

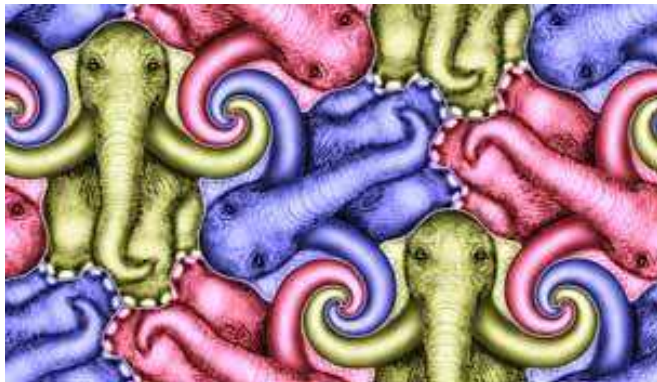
Pavages Pavages Pa
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ges Pavages Pavage
es Pavages Pavages
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p3



Tri => Mathémat**(r)**iques!

Pavage: Ensemble de tuiles disjointes recouvrant le plan

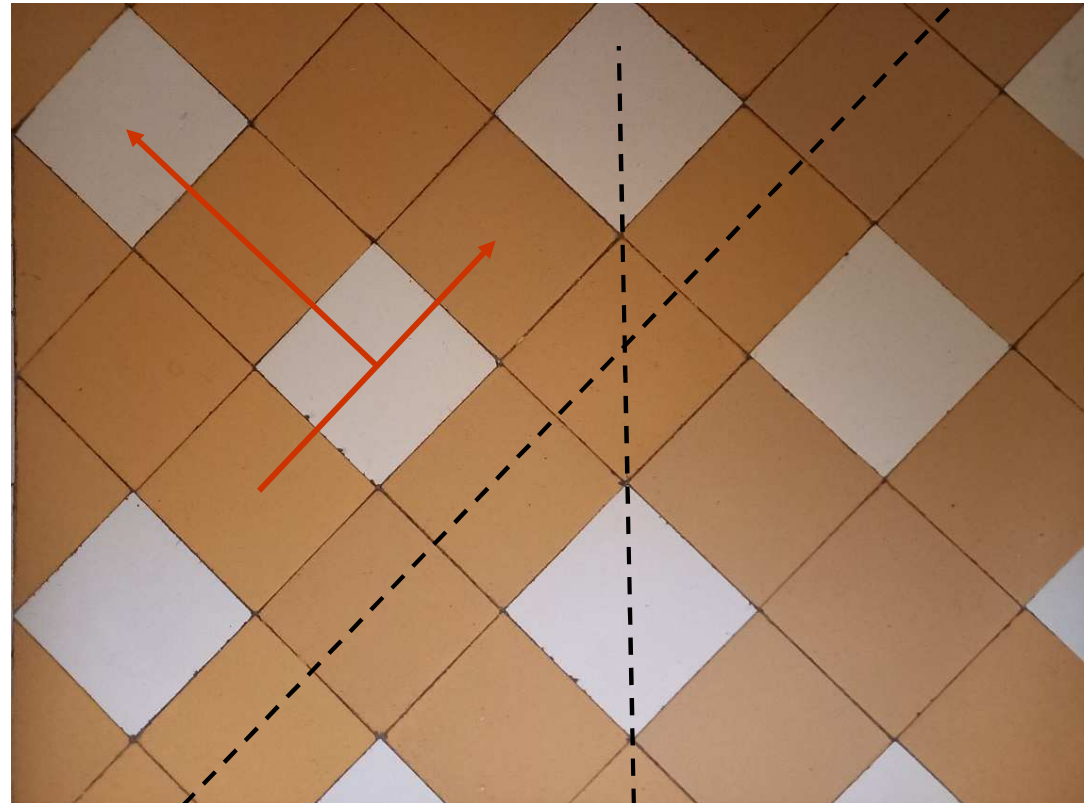
Groupe de symétrie d'un pavage P

=

Ensemble des isométries S du plan telles que $S(P) = P$.

Isométries:

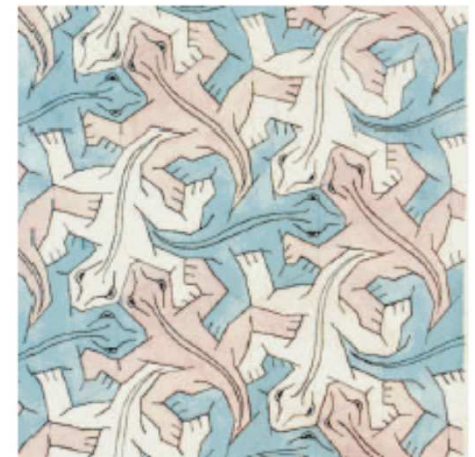
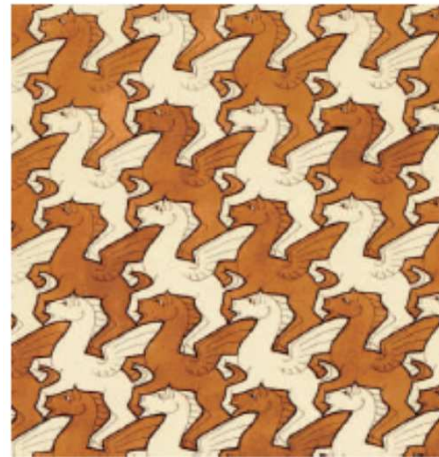
- Translations
- Rotations
- Symétries axiales



Périodique: $S(P)$ contient 2 translation indépendantes

Antiquité: motifs décoratifs

Evgraf Fedorof : 17 groupes cristallographiques du plan
(19 types de pavés)





Il existe cinq groupes d'isométries qui permettent de paver le plan sans retourner les tuiles :

- les translations
- les rotations d'angle 60, 90, 120 ou 180 degrés

*les seuls polygones réguliers qui pavent le plan sont:
le triangle équilatéral, le carré et l'hexagone.*

Retournement des tuiles (motifs)

=>

12 manières supplémentaires de paver le plan.



John H. Conway
(1937-2020)

John Conway, William Thurston

Les symétries des pavages peuvent être définies par une notation adéquate, la *signature*, composée de symboles représentant les:

- réflexions (*)
- rotations (n)
- « miracle » (x)
- « merveille » (o)

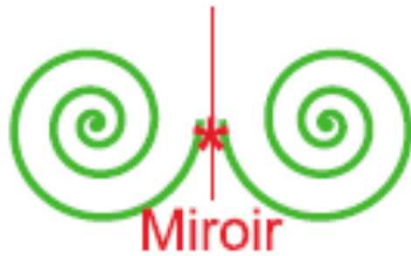
à l'exception des translations.

Translation

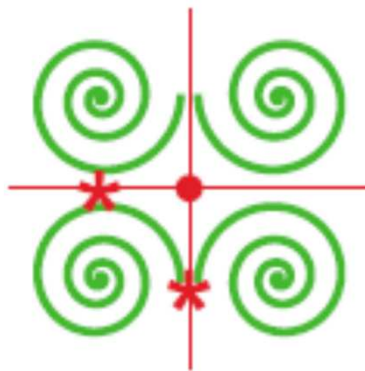
=

composition de rotations

Réflexion (*n) : isométrie indirecte = retournement

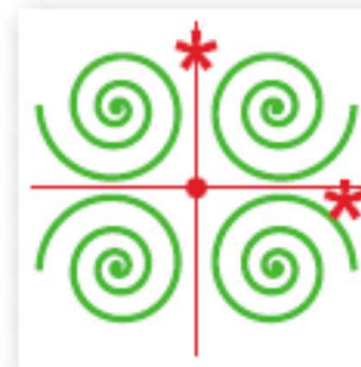


Symétrie de réflexion
notation *



Deux miroirs
notation *2

*2



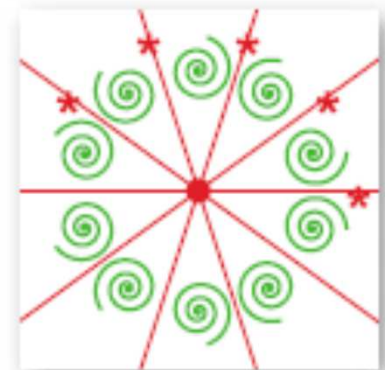
*3



*4

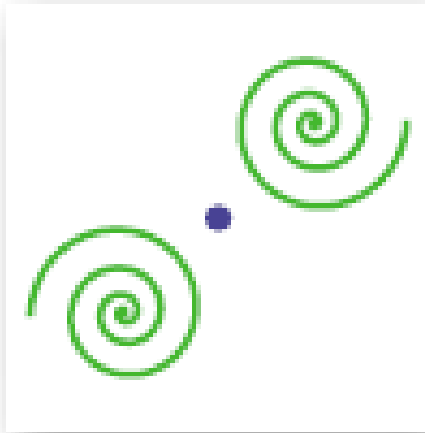


*5

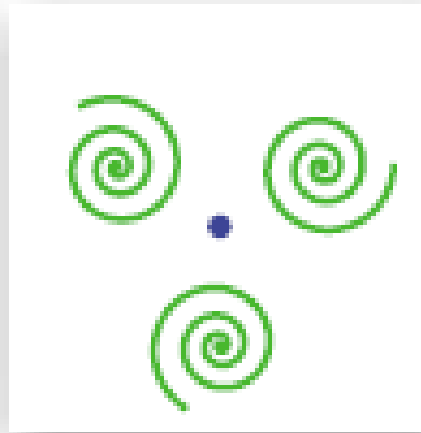


Rotation (n) : isométrie directe = déplacement

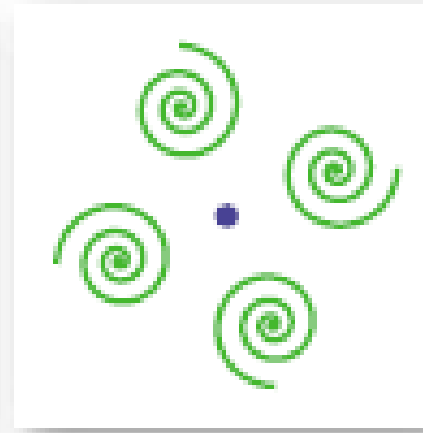
2.



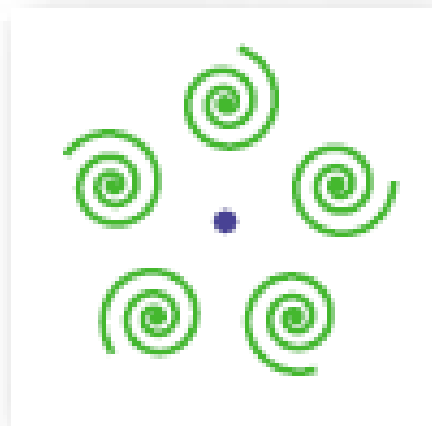
3.



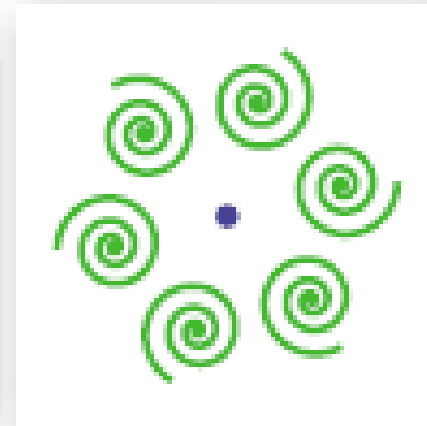
4.



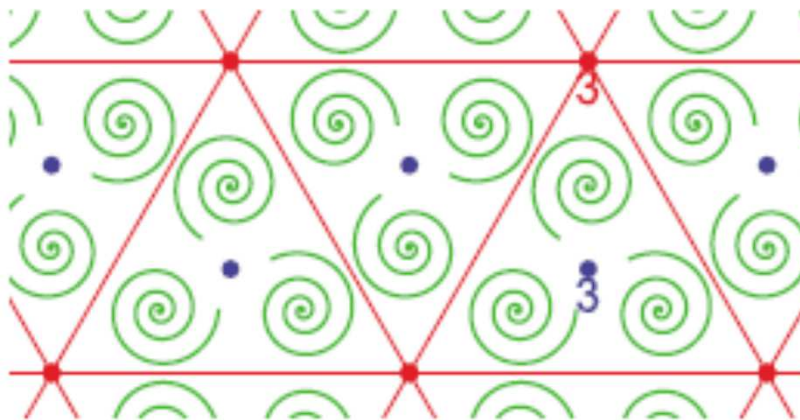
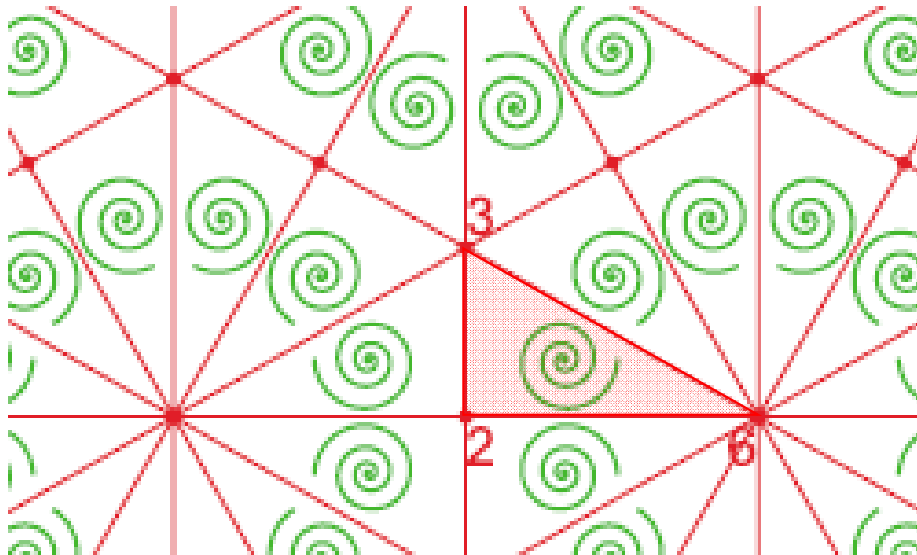
5.



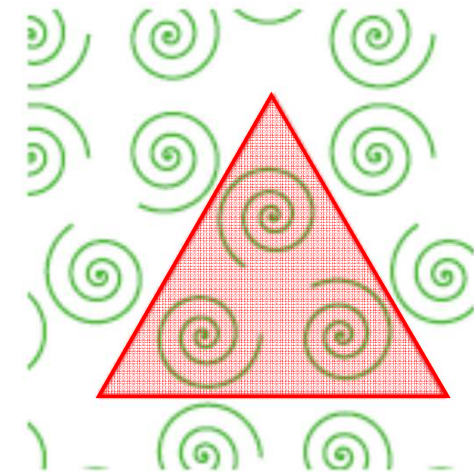
6.



$$*6 + *3 + *2 = *632$$



Miroir(s) + Rotation(s)



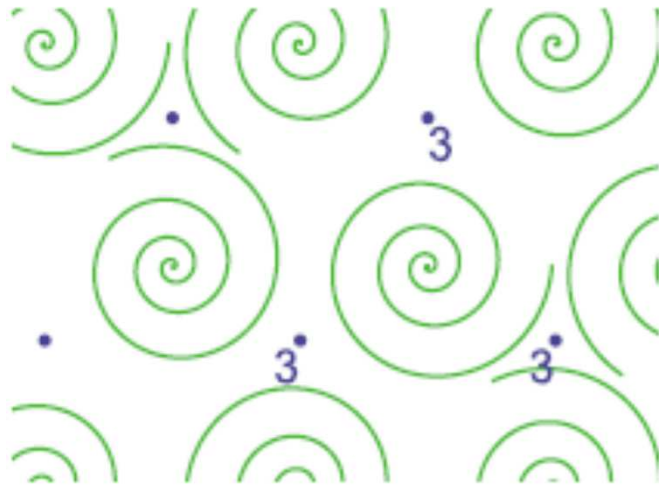
Motif fondamental

$$3. + *3 = 3*3$$

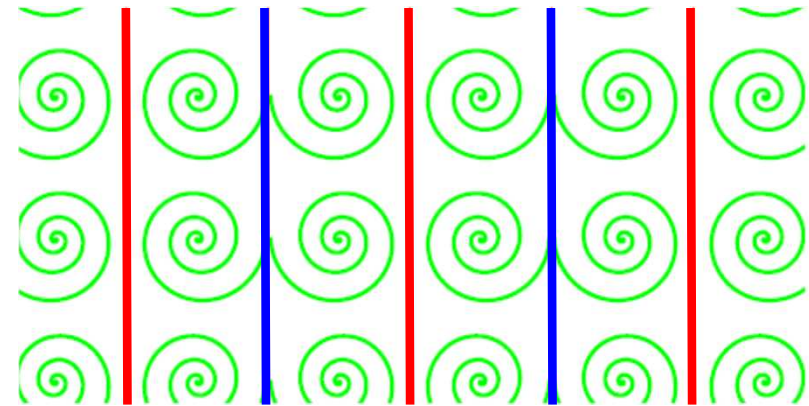


Rotations + 0.miroir

2 types de miroirs //



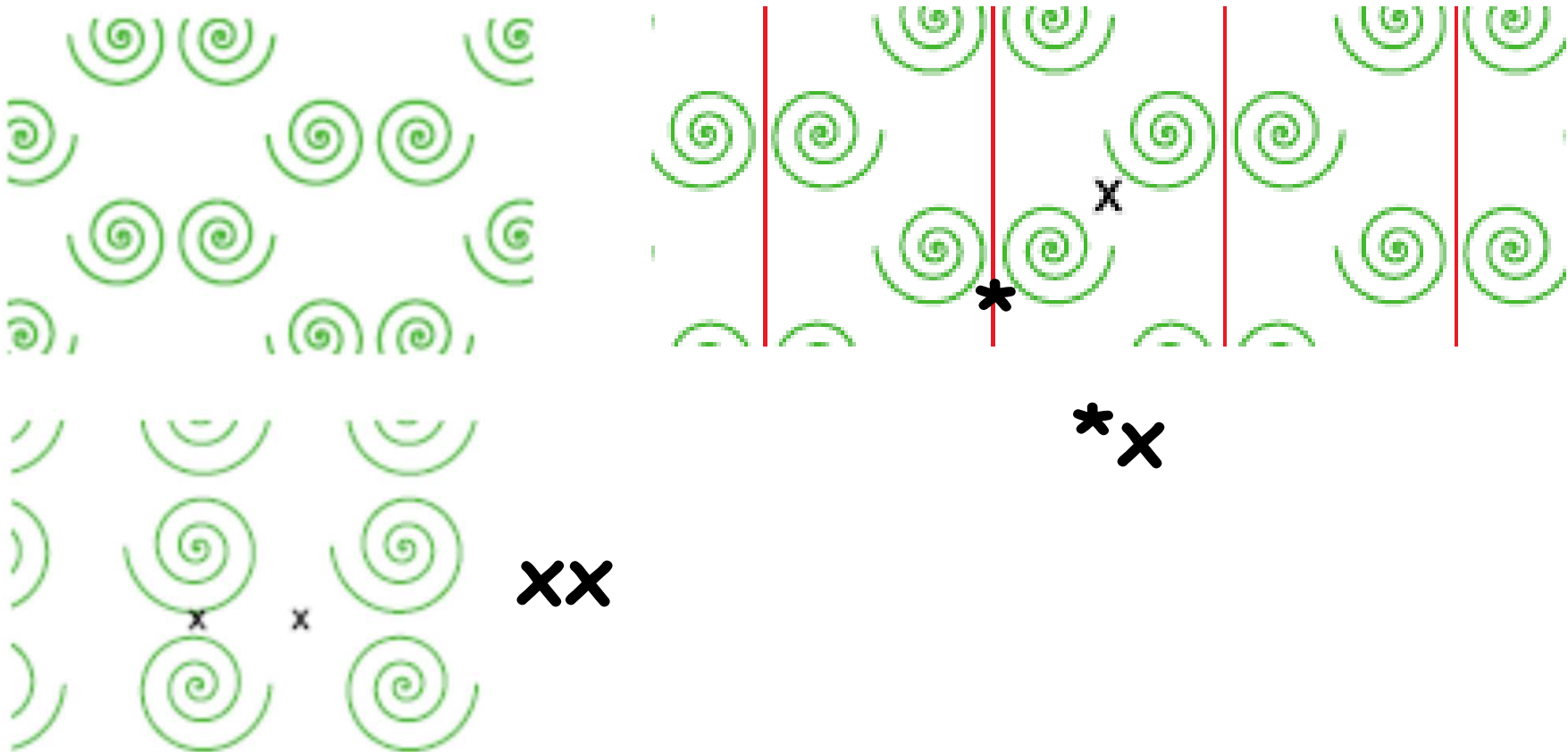
333



Une seule sorte de miroir
 Inversion des spirales entre deux miroirs

=

Miracle, noté x

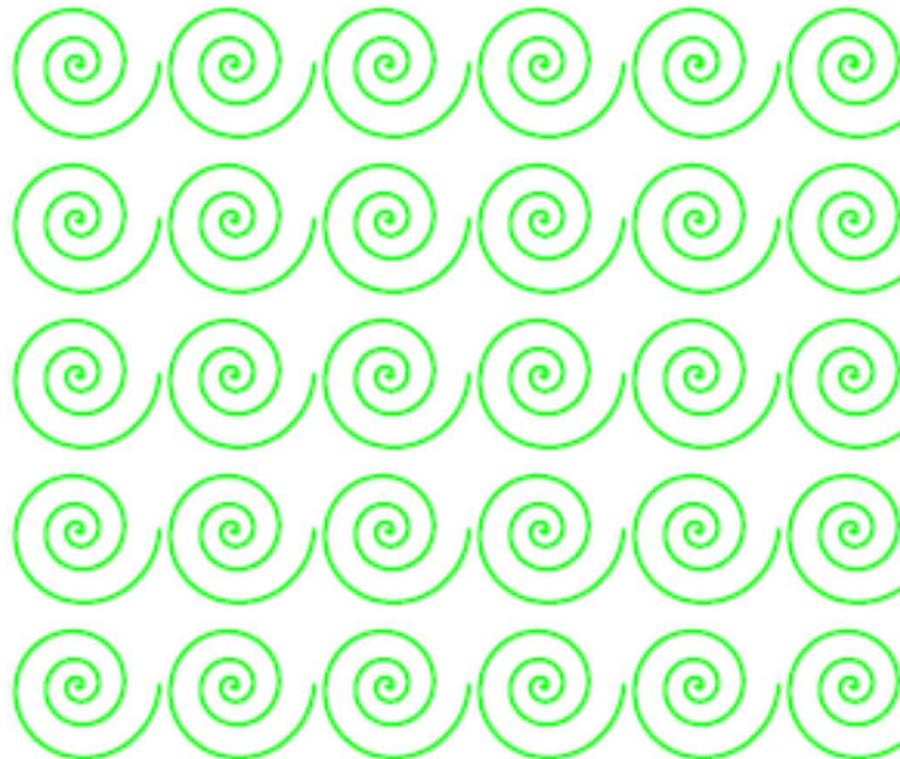


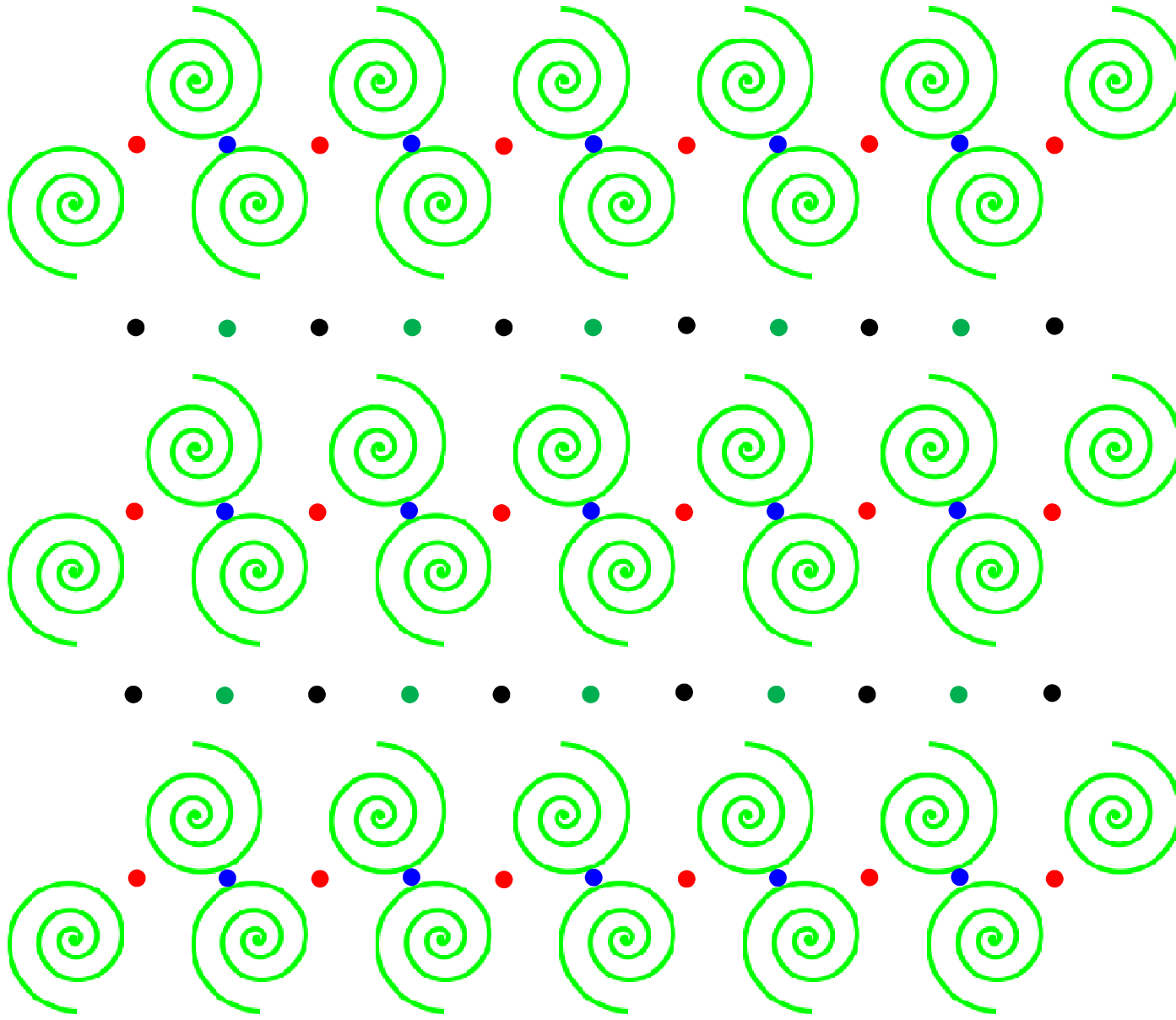
Pondération des signatures

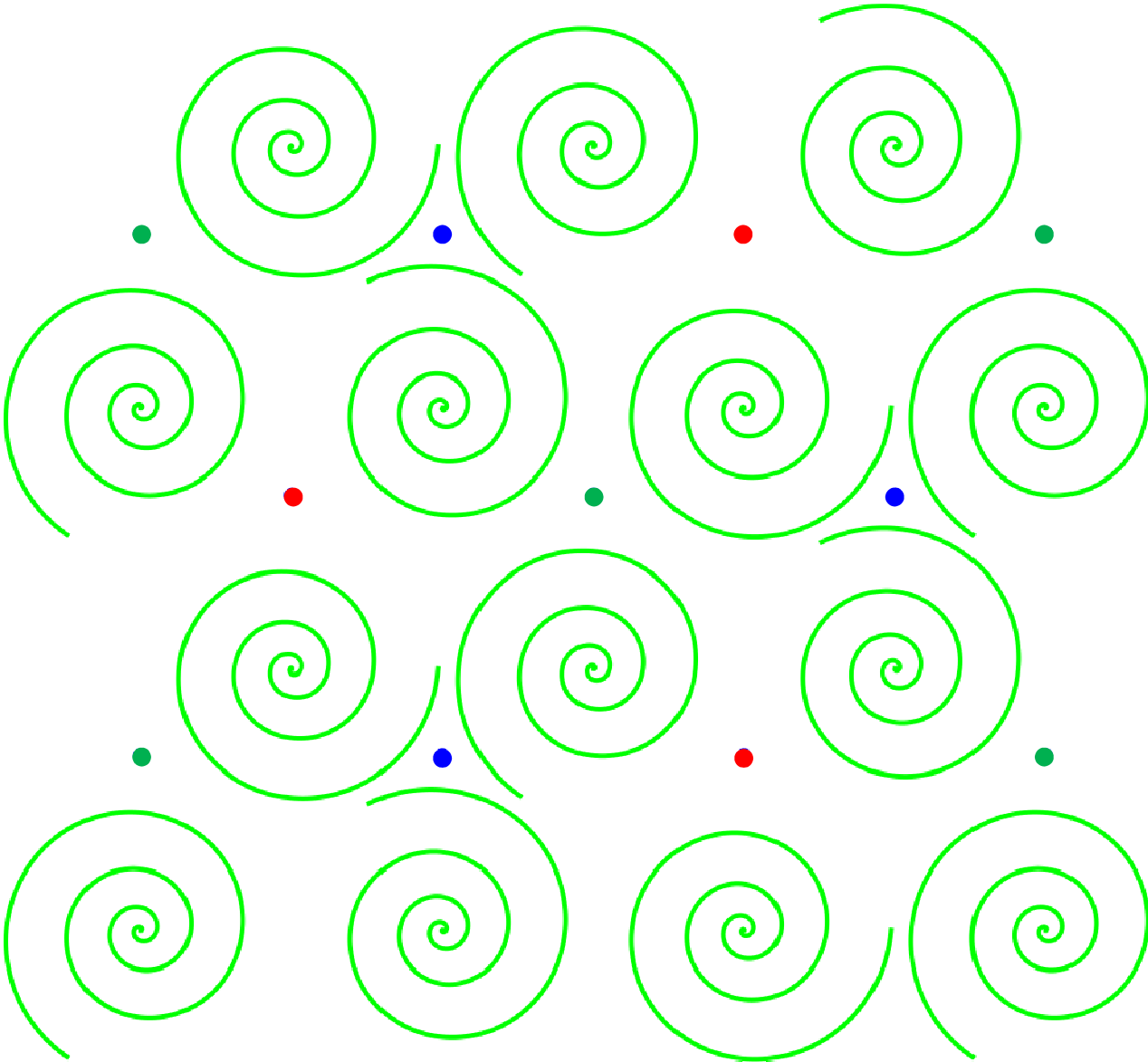


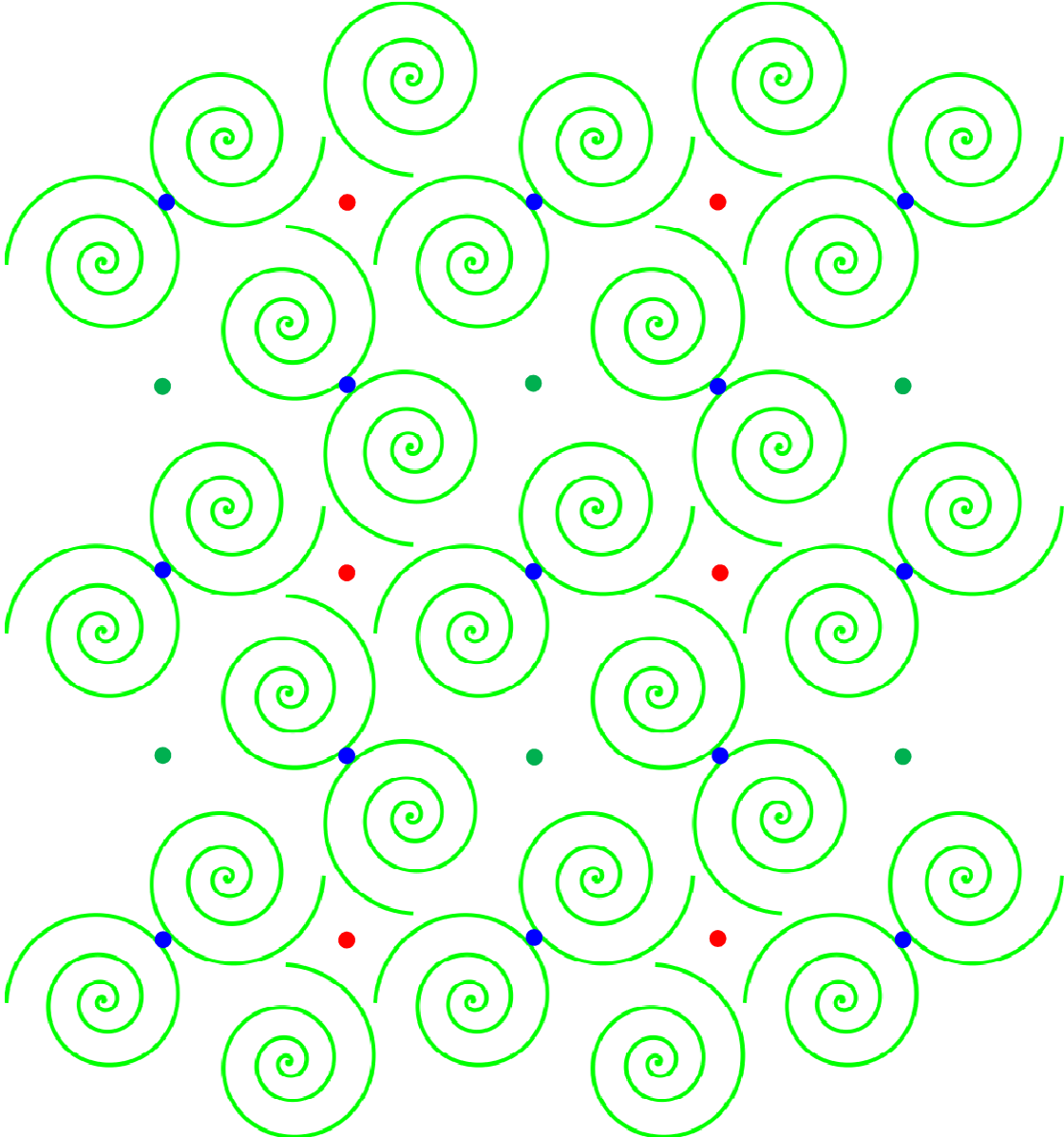
<i>Symbole</i>	<i>poids</i>	<i>Symbole</i>	<i>poids</i>
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
⋮	⋮	⋮	⋮
N	$\frac{N-1}{N}$	N	$\frac{N-1}{2N}$
∞	1	∞	$\frac{1}{2}$

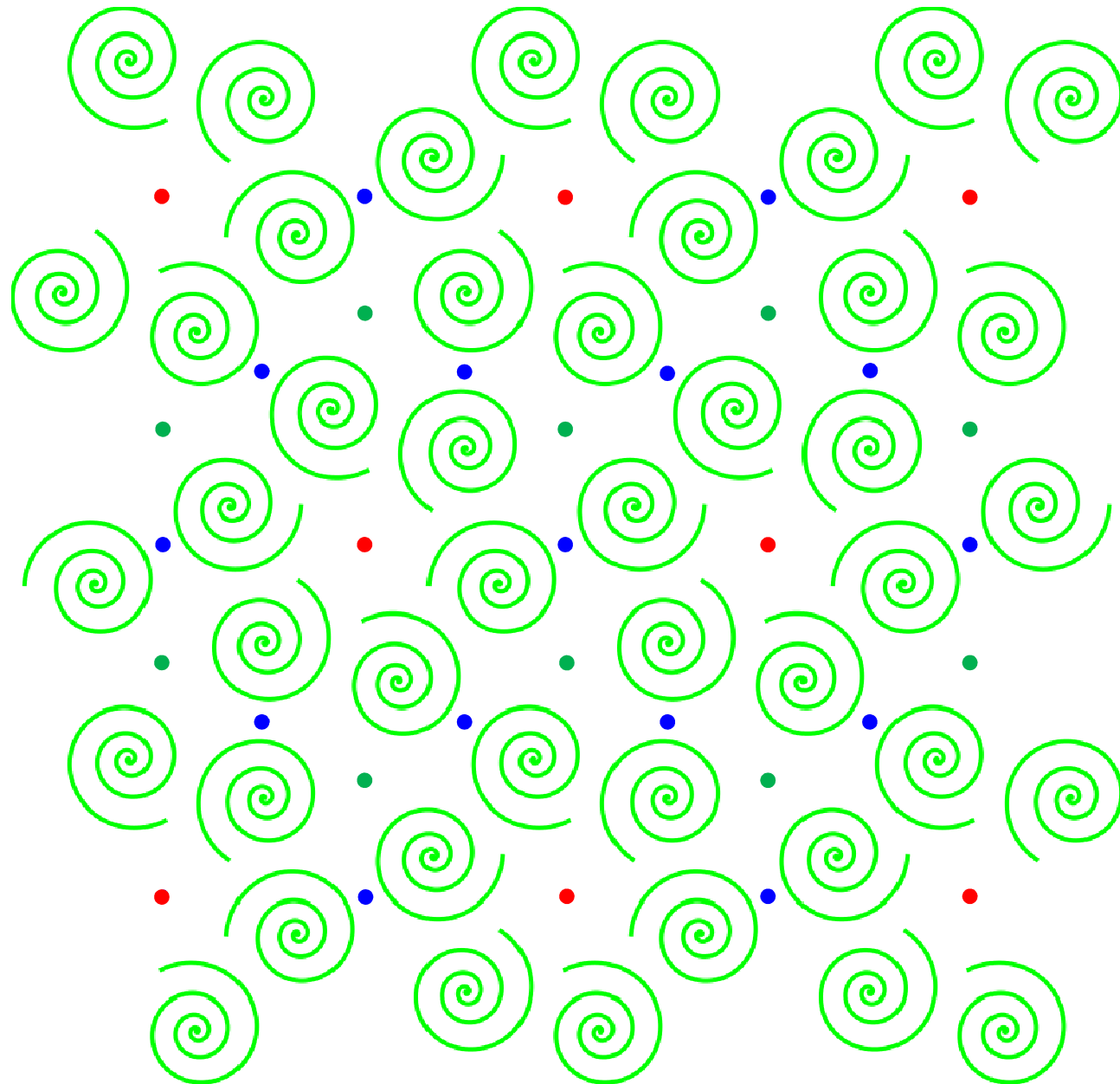
**5 groupes isométries directes (rotations)
(Conway: sans réflexion et *miracle*)**













Calcul du poids des signatures:

- **0** : 2 (par définition)

- **2222** : $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$

- **333** : $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 2$

- **442** : $\frac{3}{4} + \frac{3}{4} + \frac{1}{2} = 2$

- **632** : $\frac{5}{6} + \frac{2}{3} + \frac{1}{2} = 2$

Si $p(n\dots m) = 2$, alors $p(*n\dots m) = 1 + p(n\dots m)/2 = 2$

\Rightarrow 5 nouveaux groupes

- ***2222**
- ***333**
- ***442**
- ***632**
- ****** (*seule possibilité avec plus d'une **)



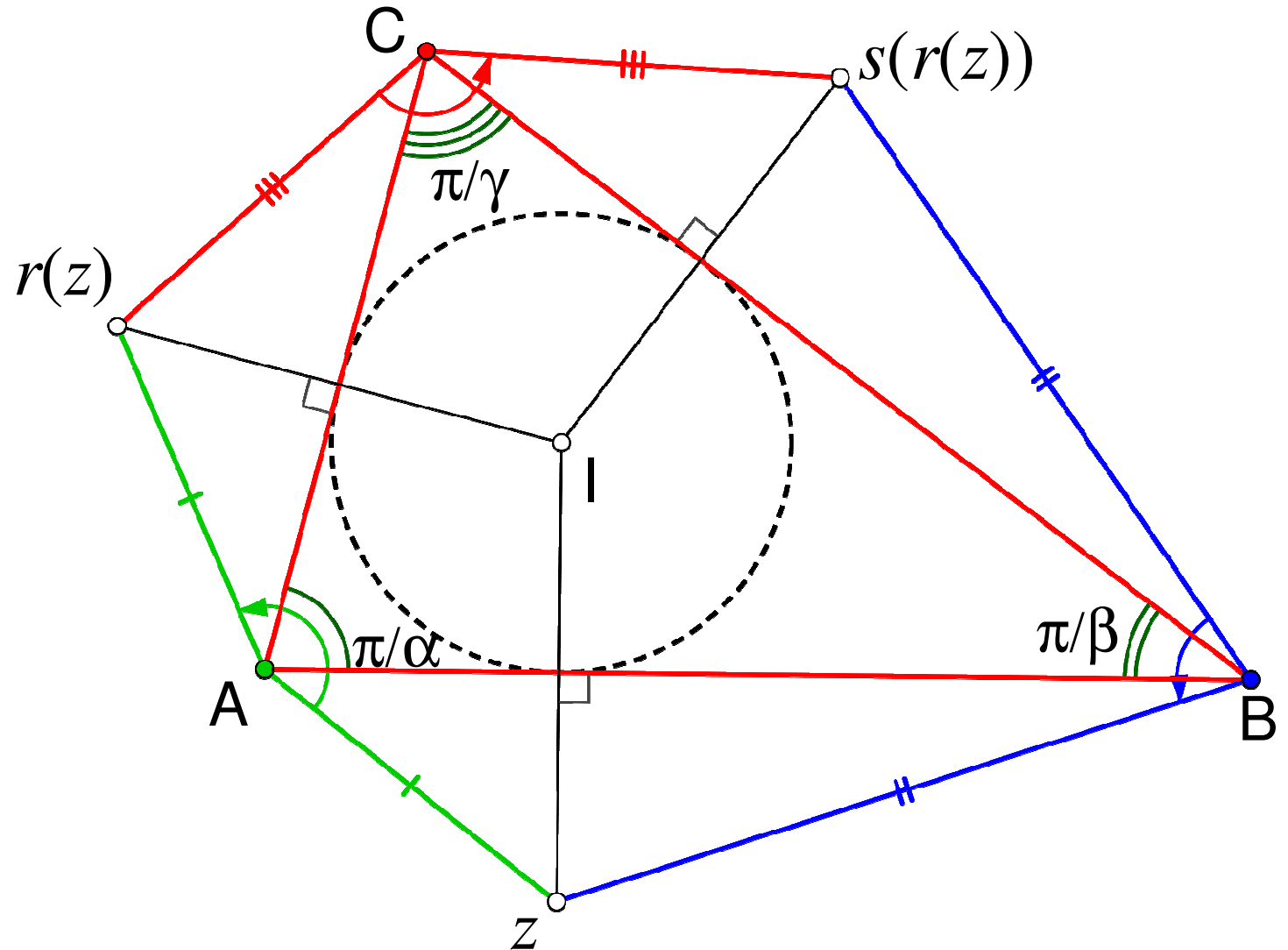
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 1$$

Trois cas :

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

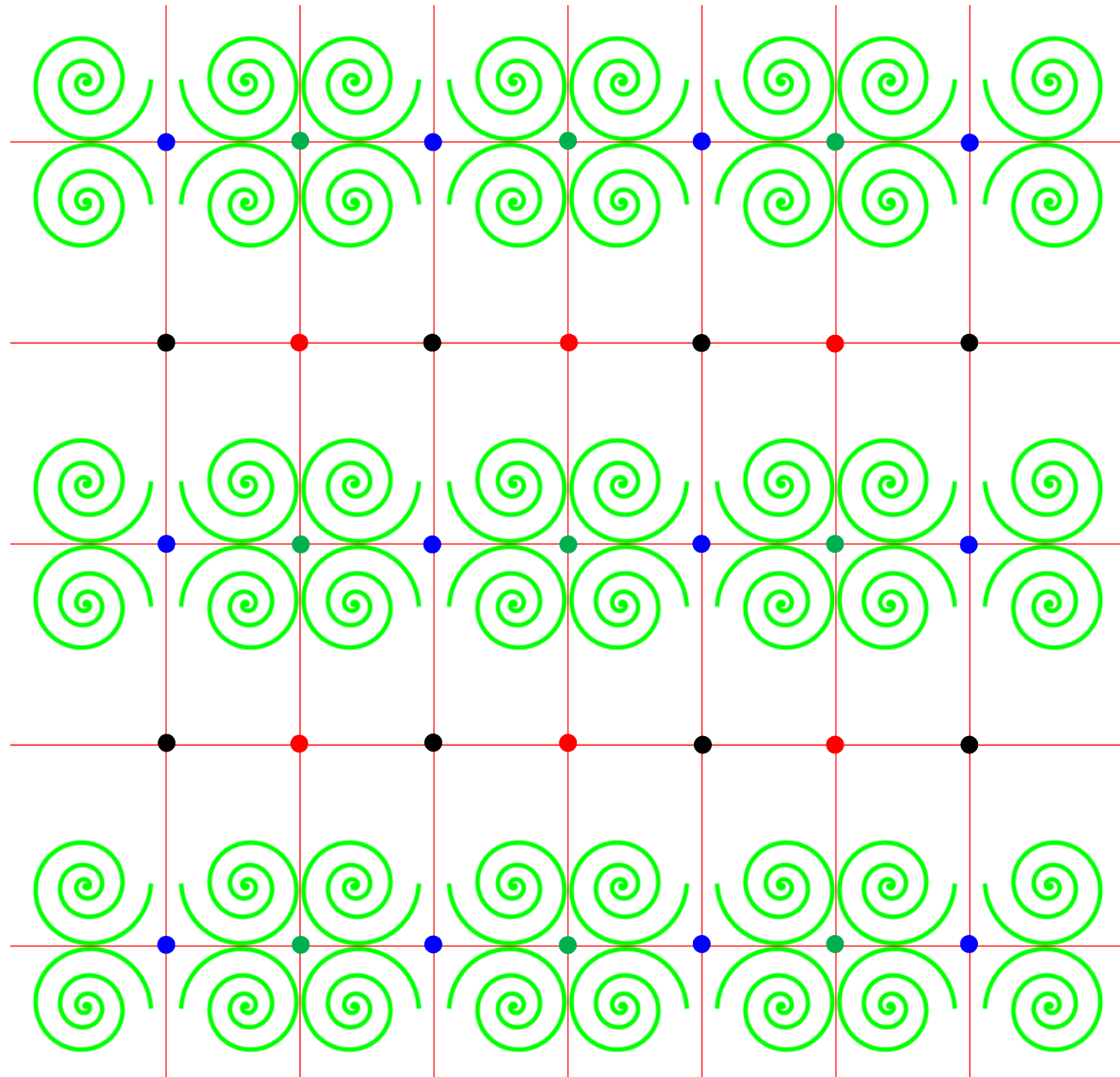
$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

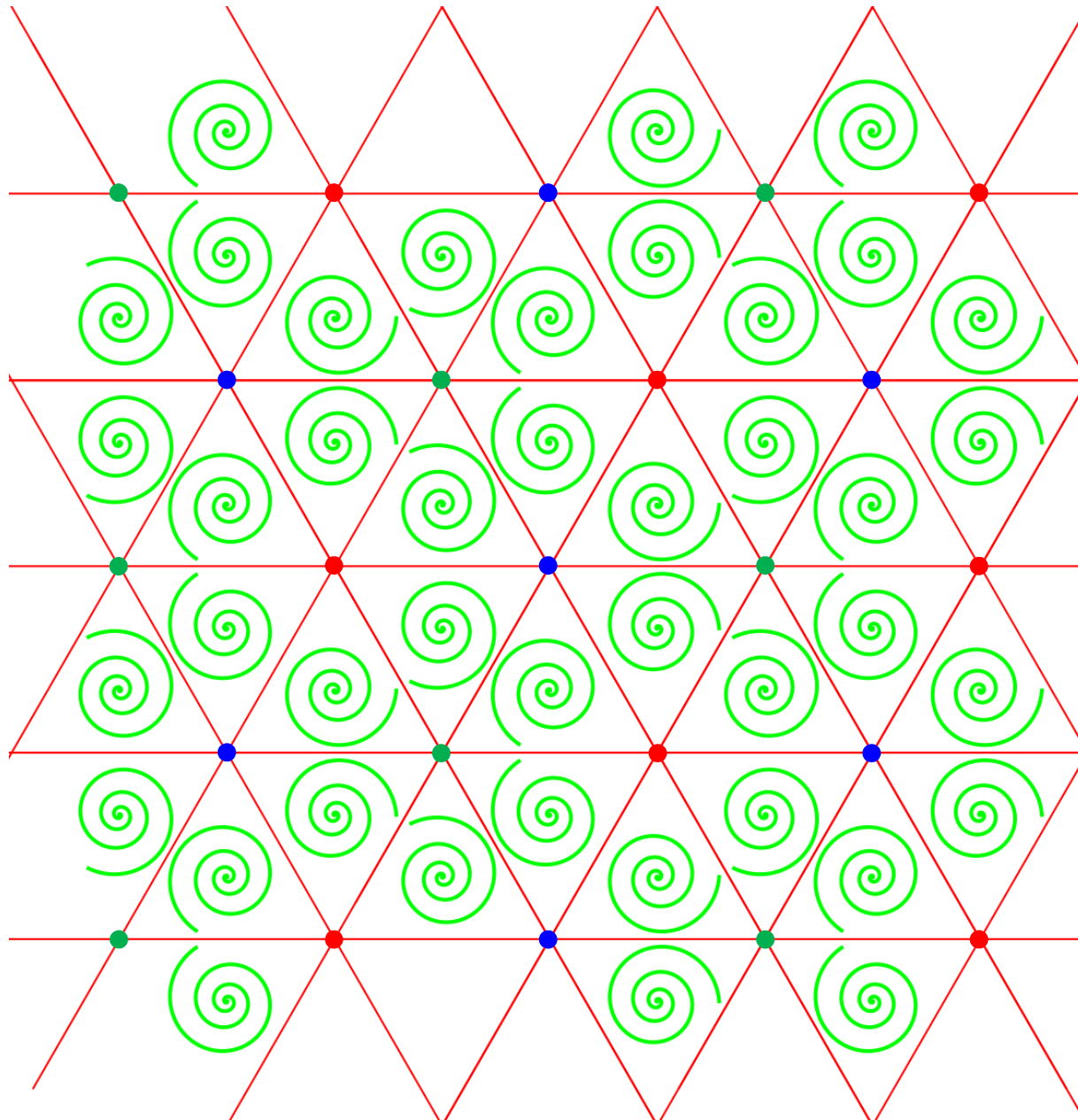




Groupes de pavages

*2222 = pmm

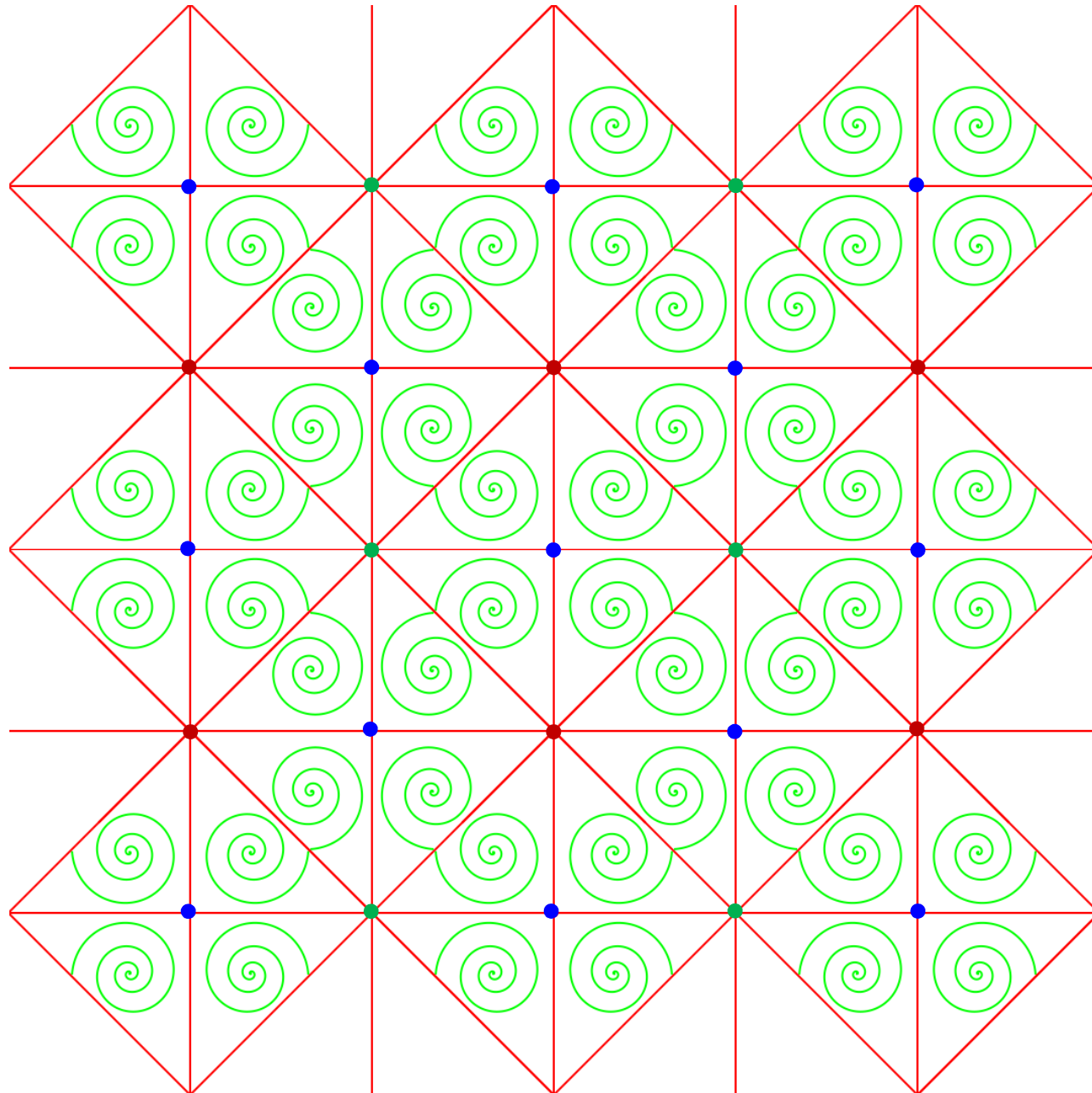


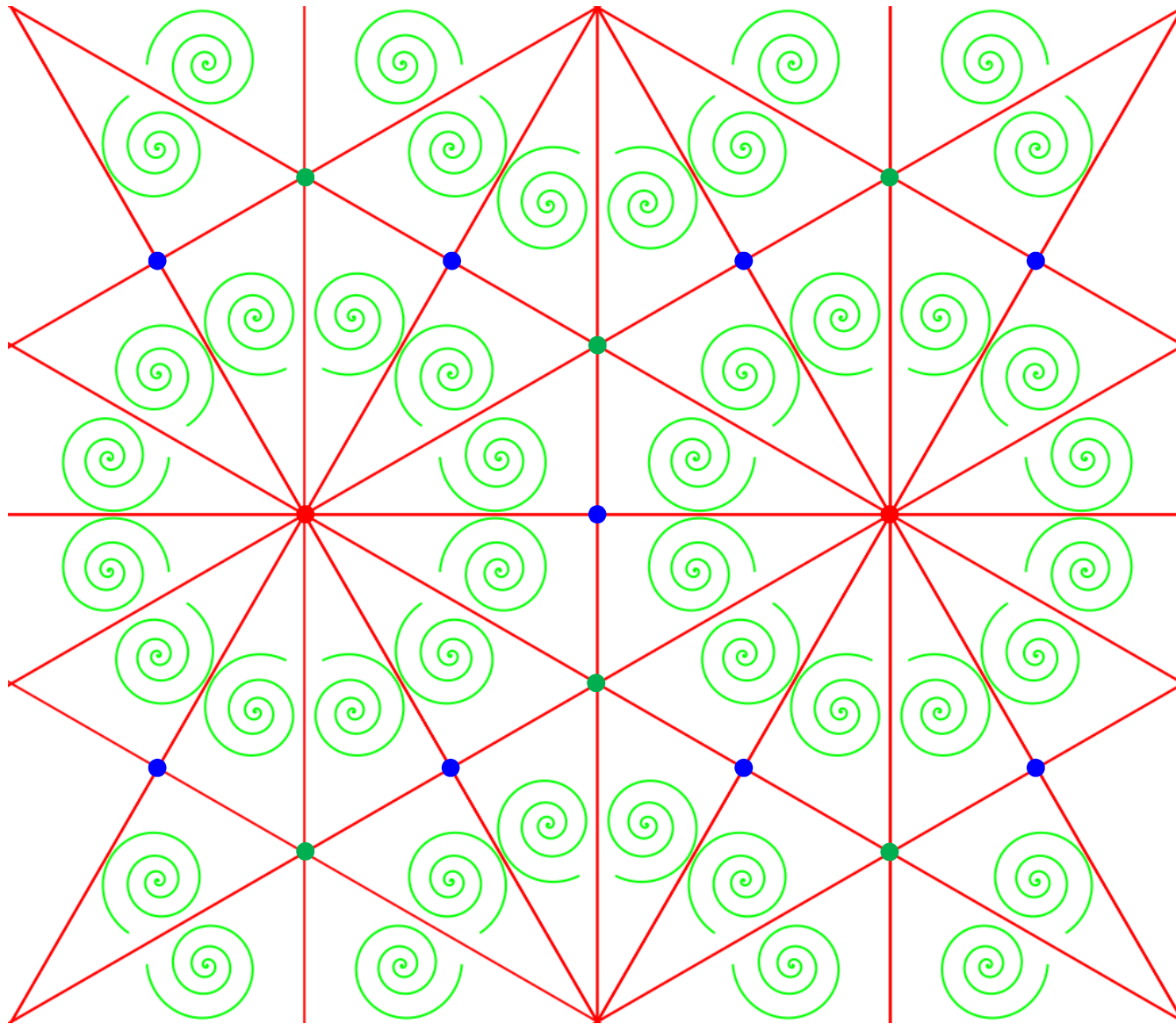




Groupes de pavages

*442 = p4m

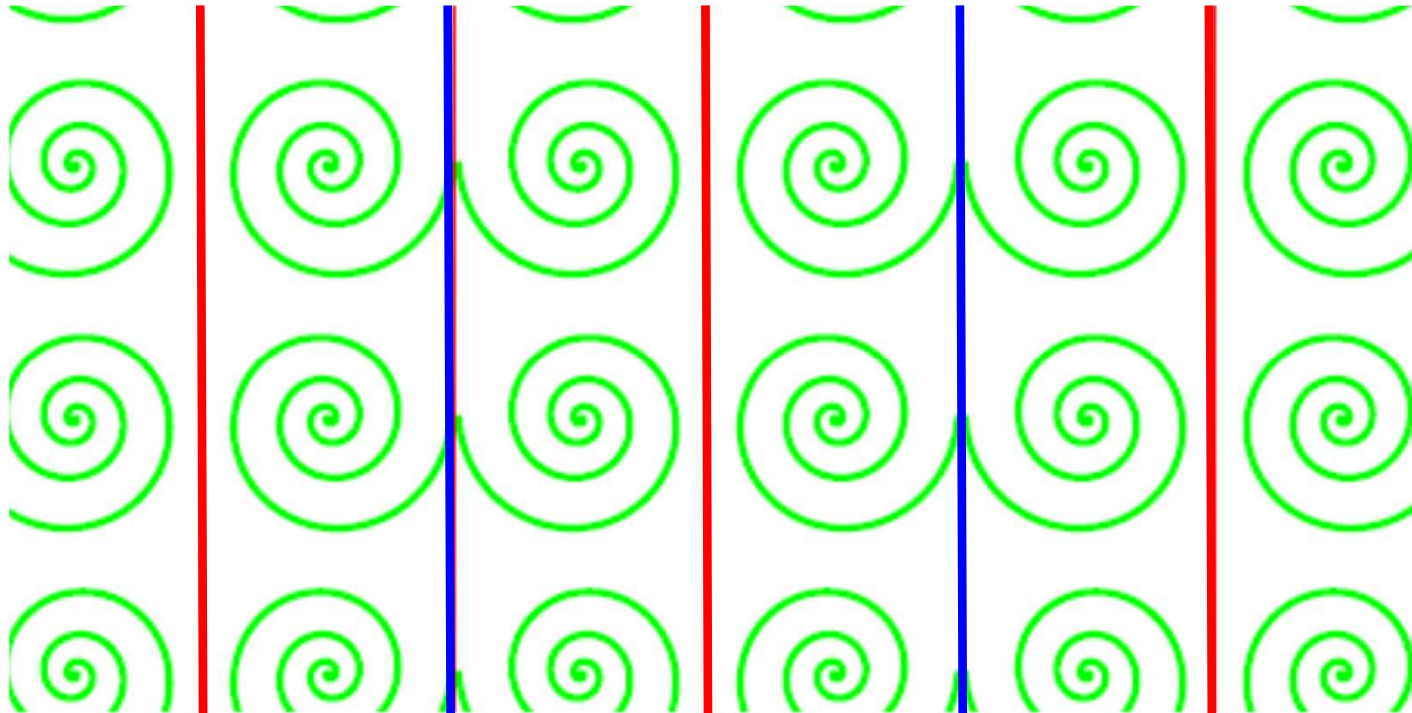


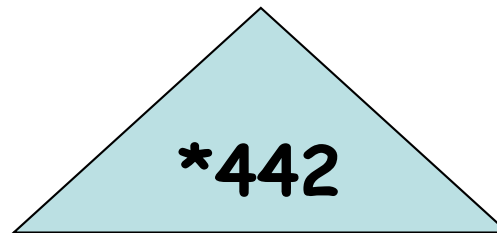
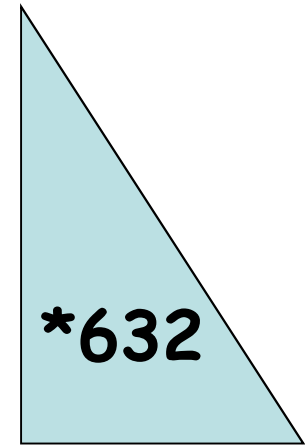
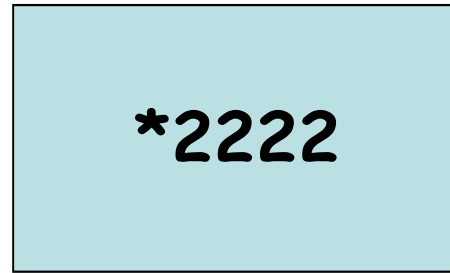
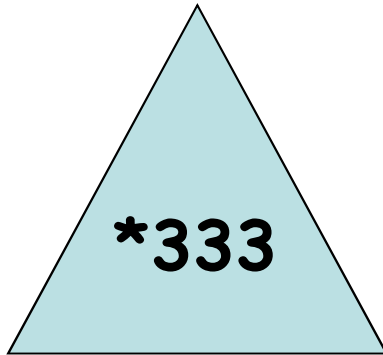




Groupes de pavages

** = pm







7 groupes hybrides (rotations, réflexions, *miracles*)

On ne change pas la valeur de la signature en:

- remplaçant n^* par $*nn$
- remplaçant x par $*$

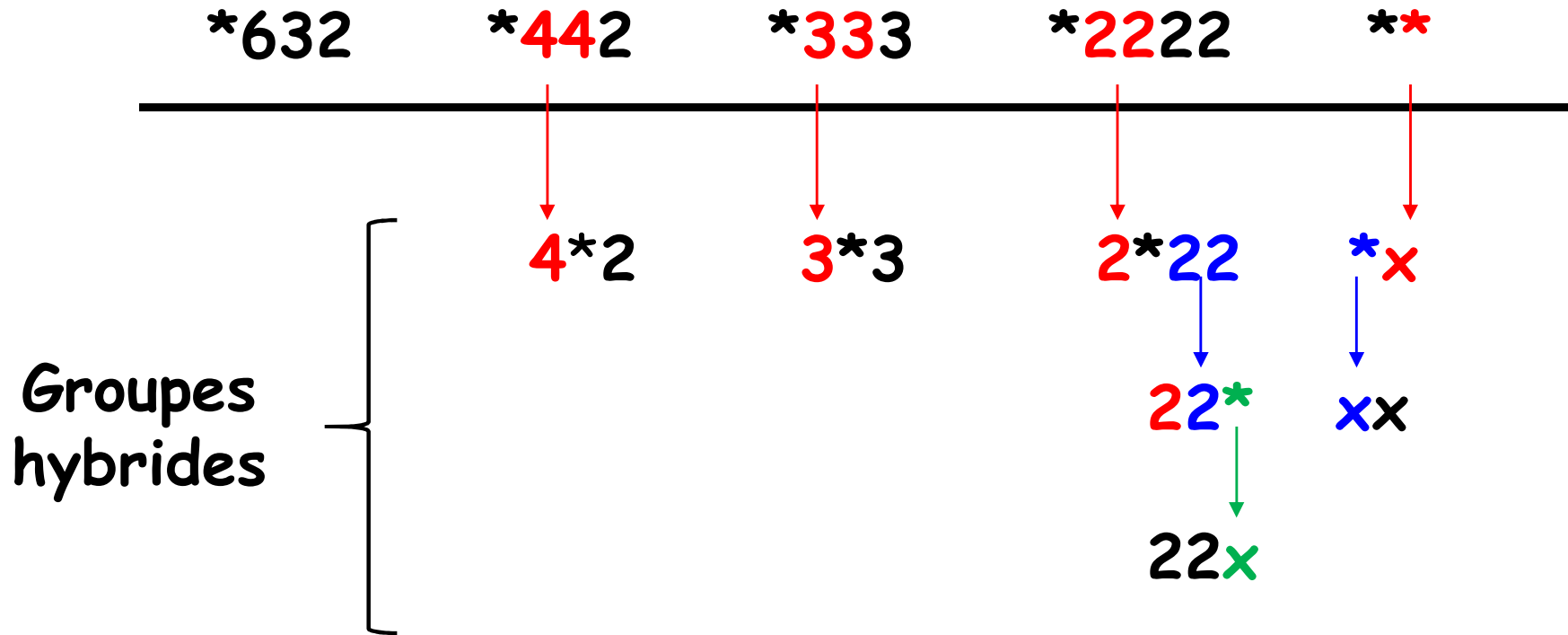
De même avec l'opération inverse:

- remplacer $*nn$ par n^*
- remplacer $*$ par x



7 groupes hybrides (rotations, réflexions, *miracles*)

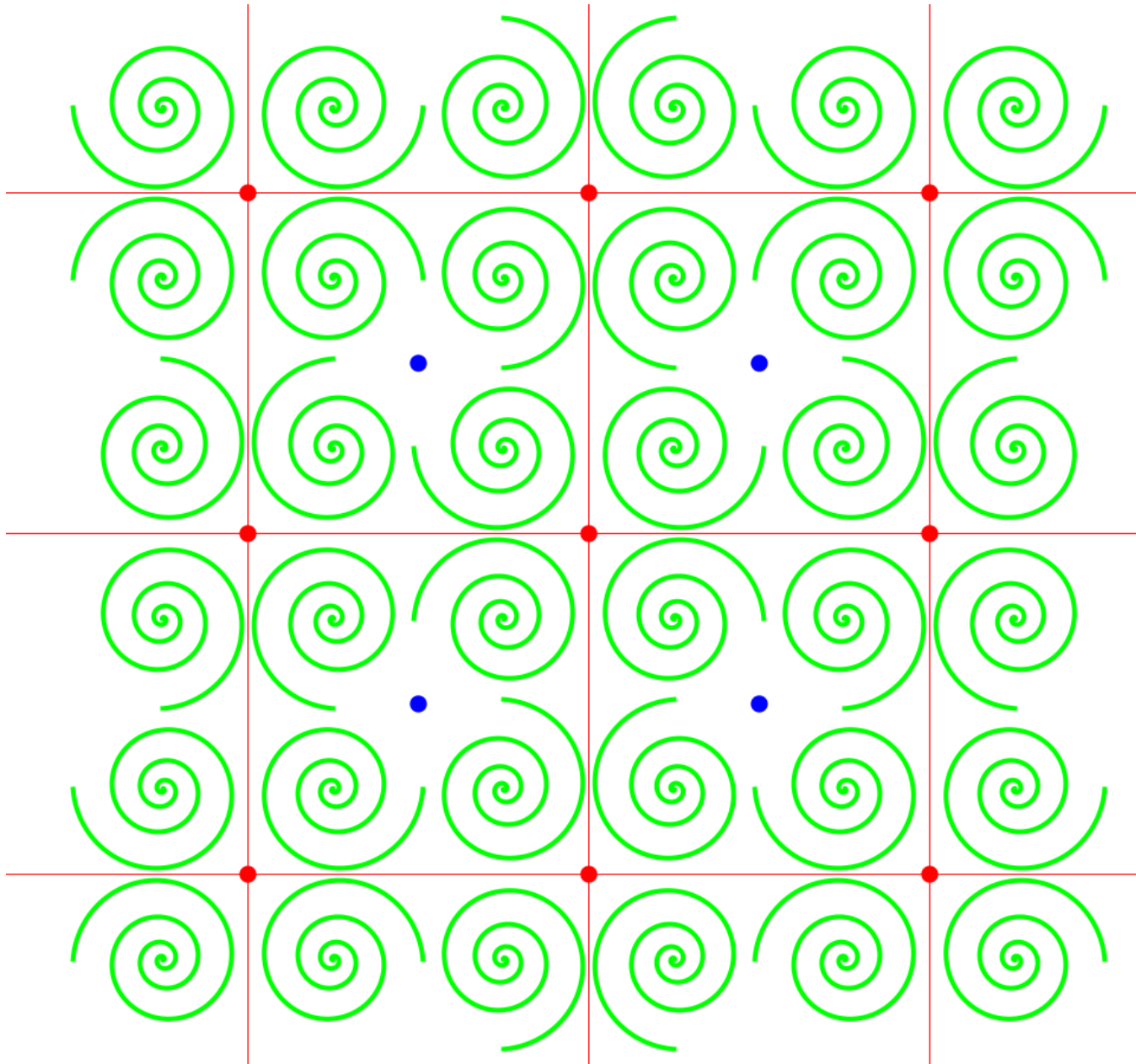
Jeu d'écriture

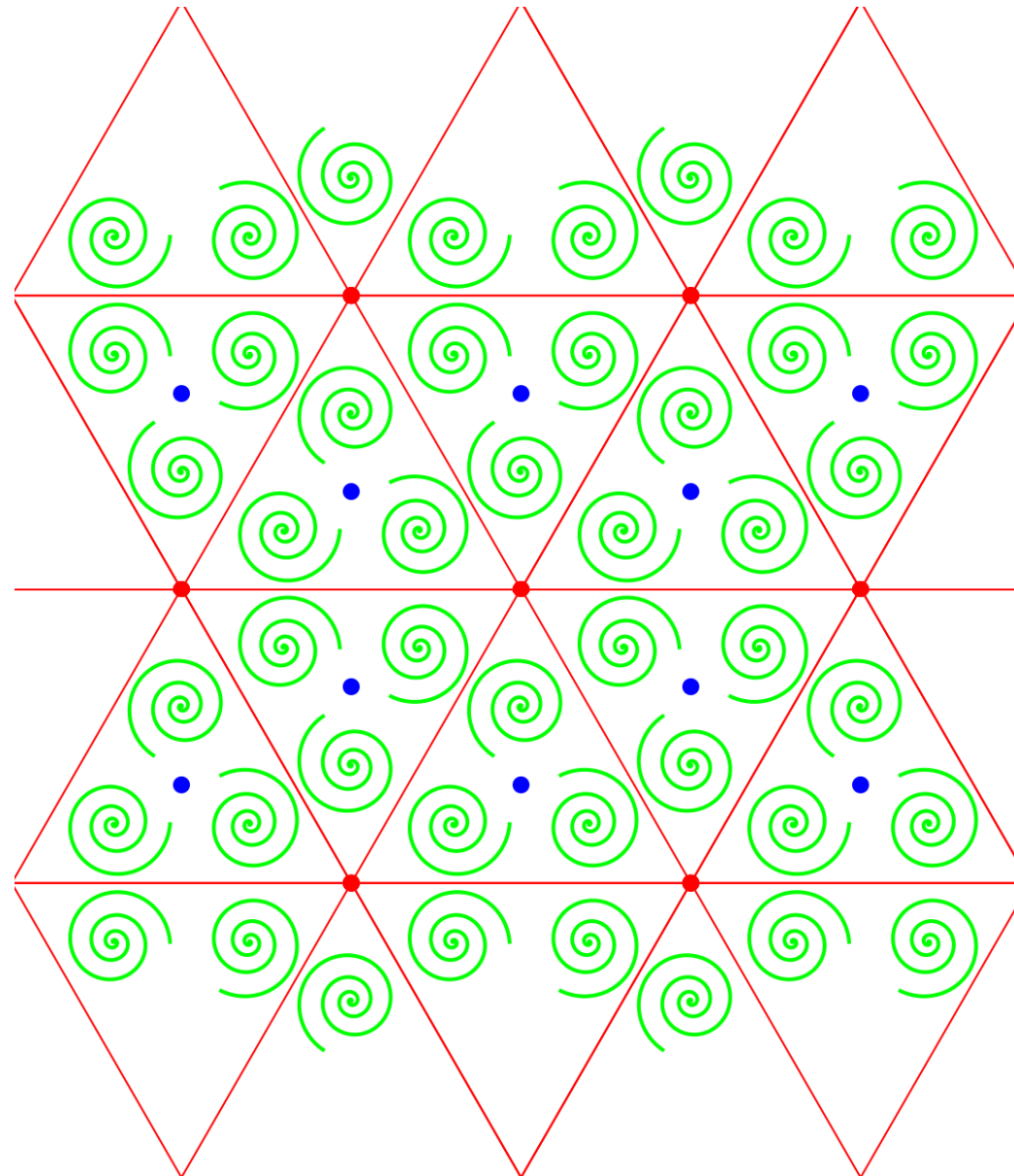




Groupes de pavages

$$4*2 = p4g$$

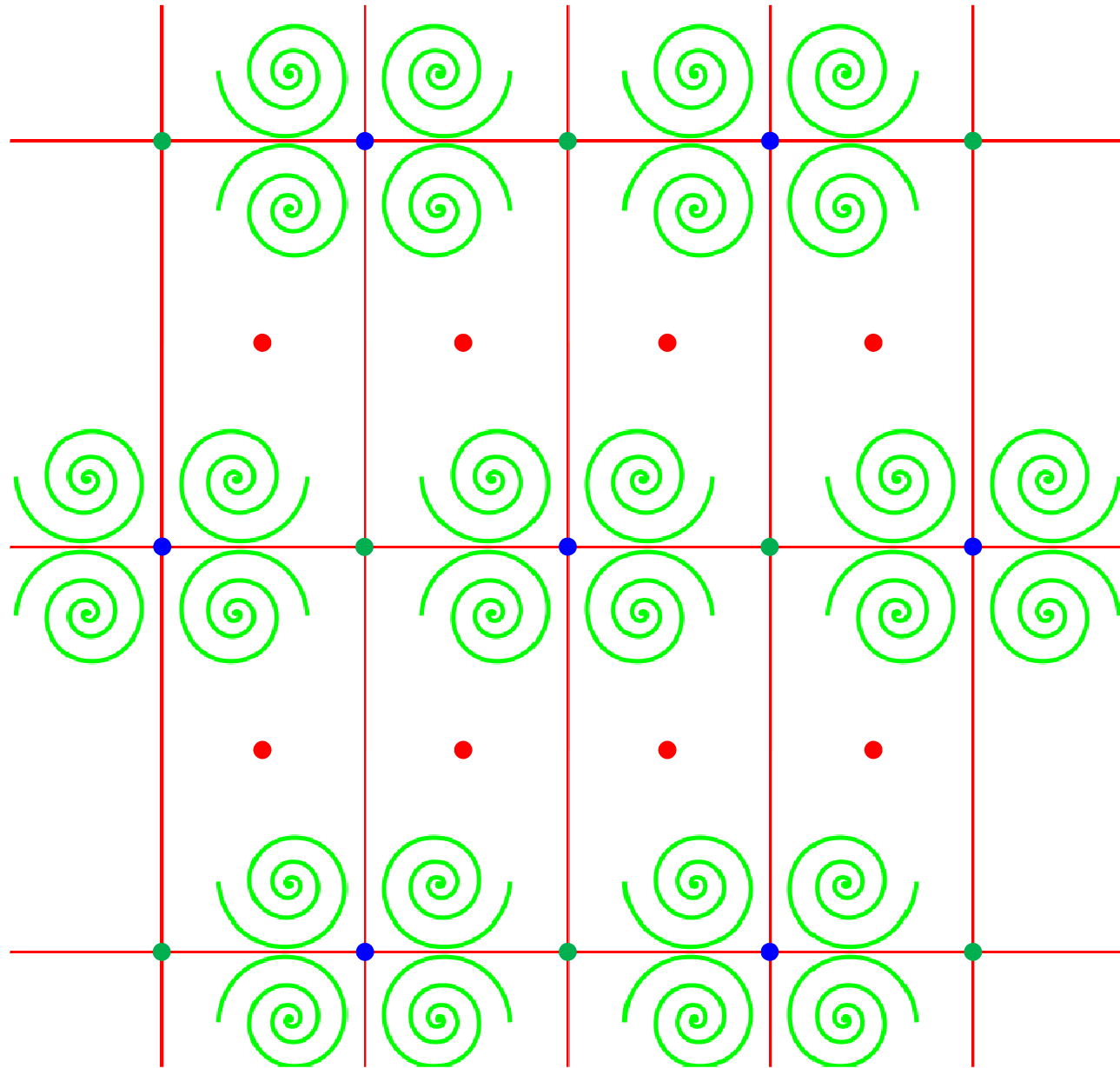


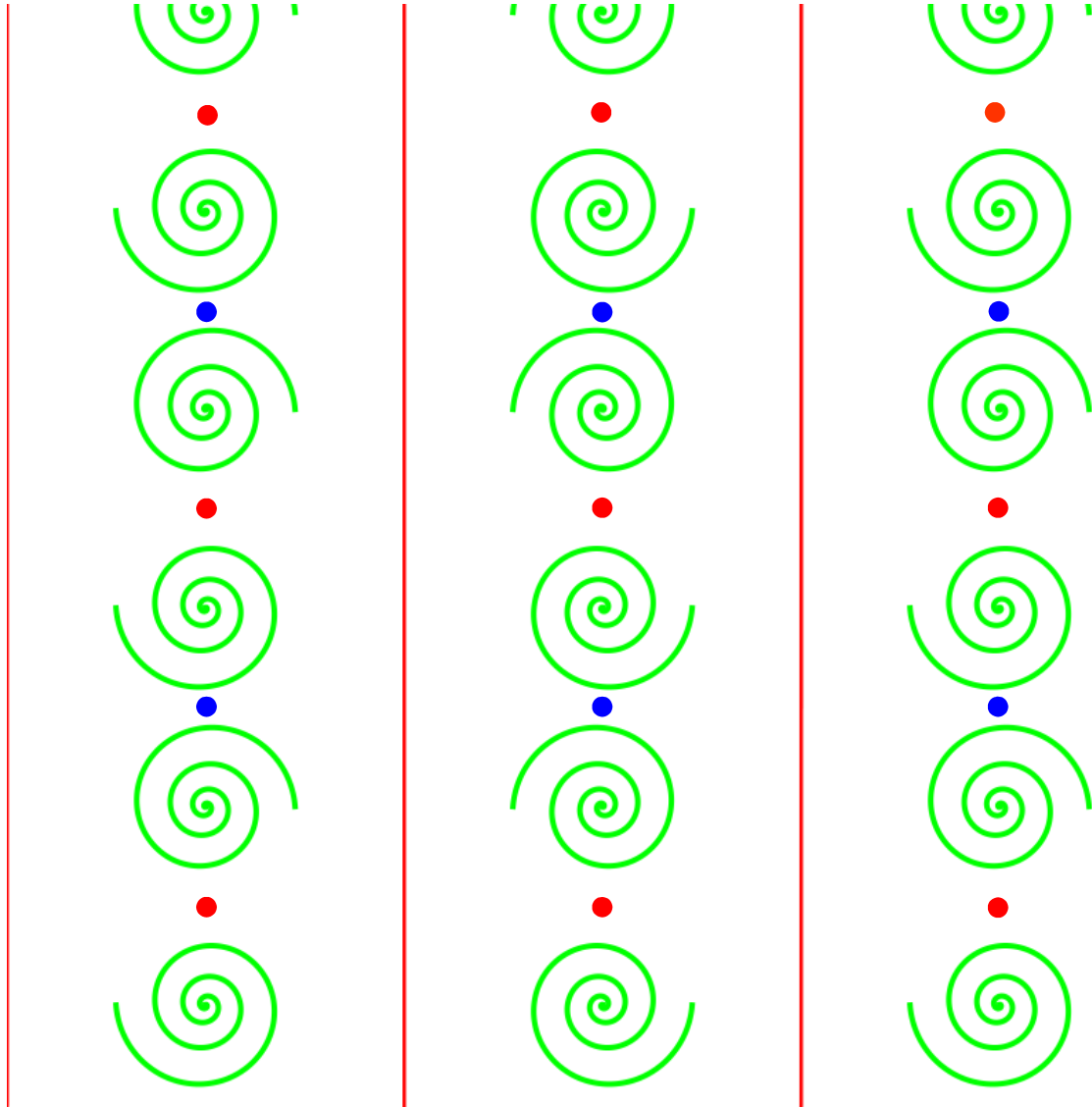




Pavé, pas pris

$2*22 = cmm$

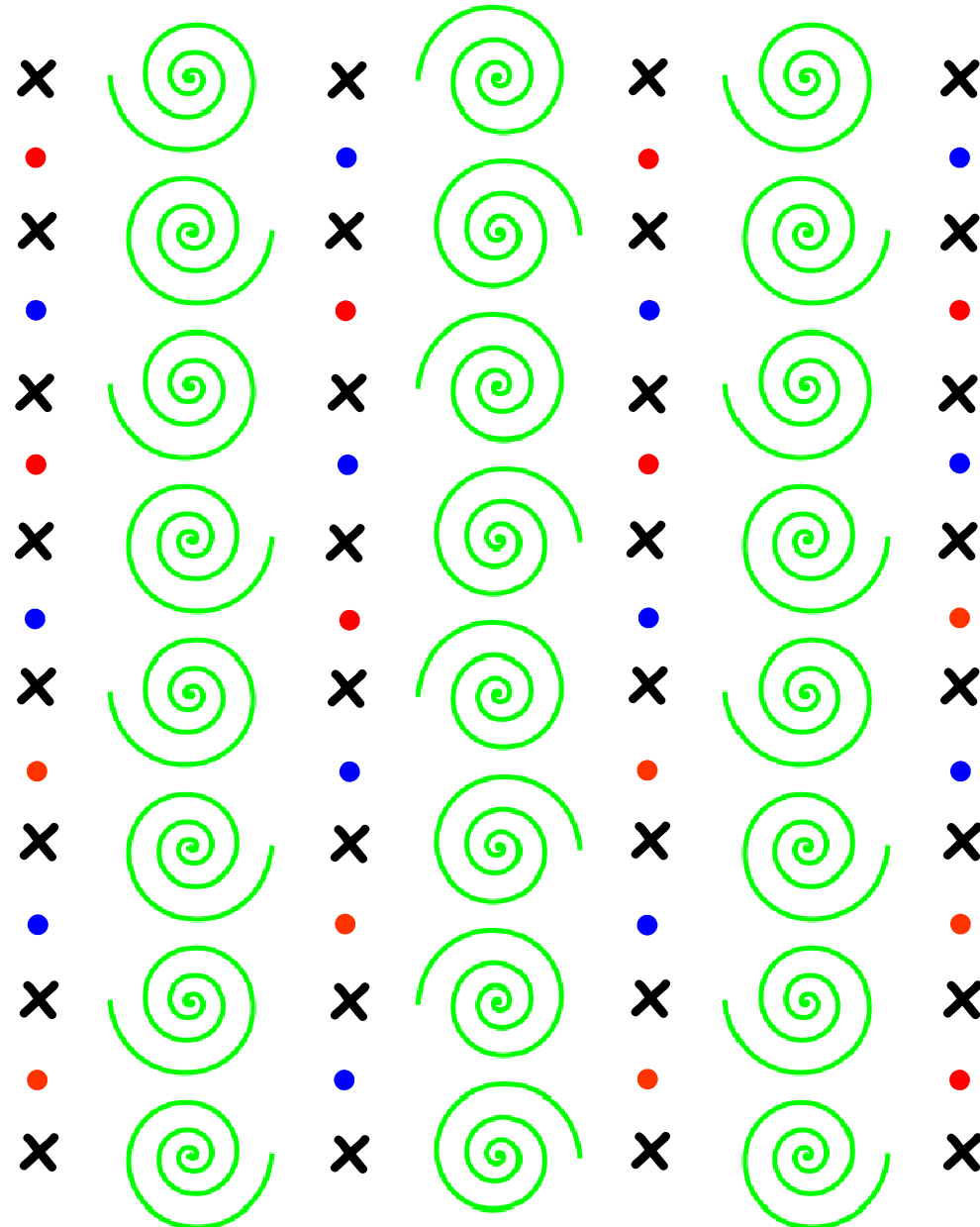






Groupes de pavages

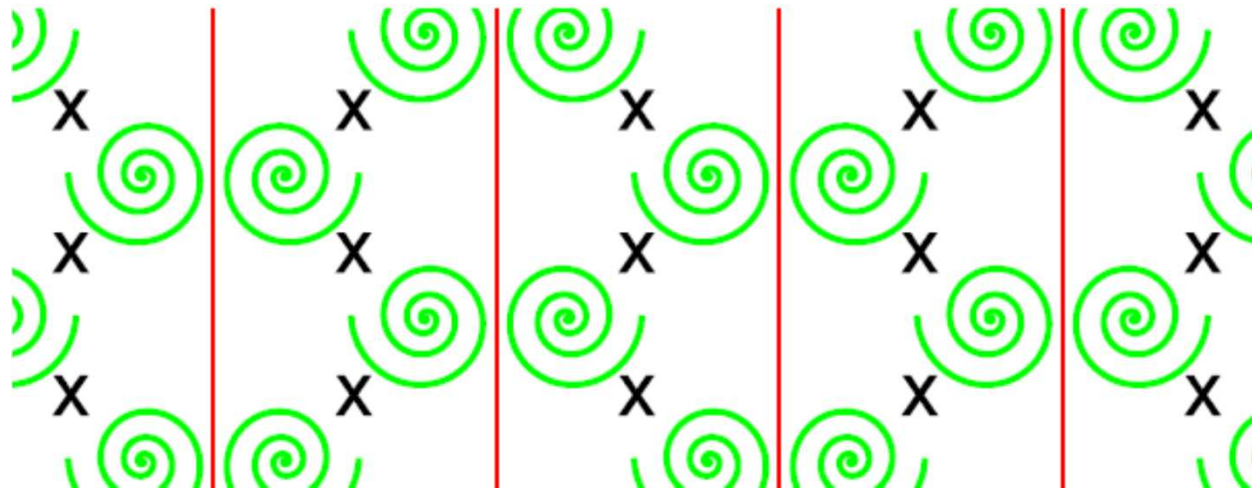
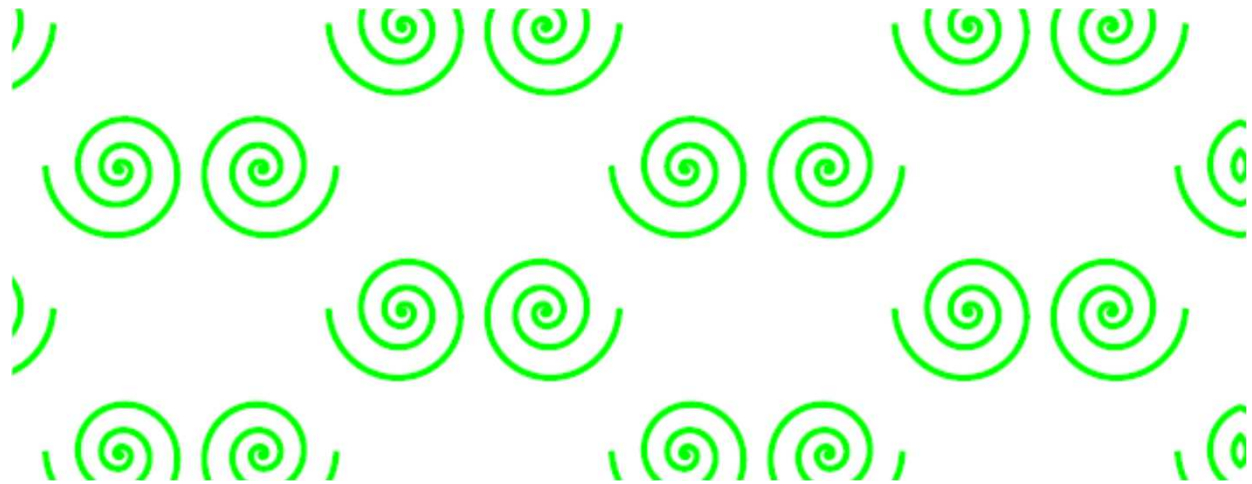
$22x = pgg$

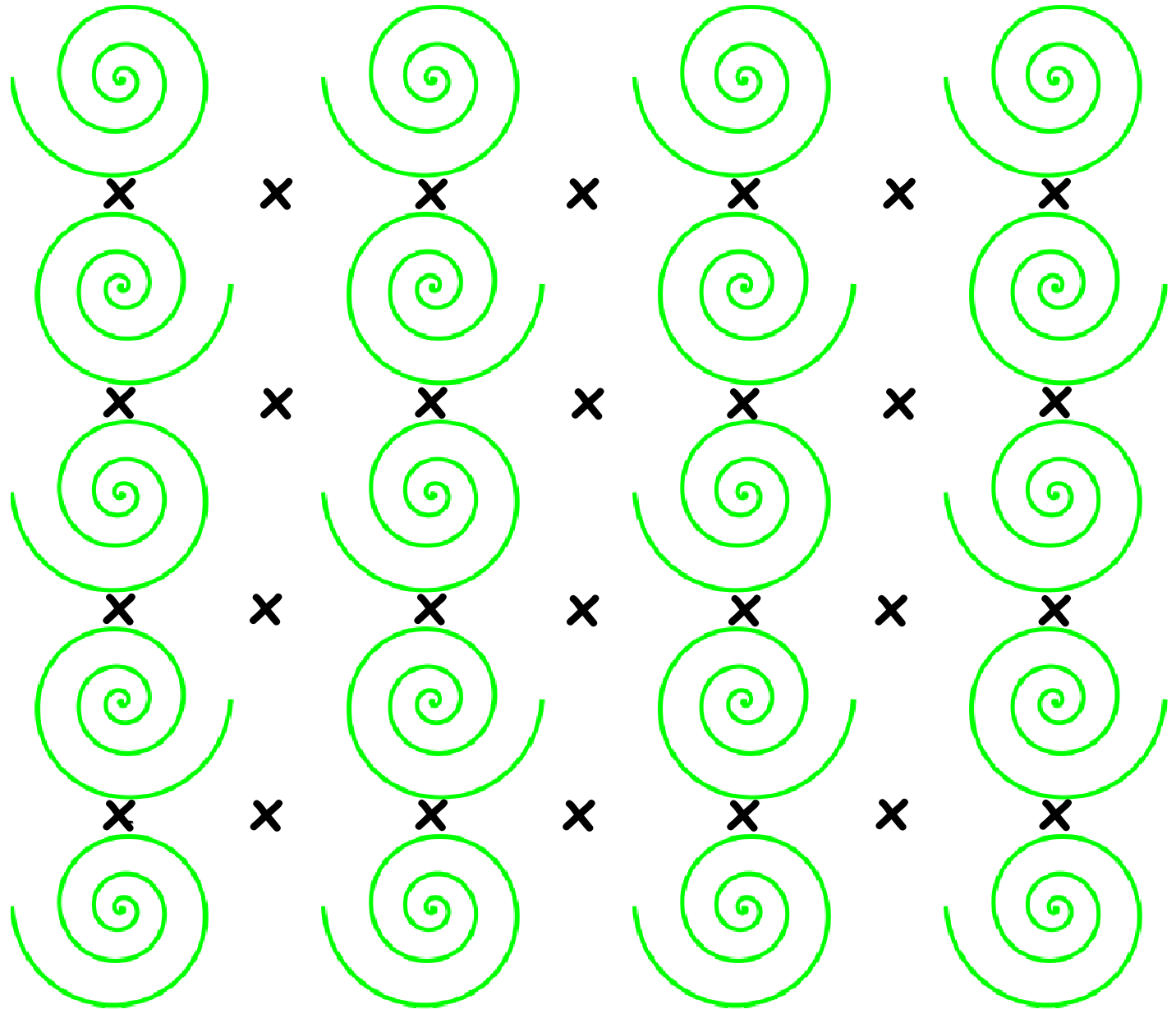


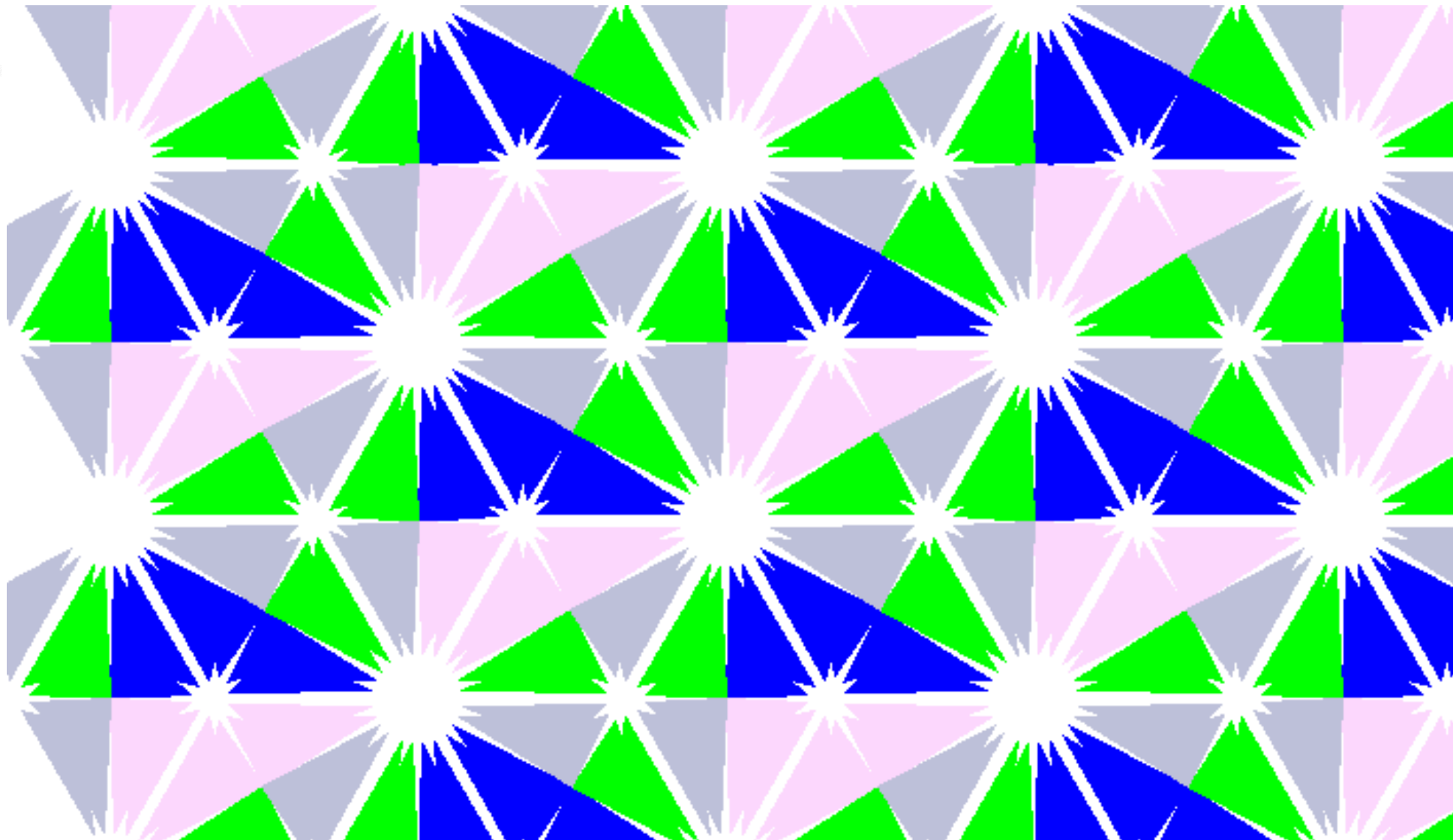


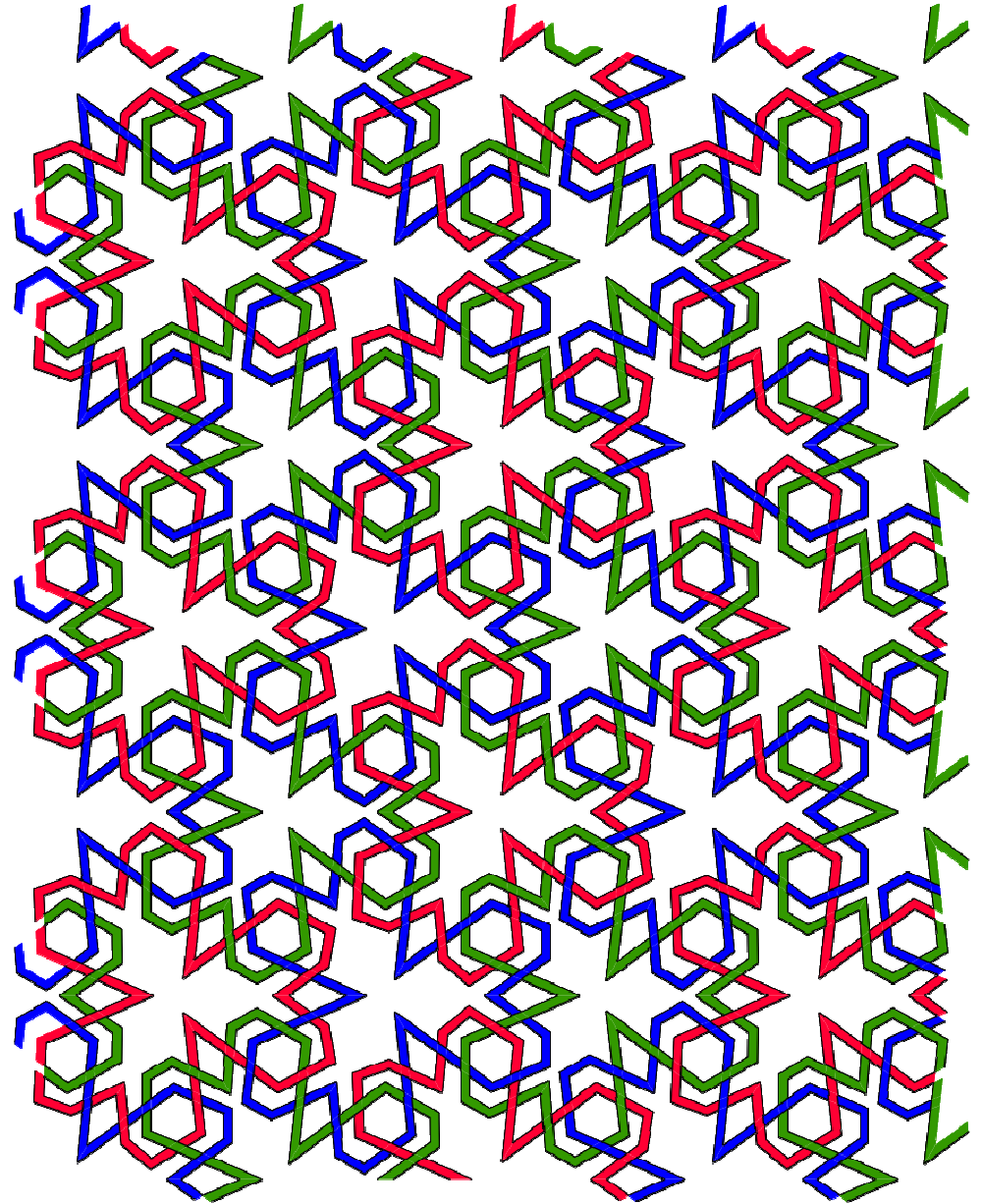
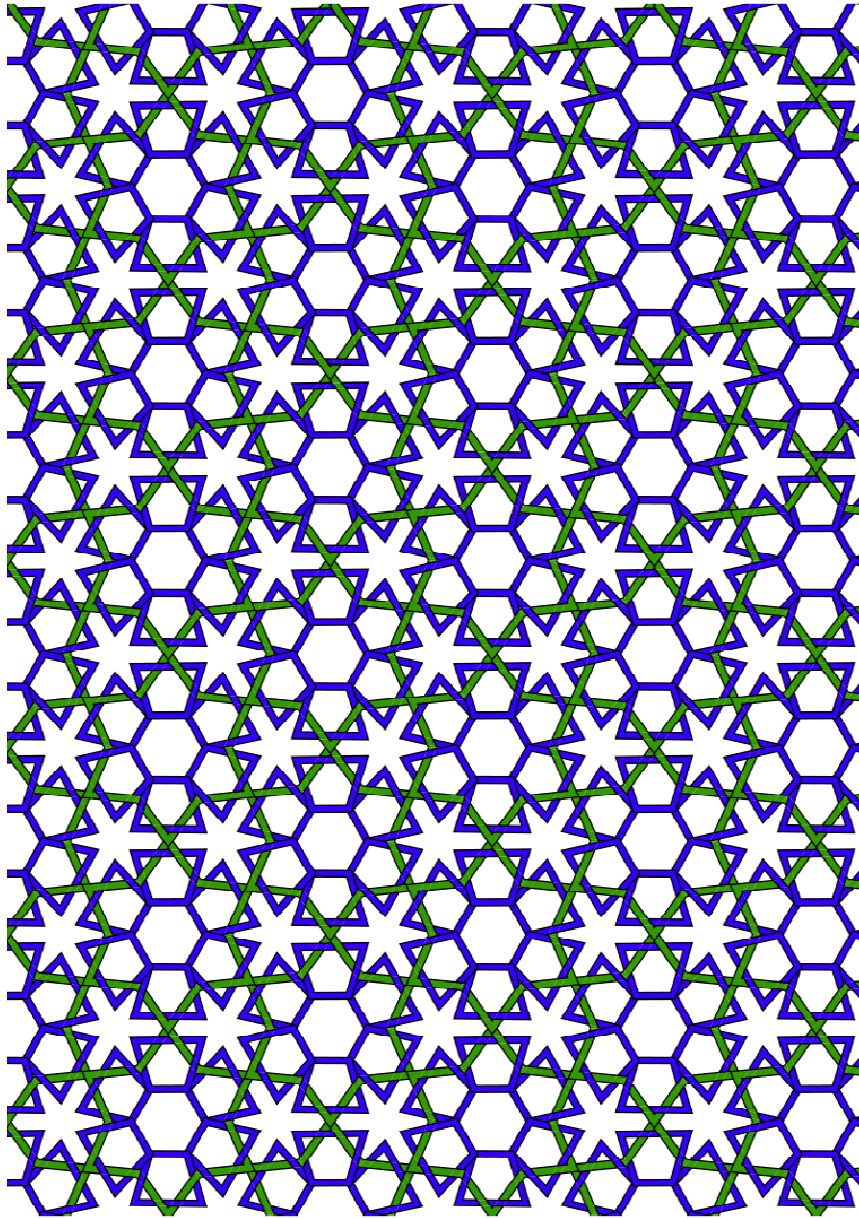
Groupes de pavages

*x = cm











The screenshot shows a software window titled "Tiling:p3m1:". The interface is split into two main areas. On the left, a large triangle is shown with a smaller purple triangle inside it, and a green line segment is drawn across it. Below this is a toolbar with several icons for drawing and editing. On the right, a repeating pattern of blue and green lines is shown, forming a complex geometric design. The pattern consists of interlocking shapes that create a series of white star-like voids. The text "Nothing selected" is visible in the center of the interface.



Tiling:p31m:
File Options

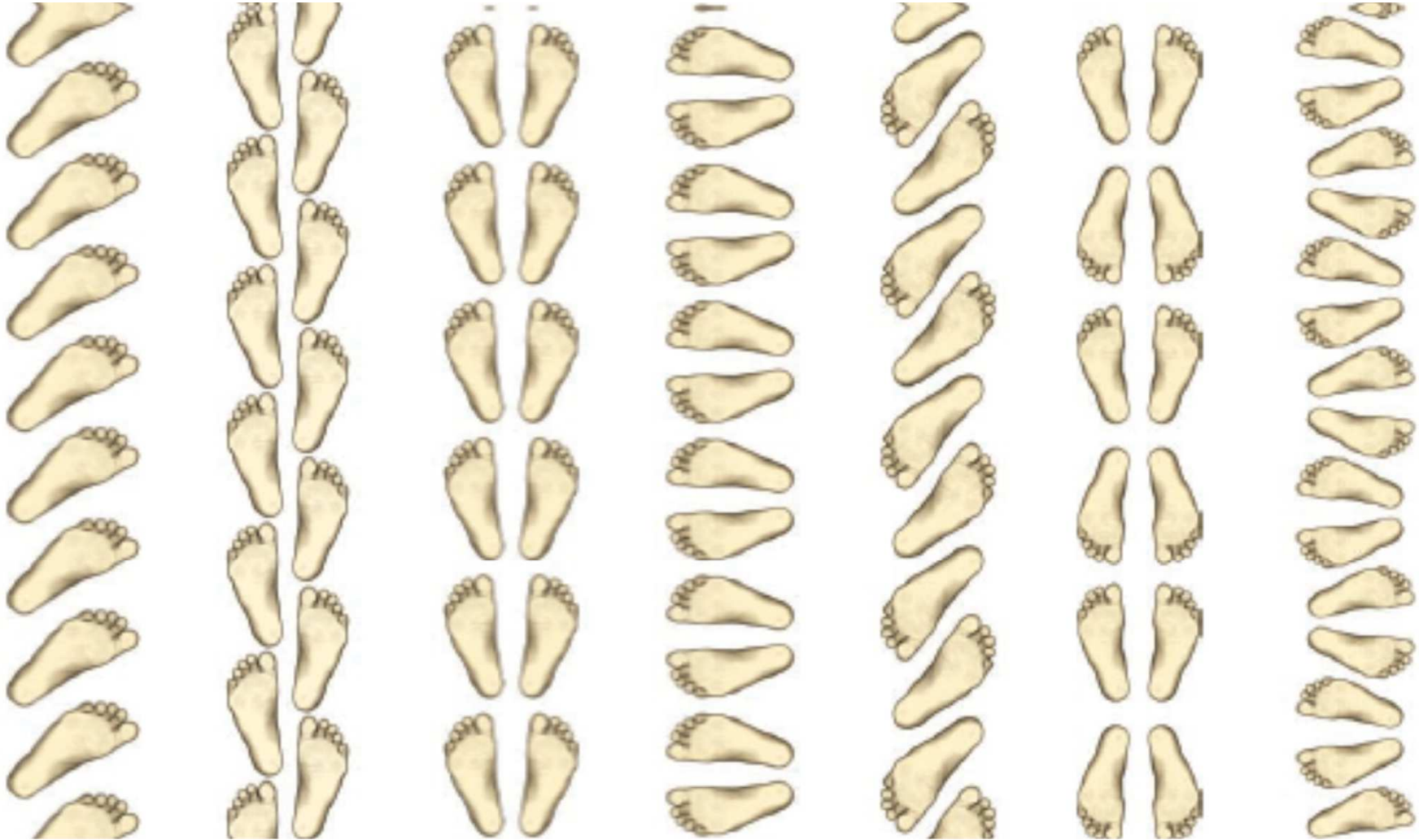
Tools

Selected line
Thick Wide
Thin Narrow
Delete



Plus petit angle de rotation	Contient une réflexion ?			
	Oui		Non	
$360^\circ / 6 = 60^\circ$	$p6m$ (*632)		$p6$ (632)	
$360^\circ / 4 = 90^\circ$	Contient un miroir à 45° ?			
	Oui : $p4m$ (*442)		Non : $p4g$ (4*2)	
$360^\circ / 3 = 120^\circ$	Contient un centre de rotation en dehors des miroirs ?			
	Oui : $p31m$ (3*3)		Non : $p3m1$ (*333)	
$360^\circ / 2 = 180^\circ$	Contient des réflexions perpendiculaires ?			Contient une réflexion glissée ?
	Oui		Non	
	Contient un centre de rotation en dehors des miroirs ?		pmg (22*)	Oui : pgg (22X)
Oui : cmm (2*22) Non : pmm (*2222)				
aucun = 360°	Contient un axe de réflexion glissée en dehors des miroirs ?			Contient une réflexion glissée ?
	Oui : cm (*X)		Non : pm (**)	Oui : pg (XX) Non : $p1$ (O)

7 types de frise





Pavage du plan = étude des sous-groupes finis du groupe des rotations autour d'un point fixe, isomorphe à $O^+(2)$

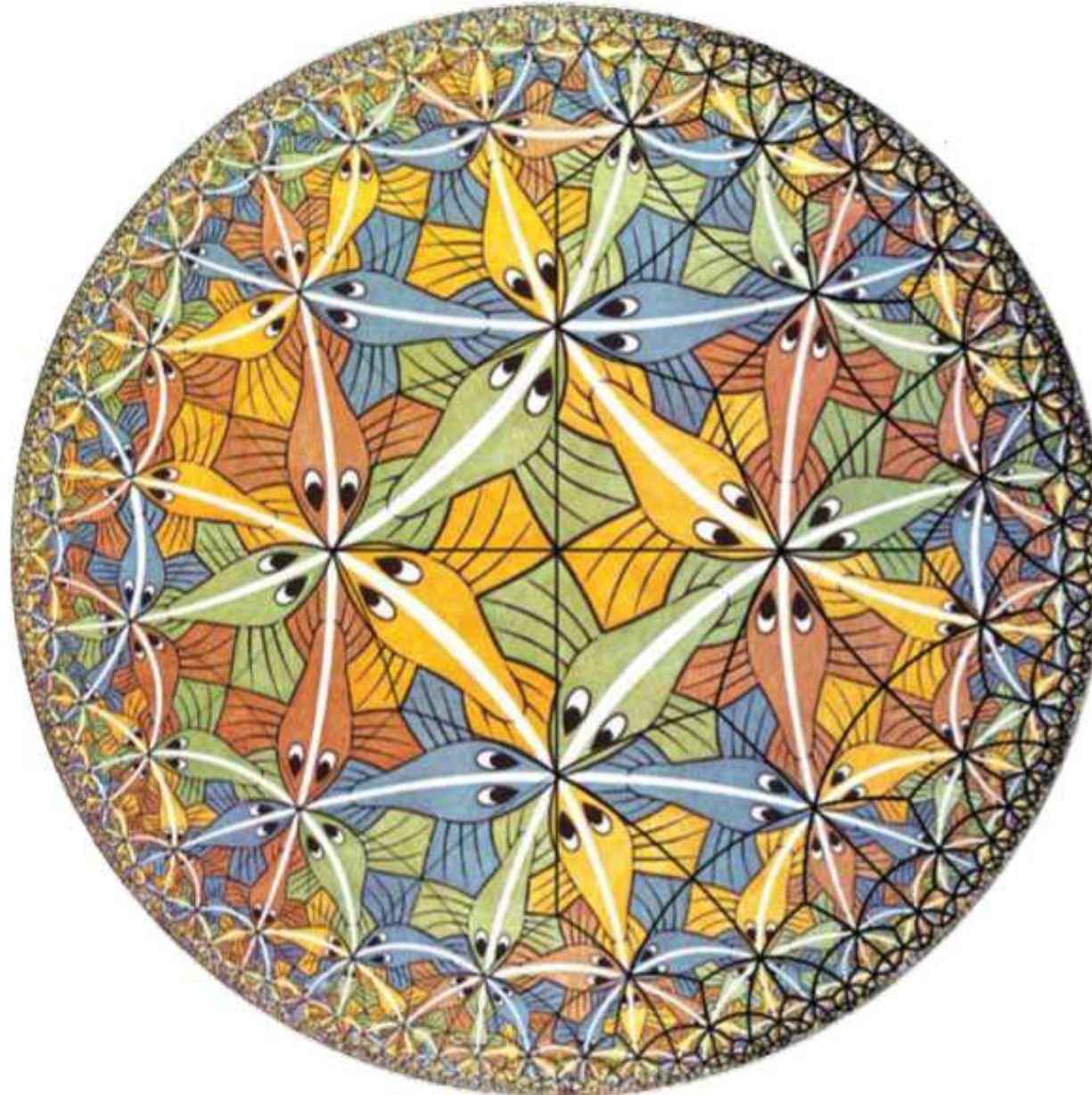
Pavage de l'espace = étude des sous-groupes finis de $O^+(3)$ = étude pavage de la sphère

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 1 \quad (\text{E})$$

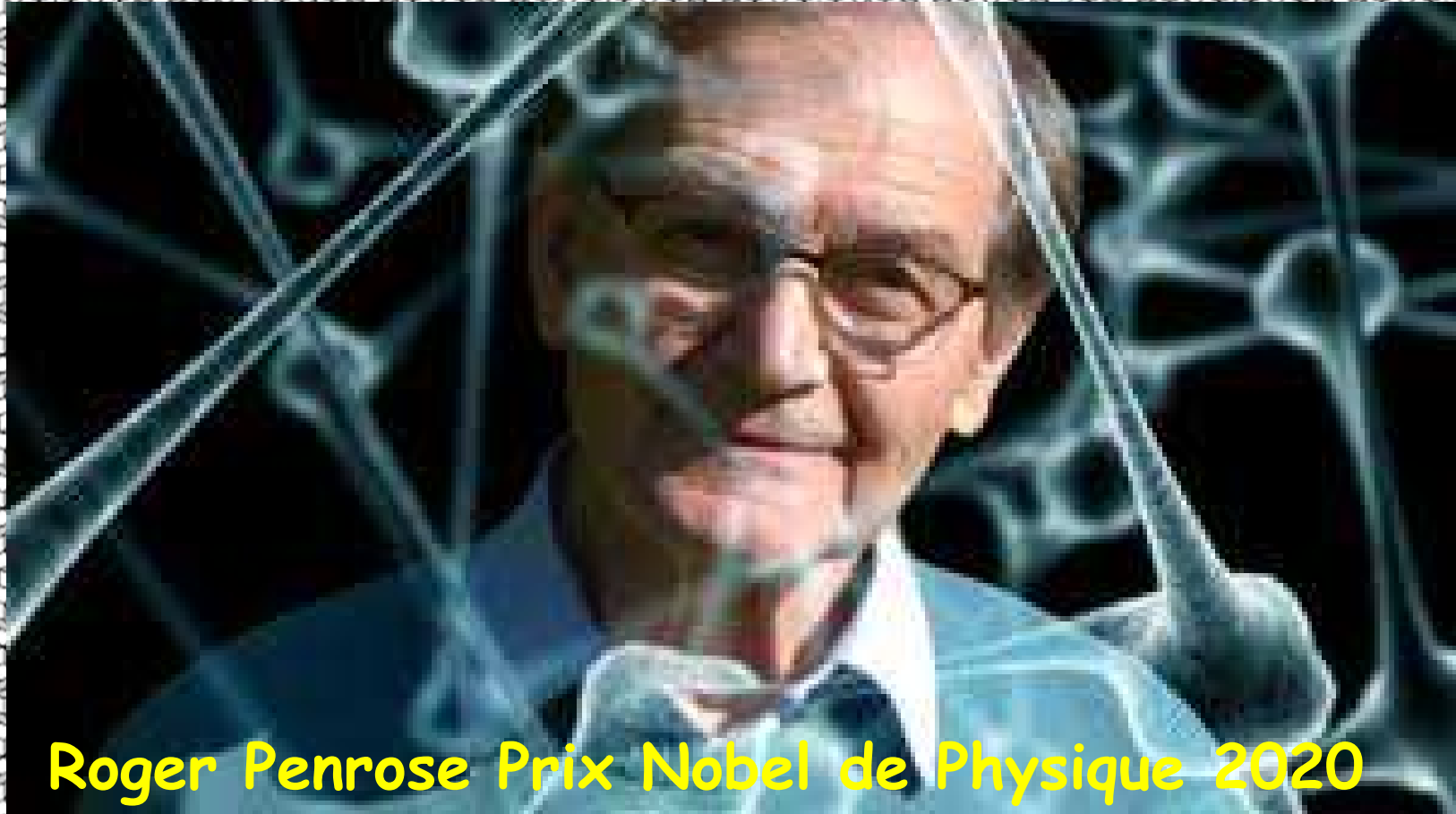
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} > 1 \quad (\text{S})$$

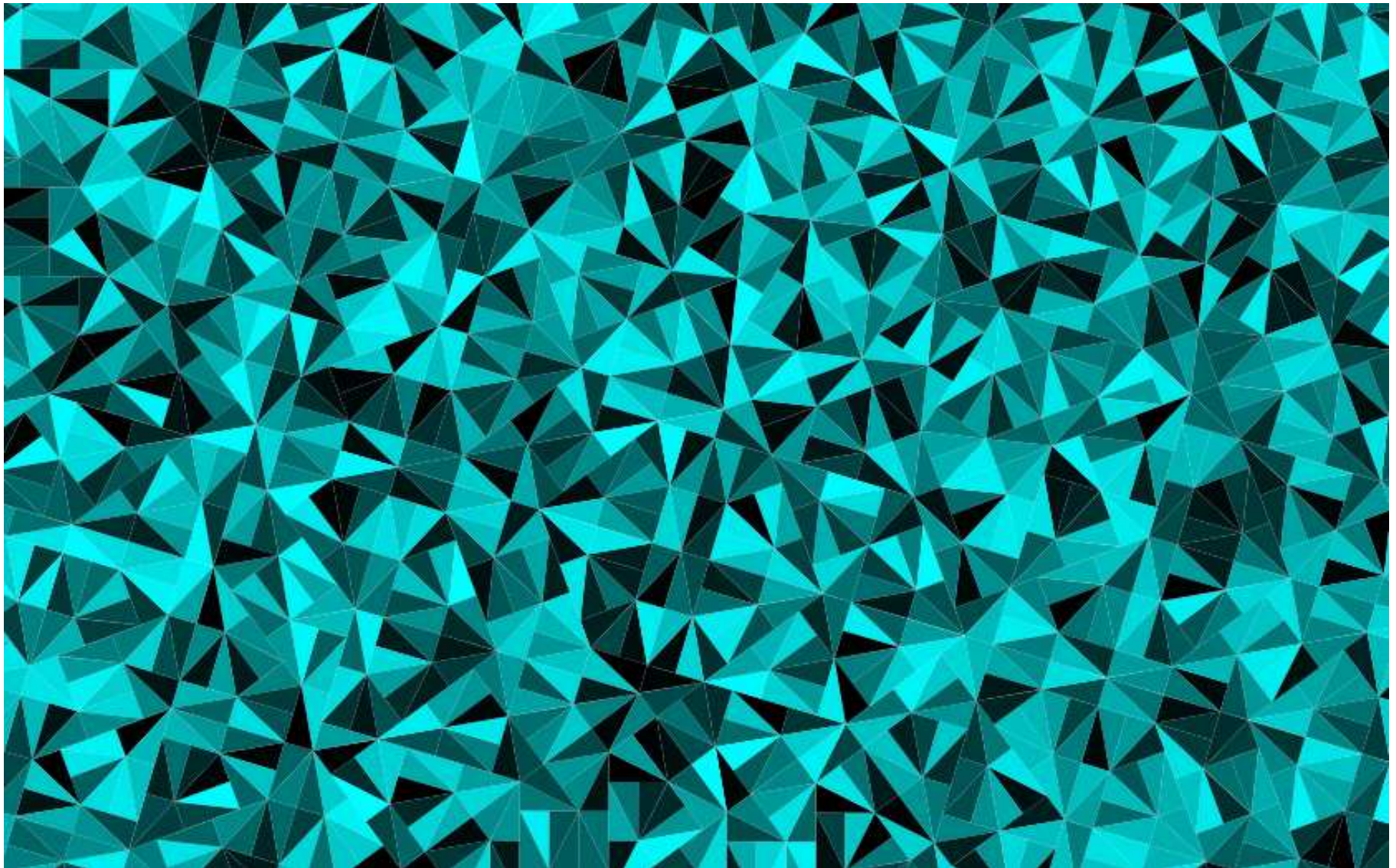
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} < 1 \quad (\text{H})$$





Pavages de Penrose





(Radin, Conway)