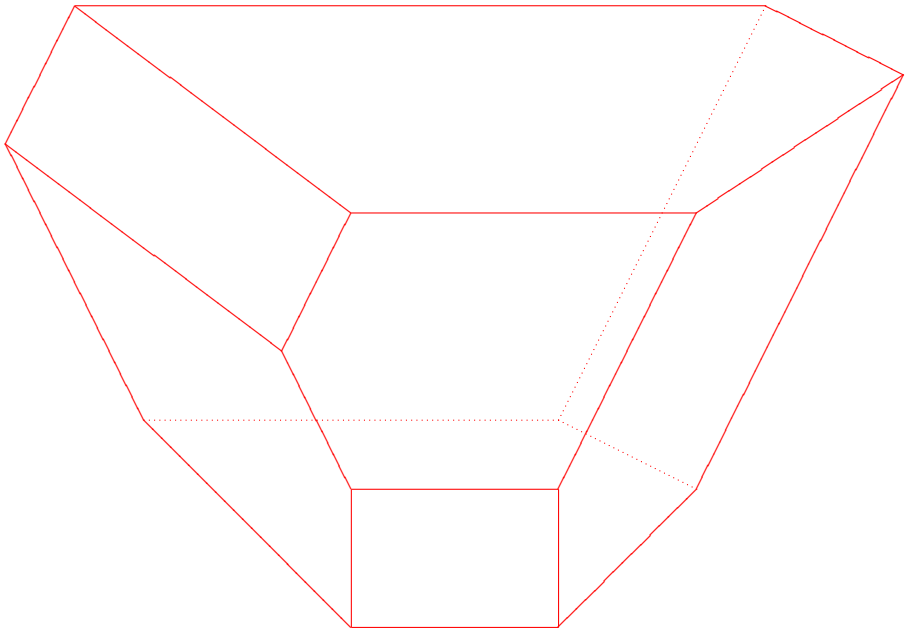
$M(t) = (1 \times 1, 2 \times 1, 3 \times 2, 1 \times 1) = (1, 2, 6, 1).$

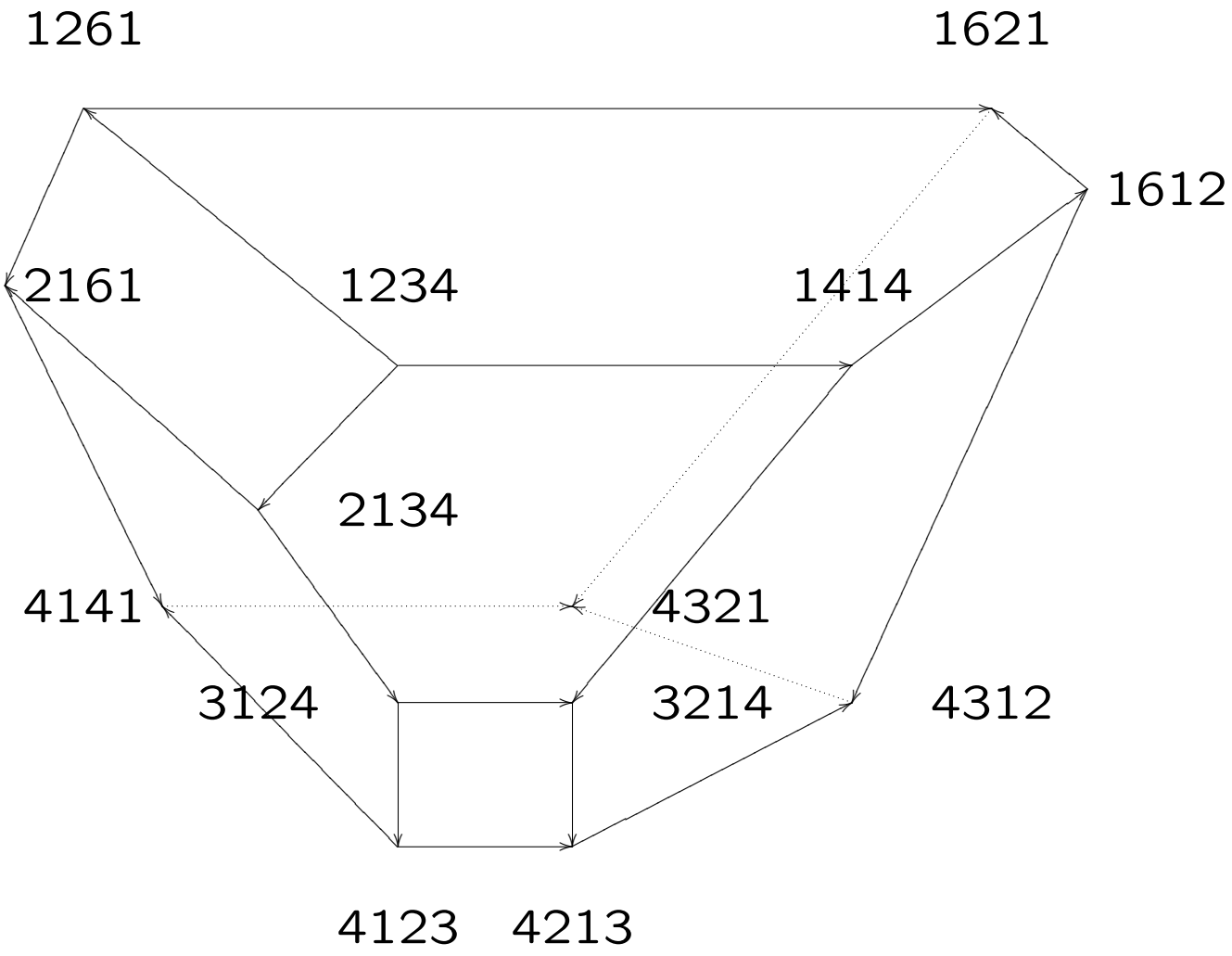
Lemme $a_1b_1 + \cdots + a_nb_n = \frac{n(n+1)}{2}$

Donc tous les points $M(t)$ sont dans un même hyperplan

Théorème L'enveloppe convexe des points $M(t)$ est le polytope de Stasheff (associaèdre)



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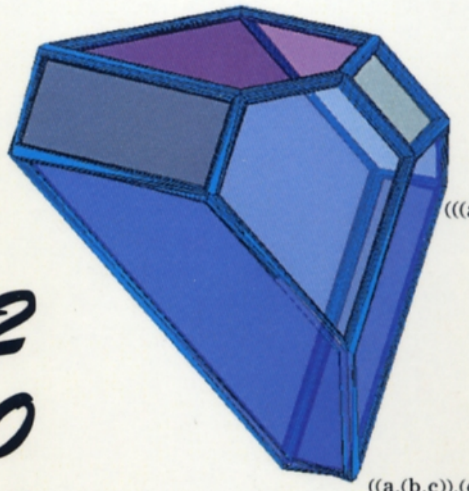
C.I.R.M.

Centre International de Rencontres Mathématiques



(((a.b).c).(d.e))

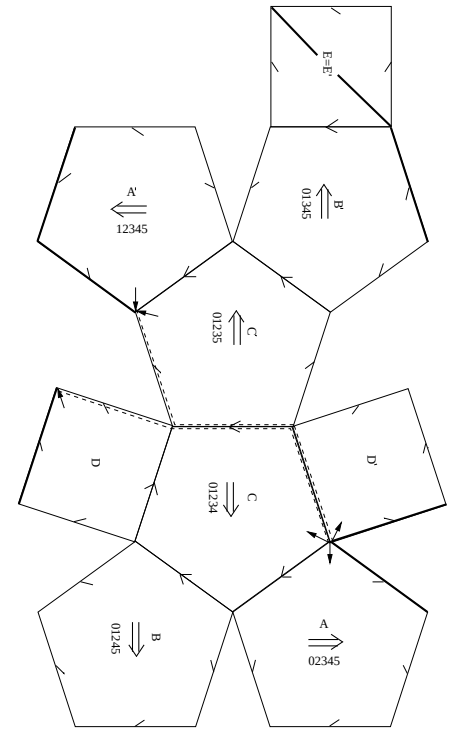
((a.(b.c)).(d.e))

2005

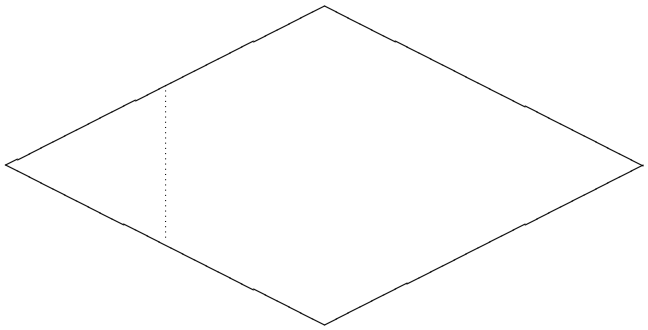
Associahèdre K3

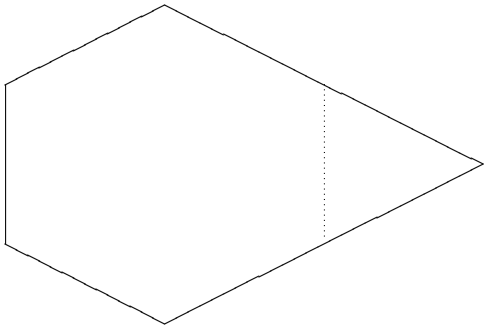
EXERCICE de trigonométrie

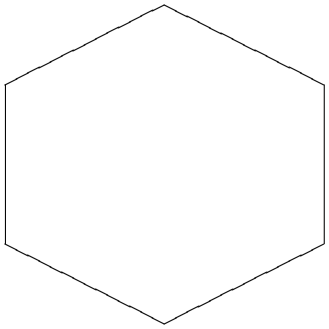
Montrer qu'on ne peut pas construire un associaédre avec des carrés (3) et des pentagones réguliers (6). Niveau terminale (?)

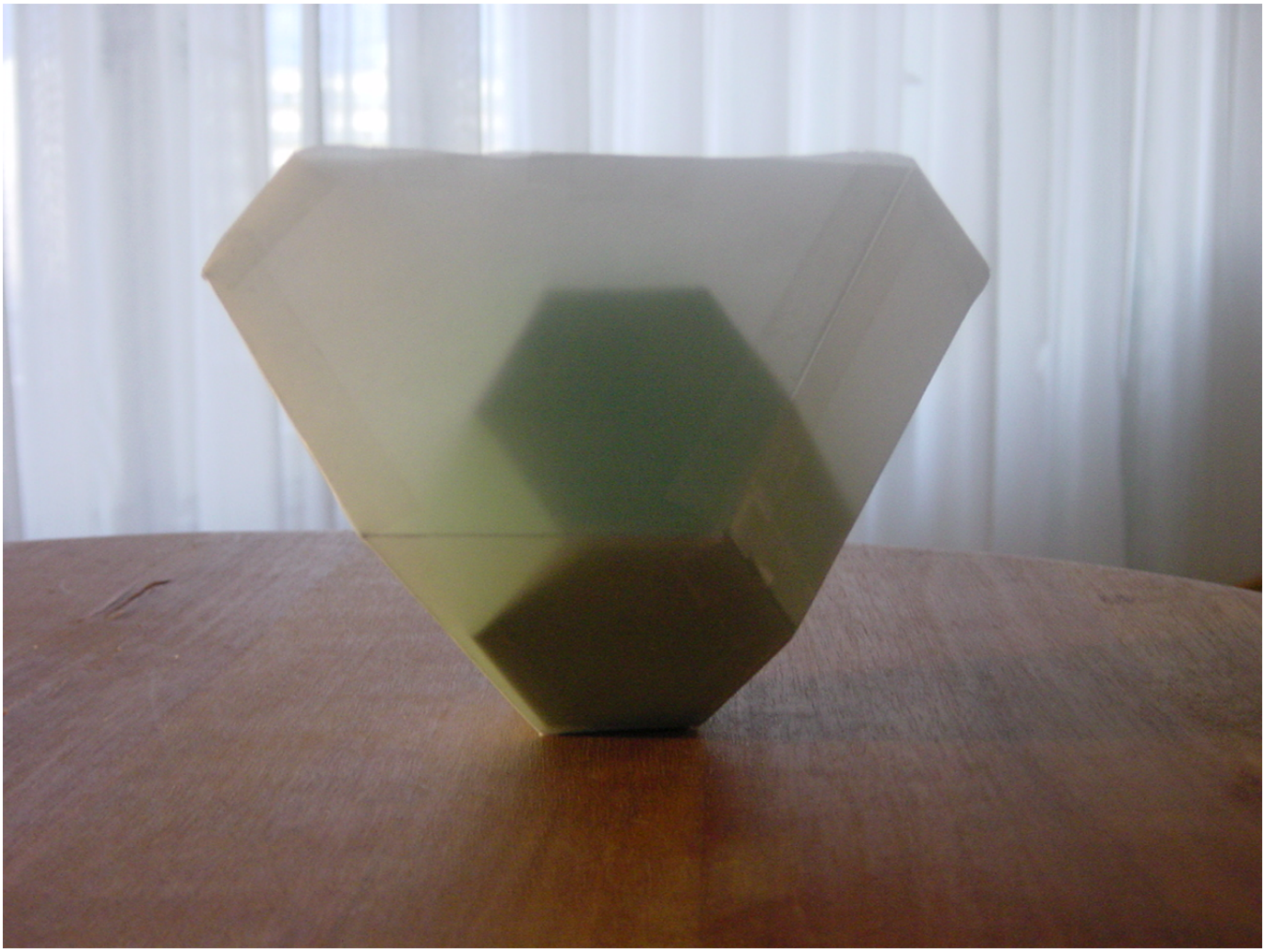


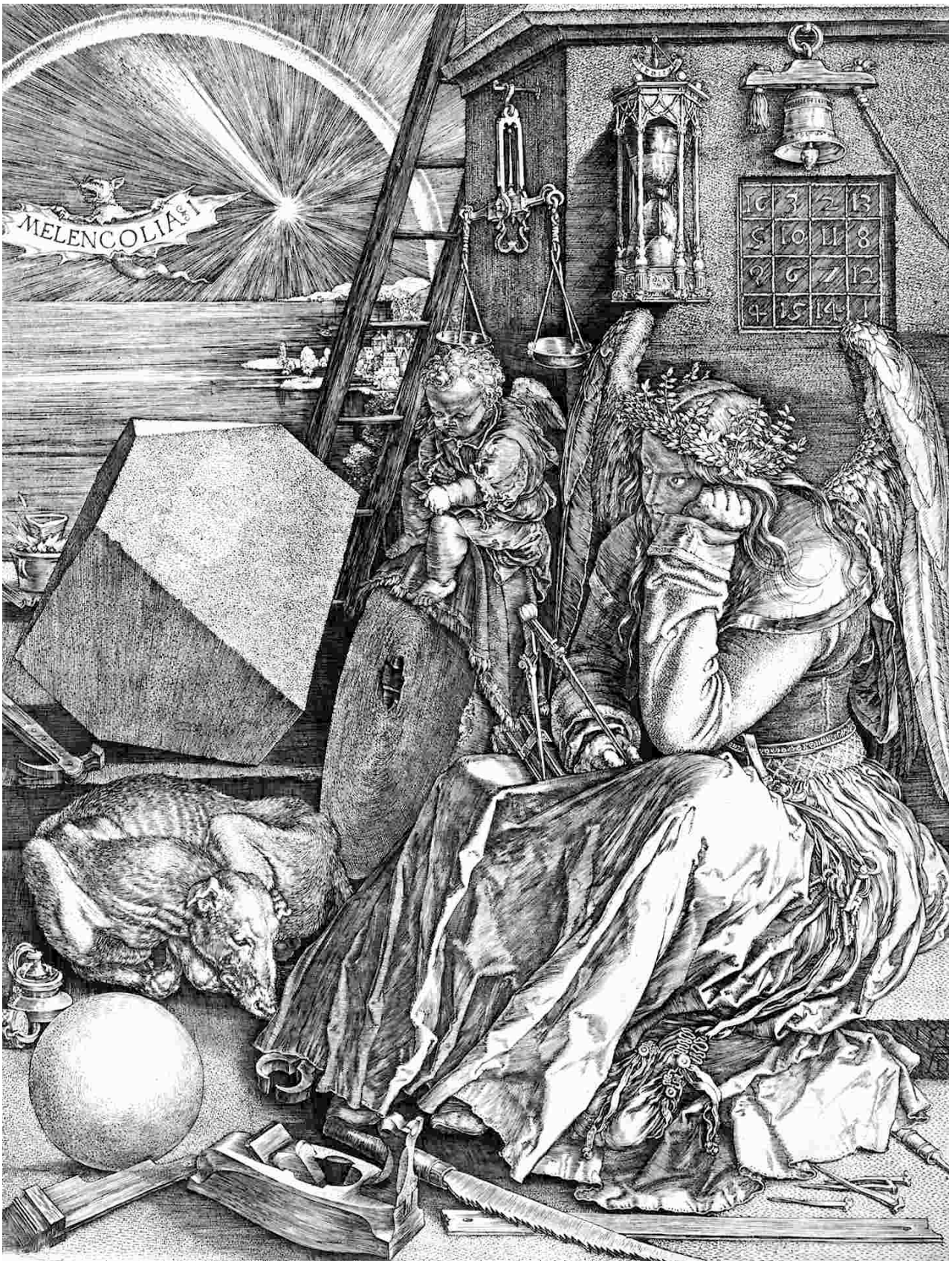
Comparaison permutoèdre associaèdre rhomboèdre





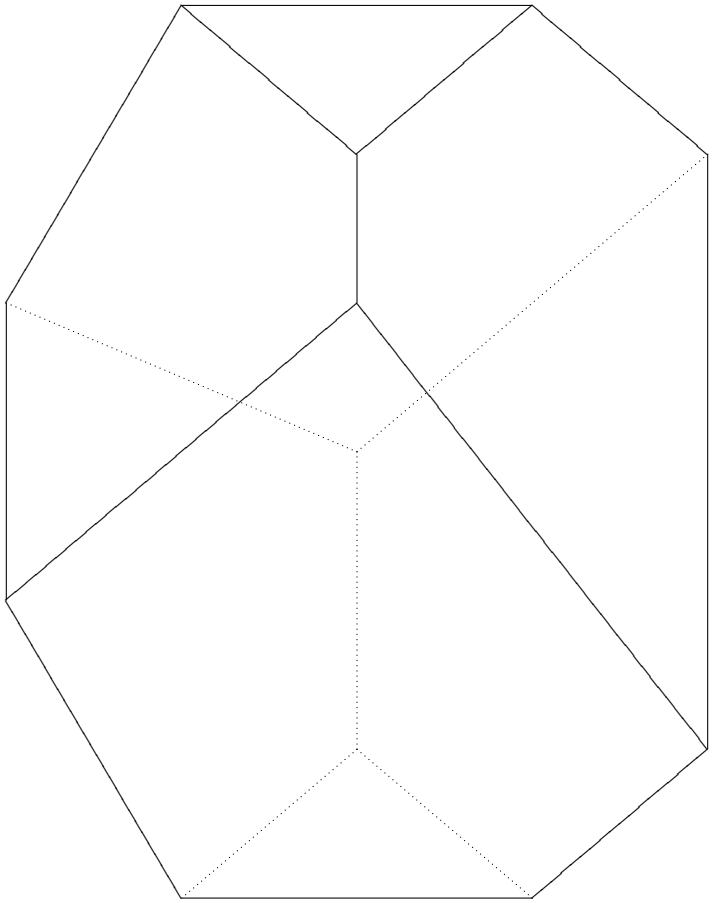


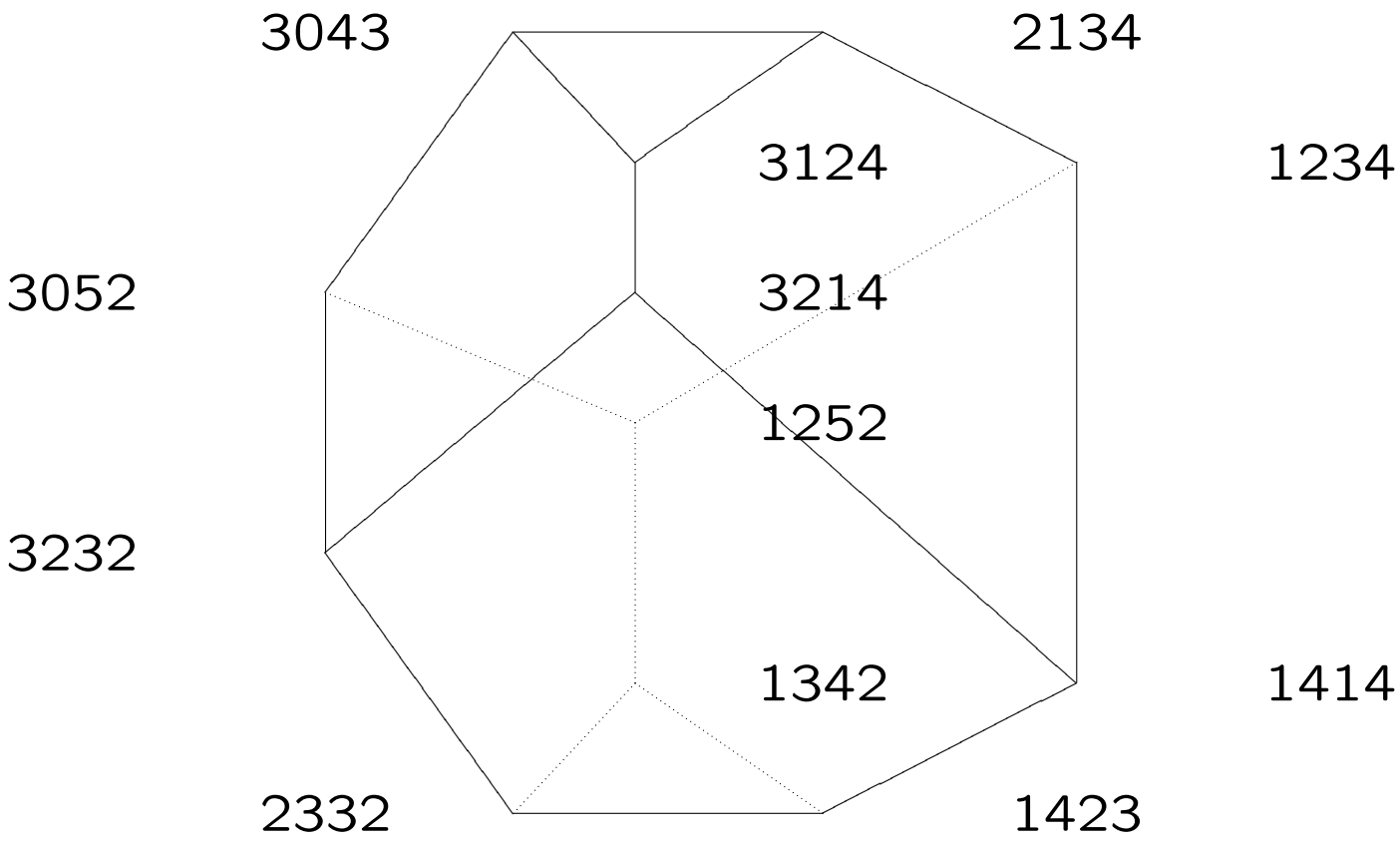



MELENCOLIA I
16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1

Albrecht Dürer Melencholia I (1514)

Polytope de Dürer = rhomboèdre tronqué

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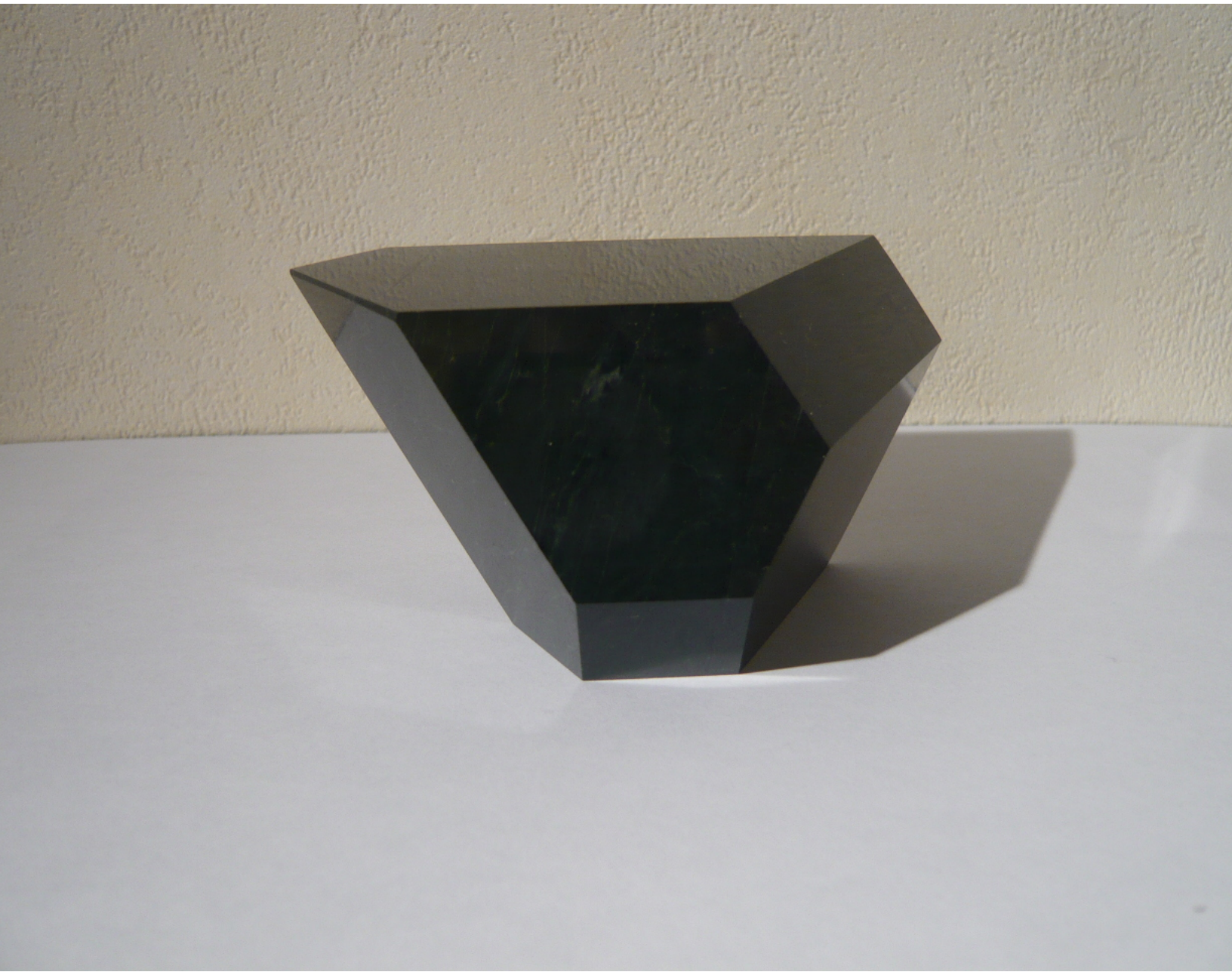

Merci pour votre attention !