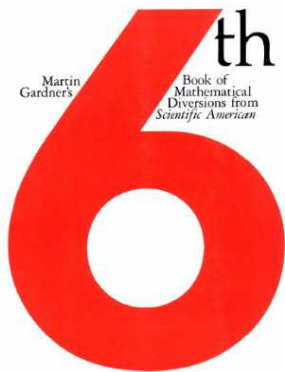


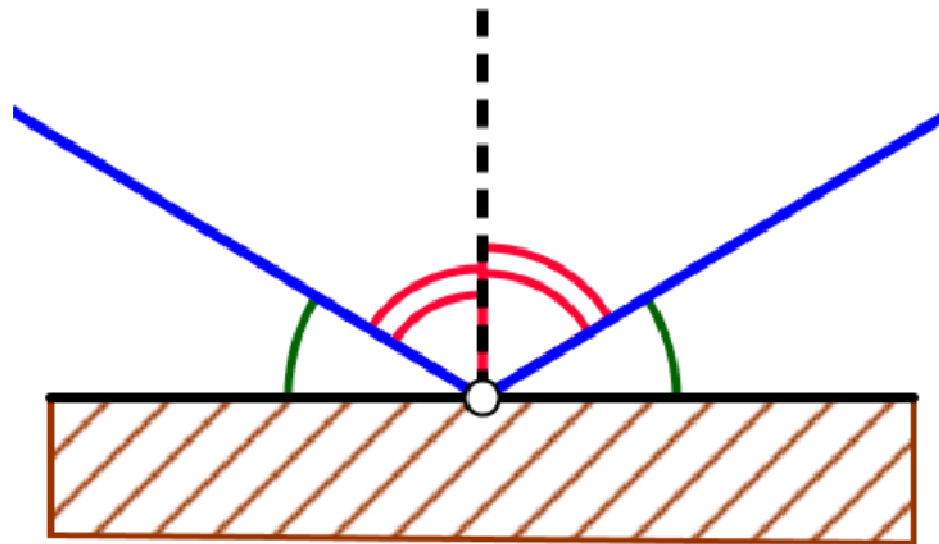


Opérations sur le billard



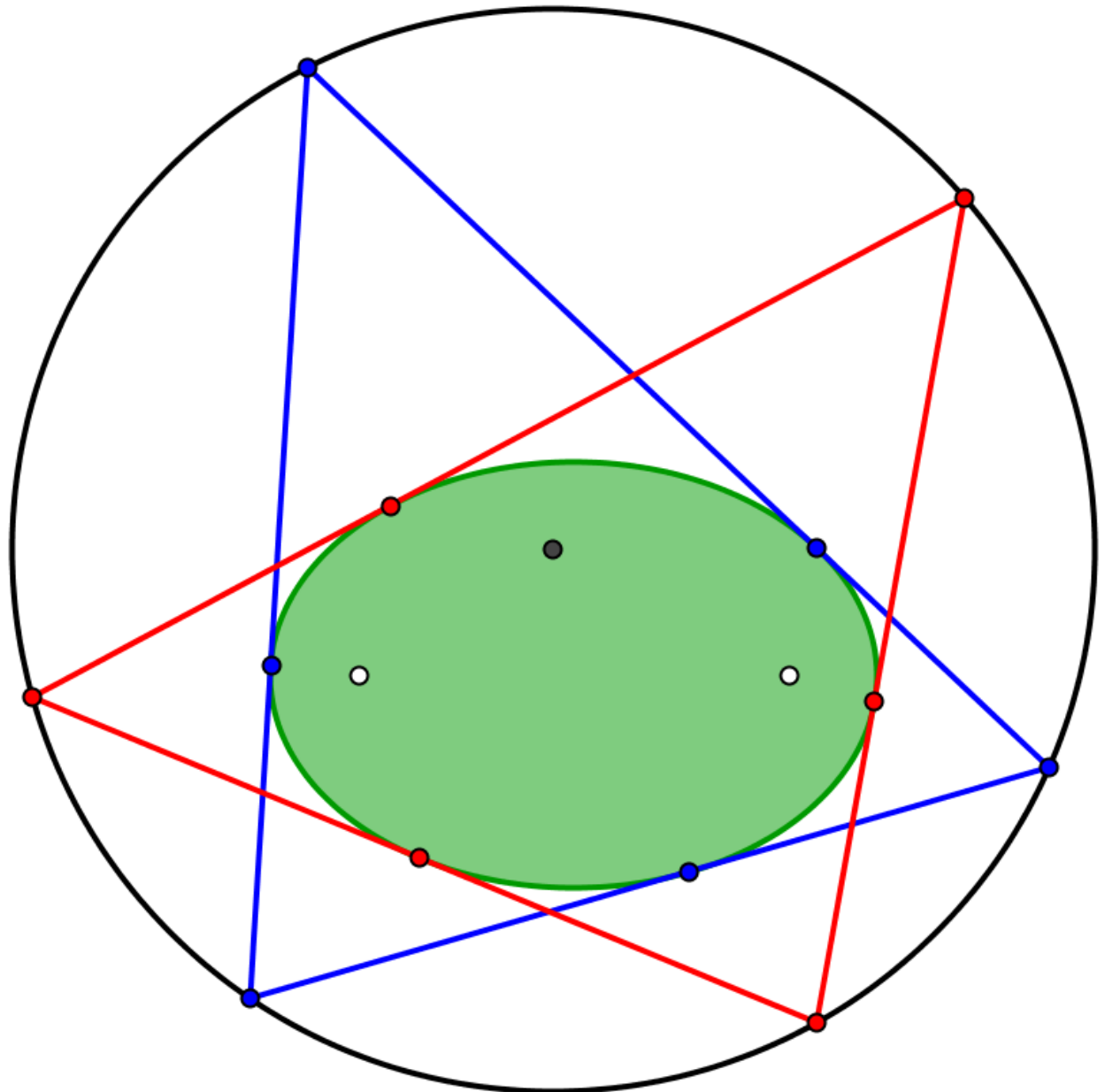


Courbe fermée quelconque Rebond spéc(tac)ulaire d'une bille ponctuelle

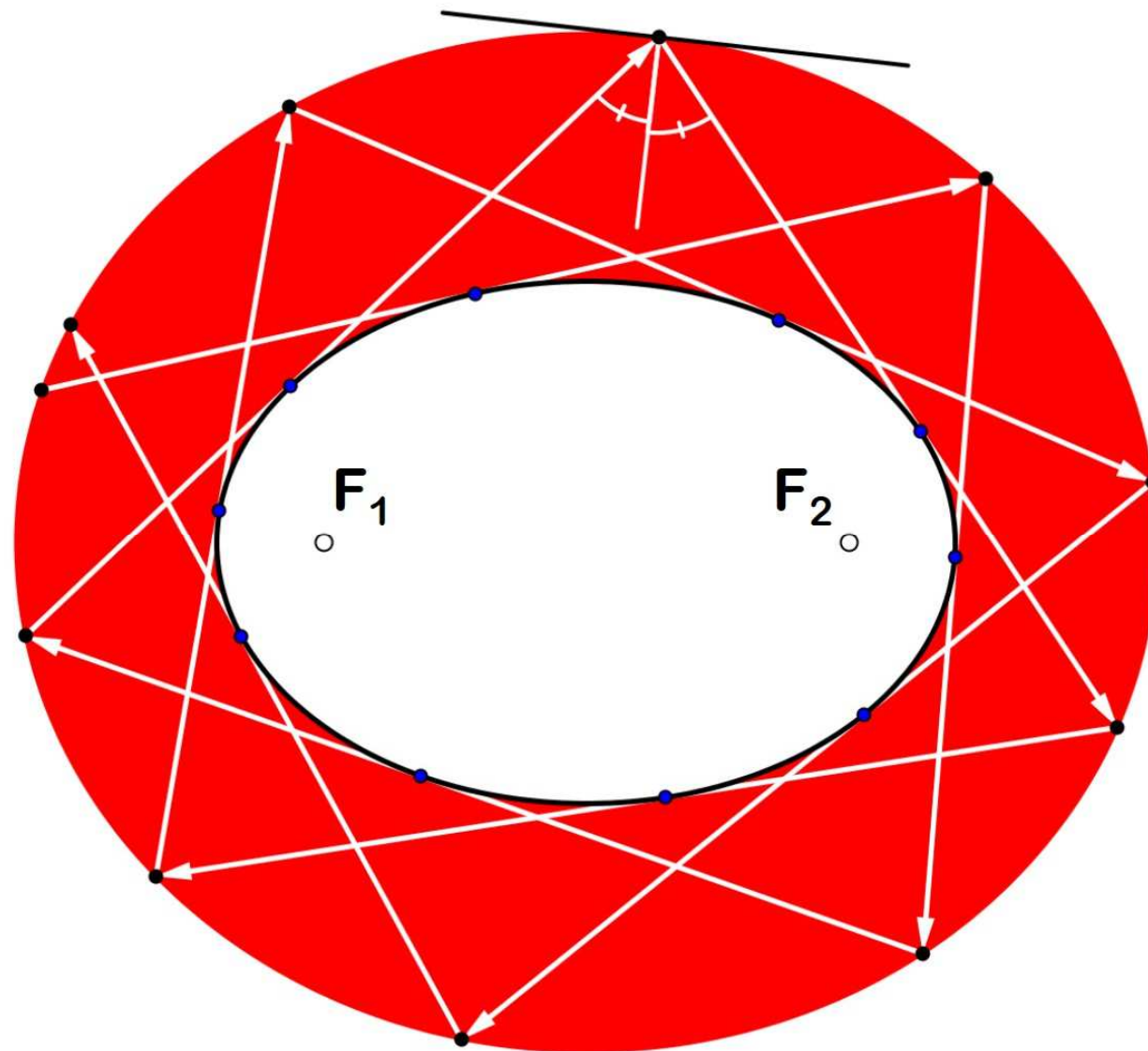




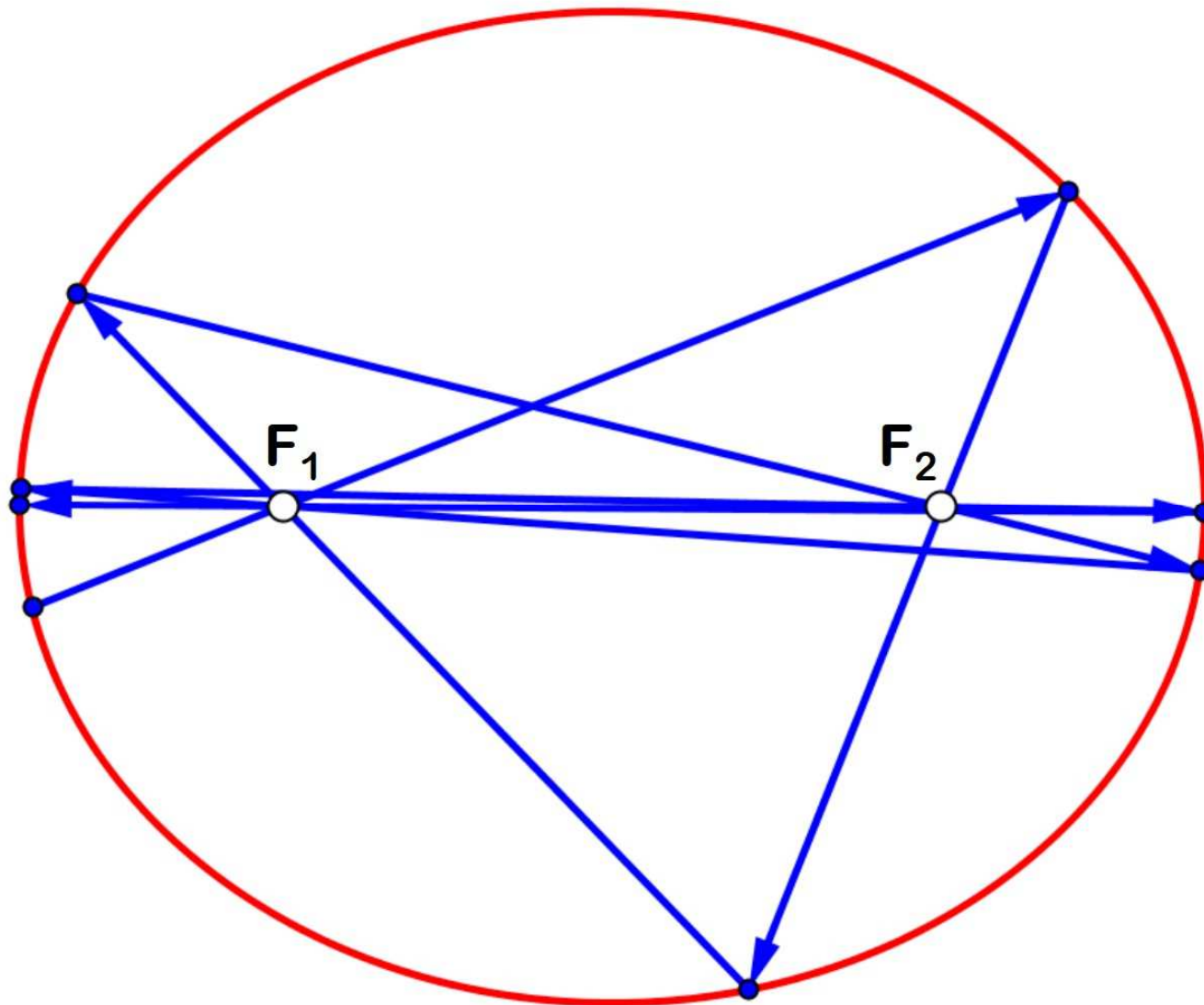
Porisme:
Propriété
géométrique
indépendante des
conditions initiales
(Poncelet, Steiner)



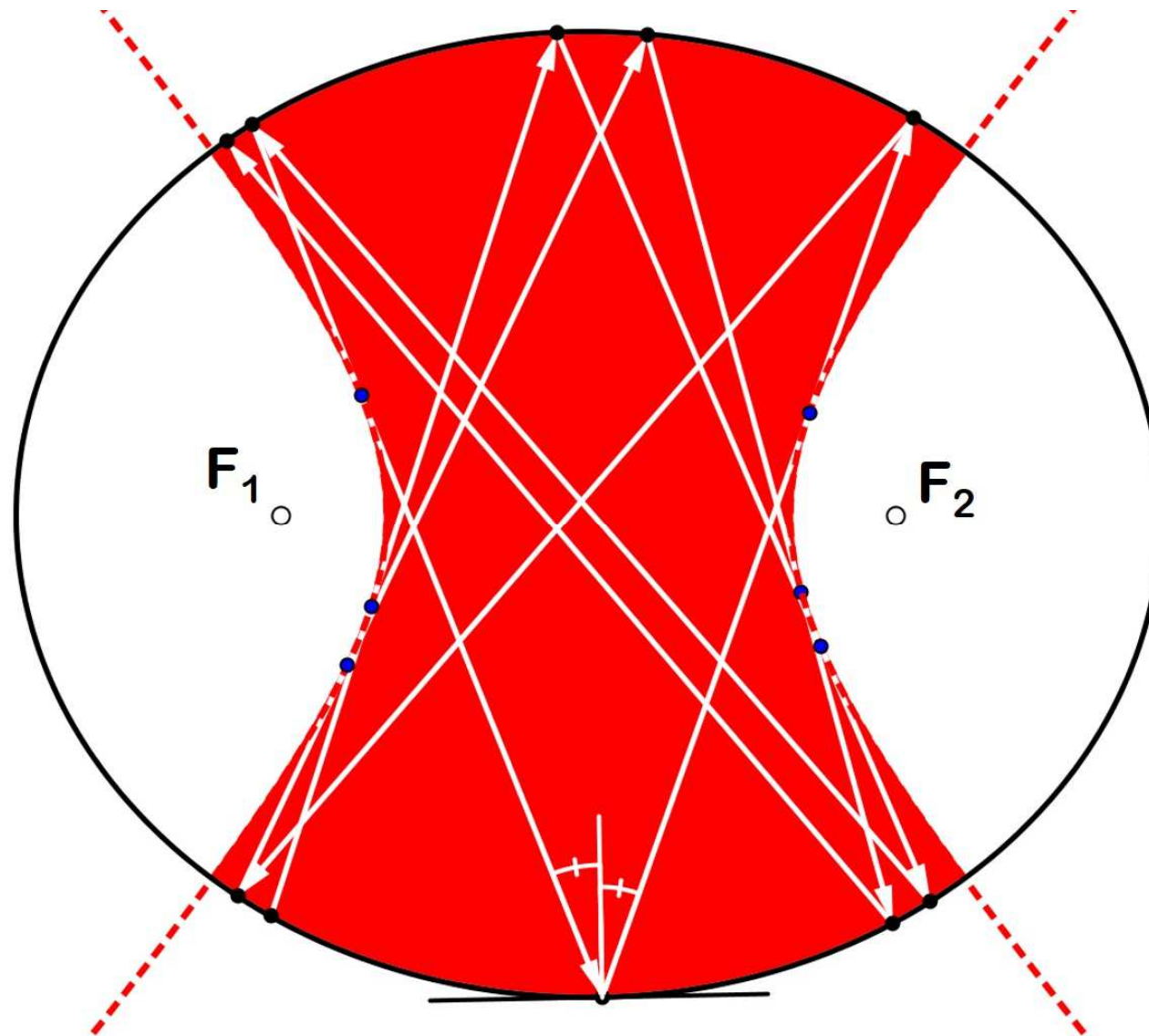
Des billiards de trajectoires



Des billiards de trajectoires



Des billiards de trajectoires



Trajectoires périodiques

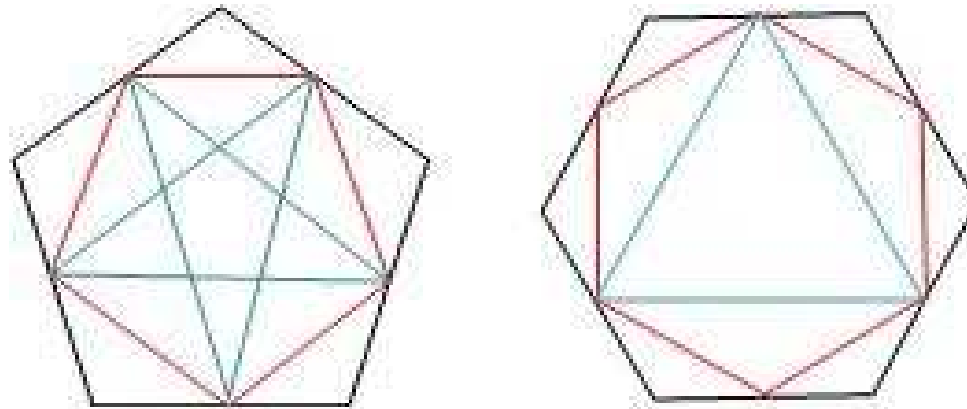


Théorème de Birkhoff:
Pour tout couple d'entiers (m, n) tels que $0 < m < n$,
il existe au moins deux lignes polygonales à n sommets
et de nombre de rotation m qui soient des trajectoires
périodiques du billard (*bien gentil*).

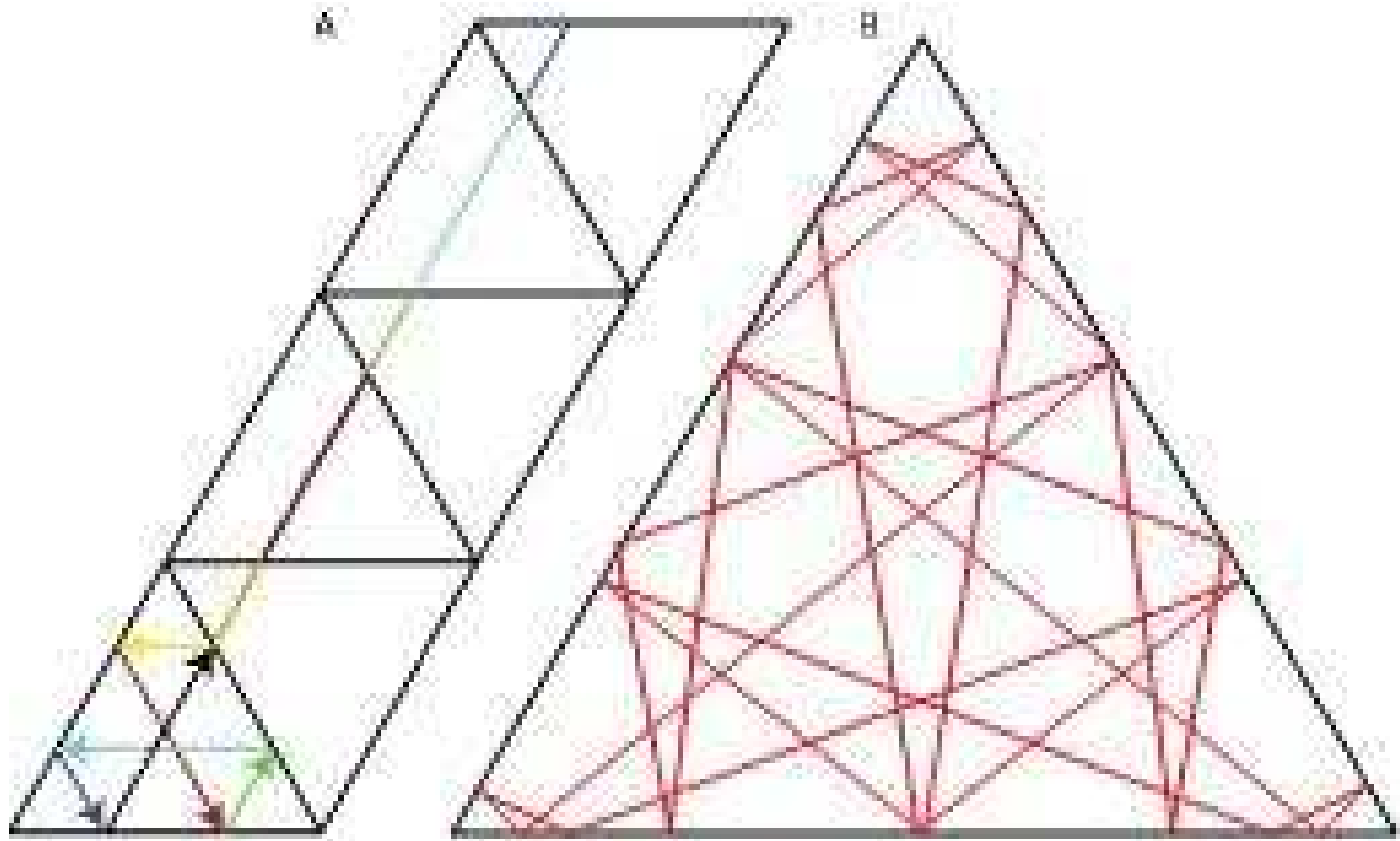
Trajectoires périodiques

Howard Alan Masur (1986):

Il existe une infinité de trajectoires périodiques dans tout polygone rationnel.



Trajectoires périodiques



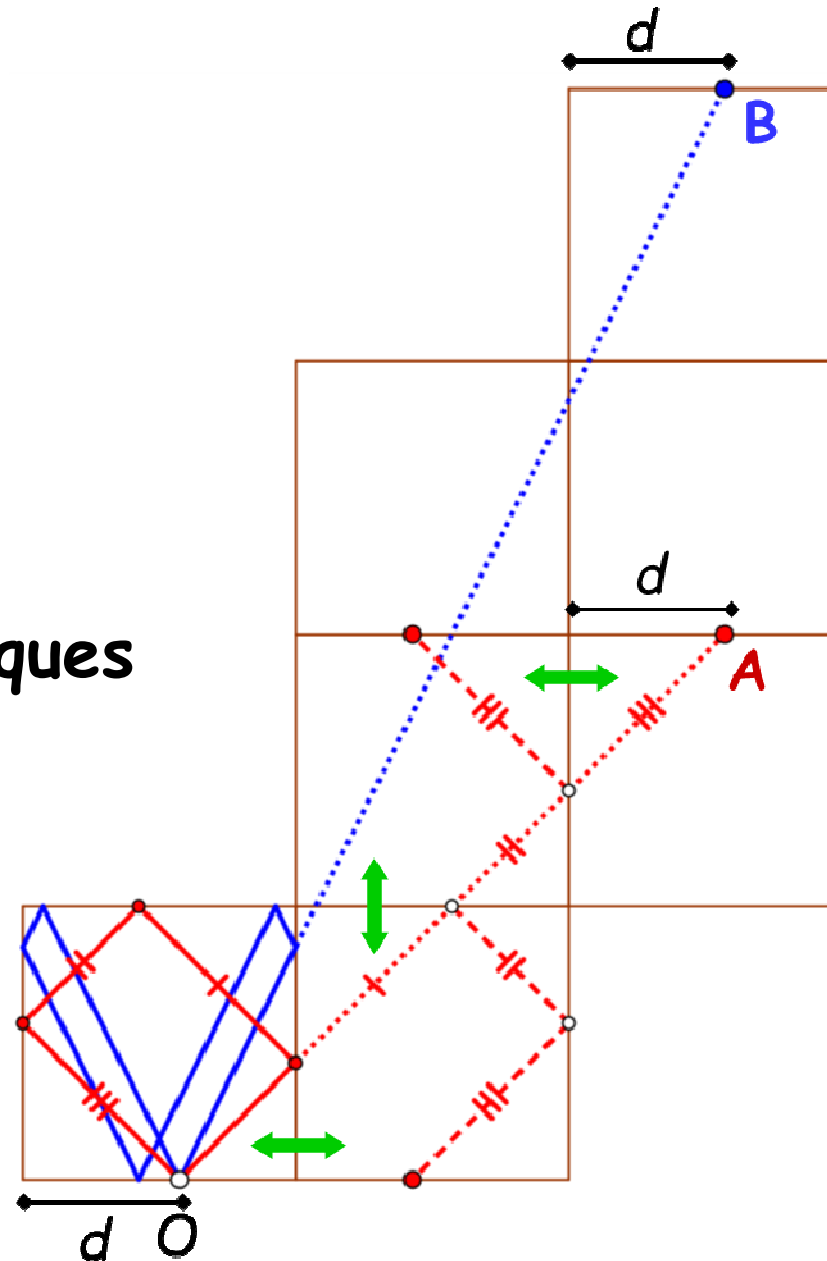
6 rebonds

22 rebonds

Trajectoires périodiques



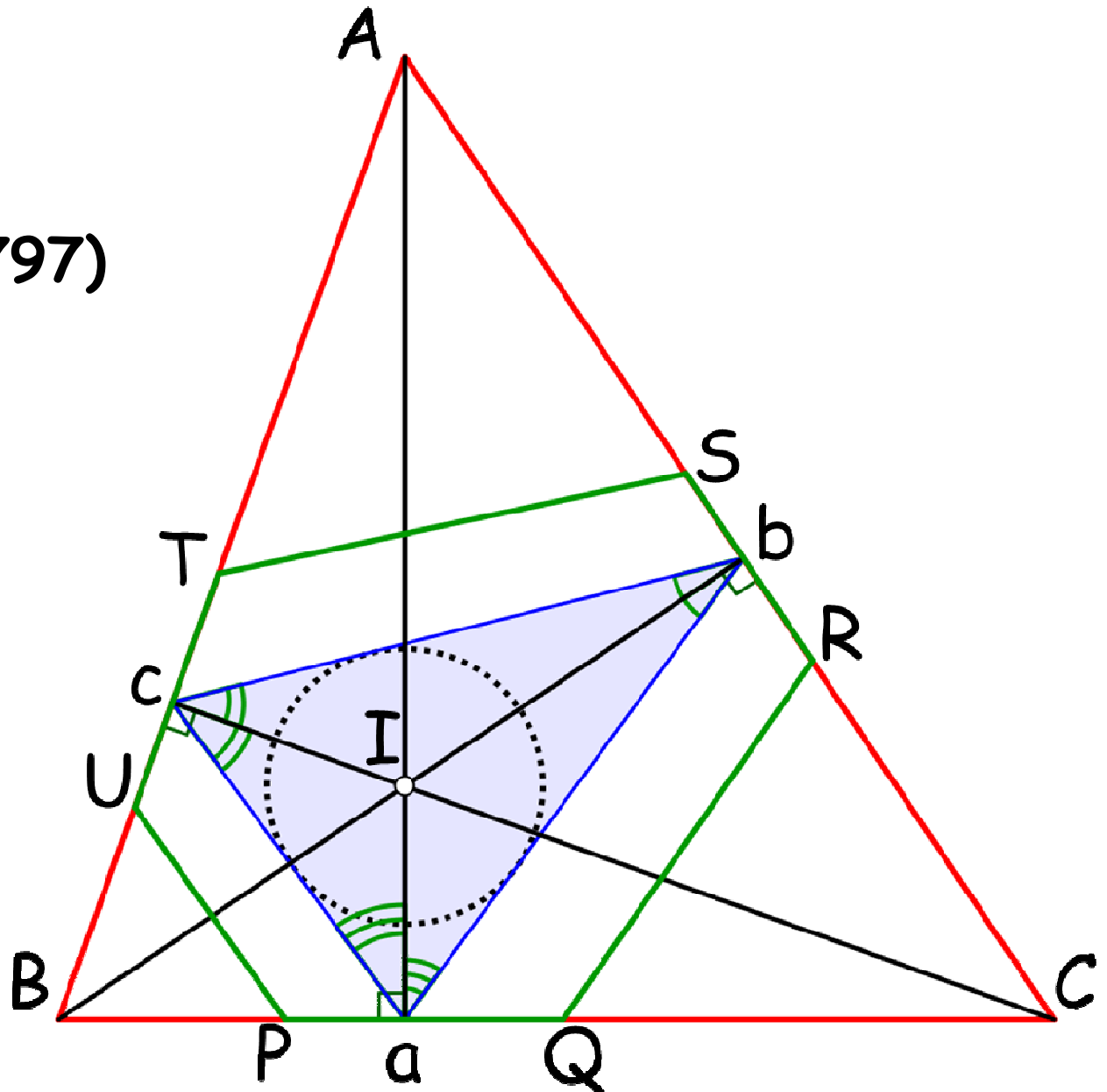
Déploiement
de
trajectoires périodiques



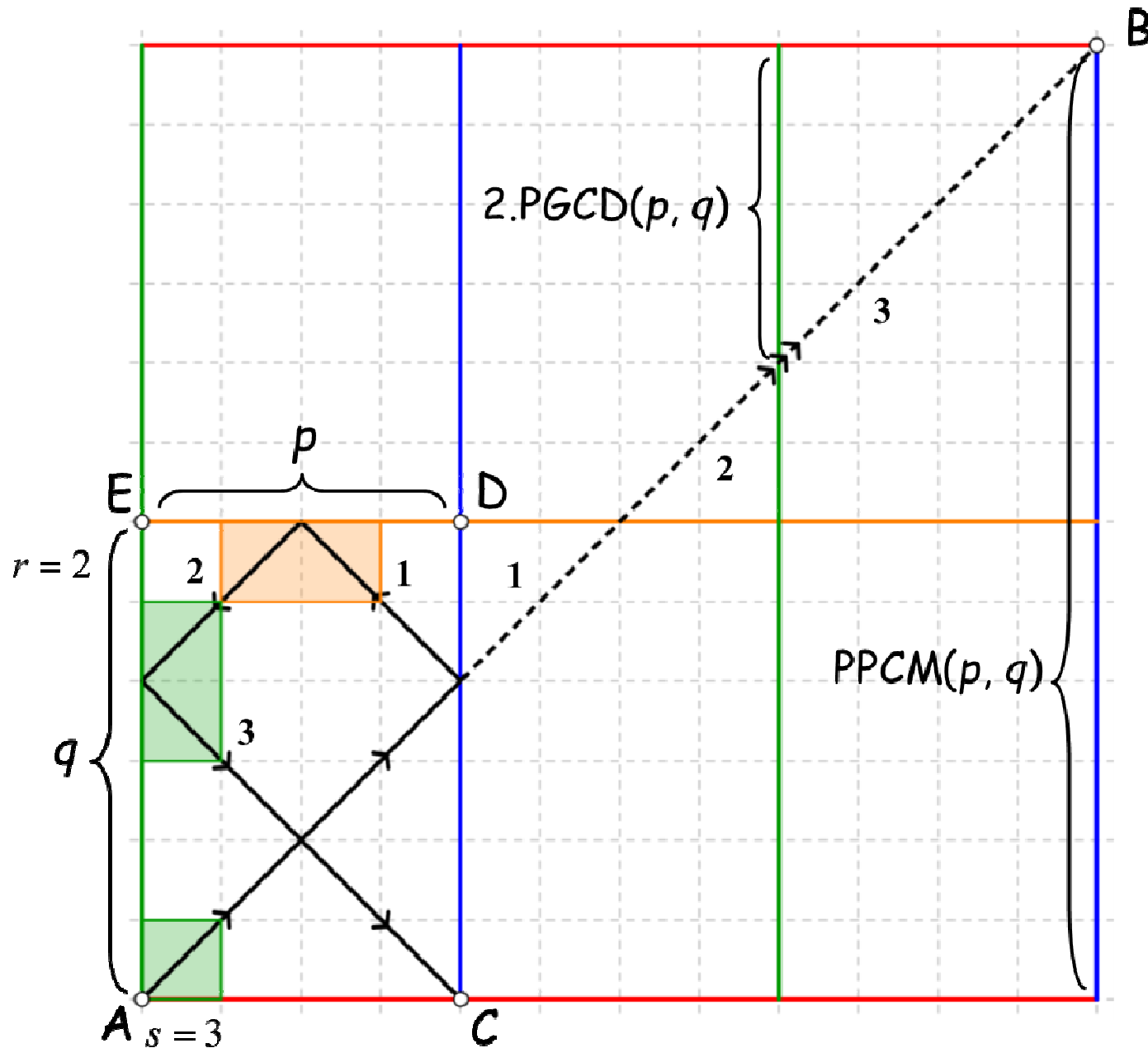


Triangle aigu:
Triangle orthique
Fagnano (1715-1797)

Triangle obtus:
Récompense
A. Katok (1944-2018)



Trajectoires encore billards



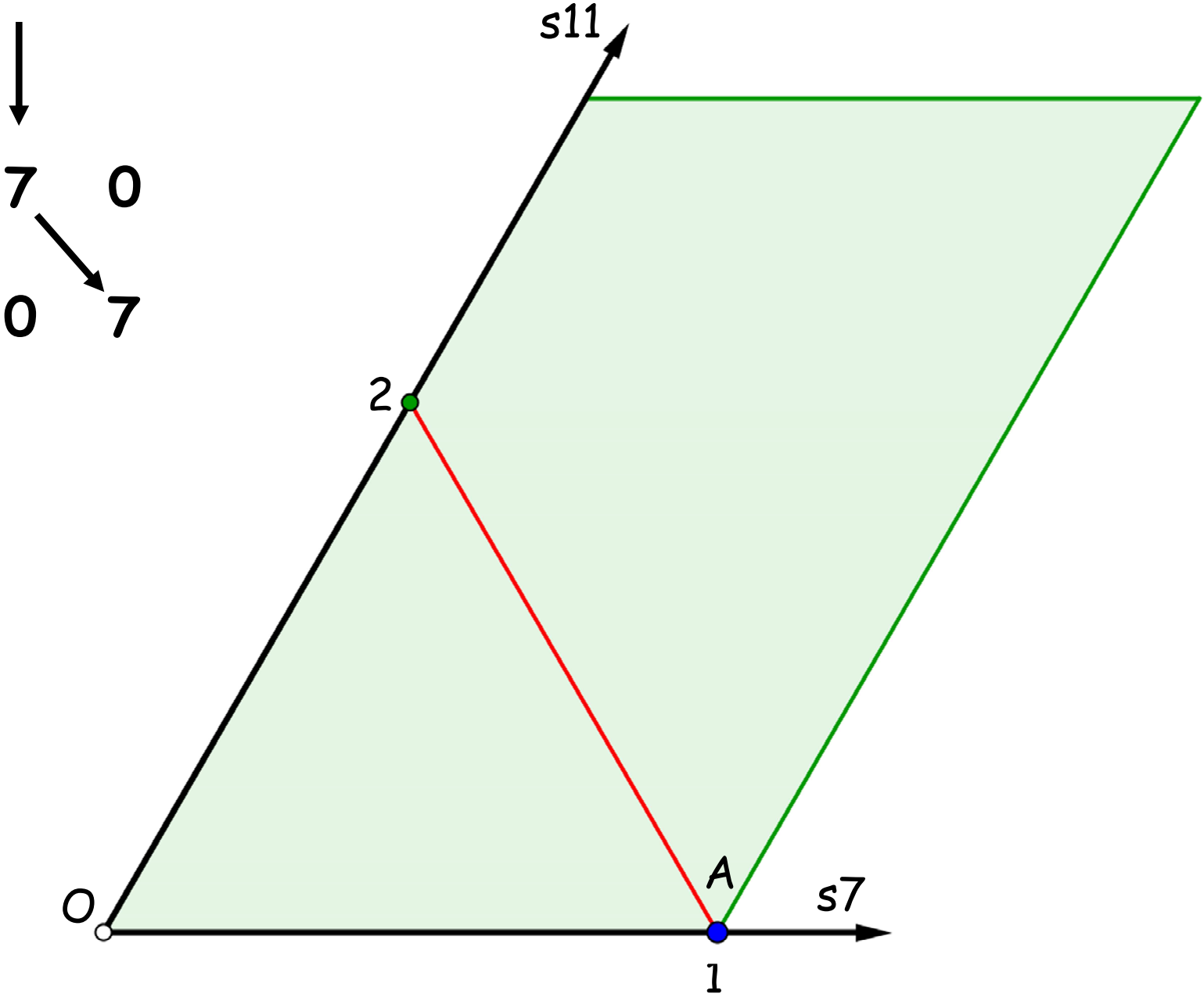


Problème ancien (*Martin dixit*):

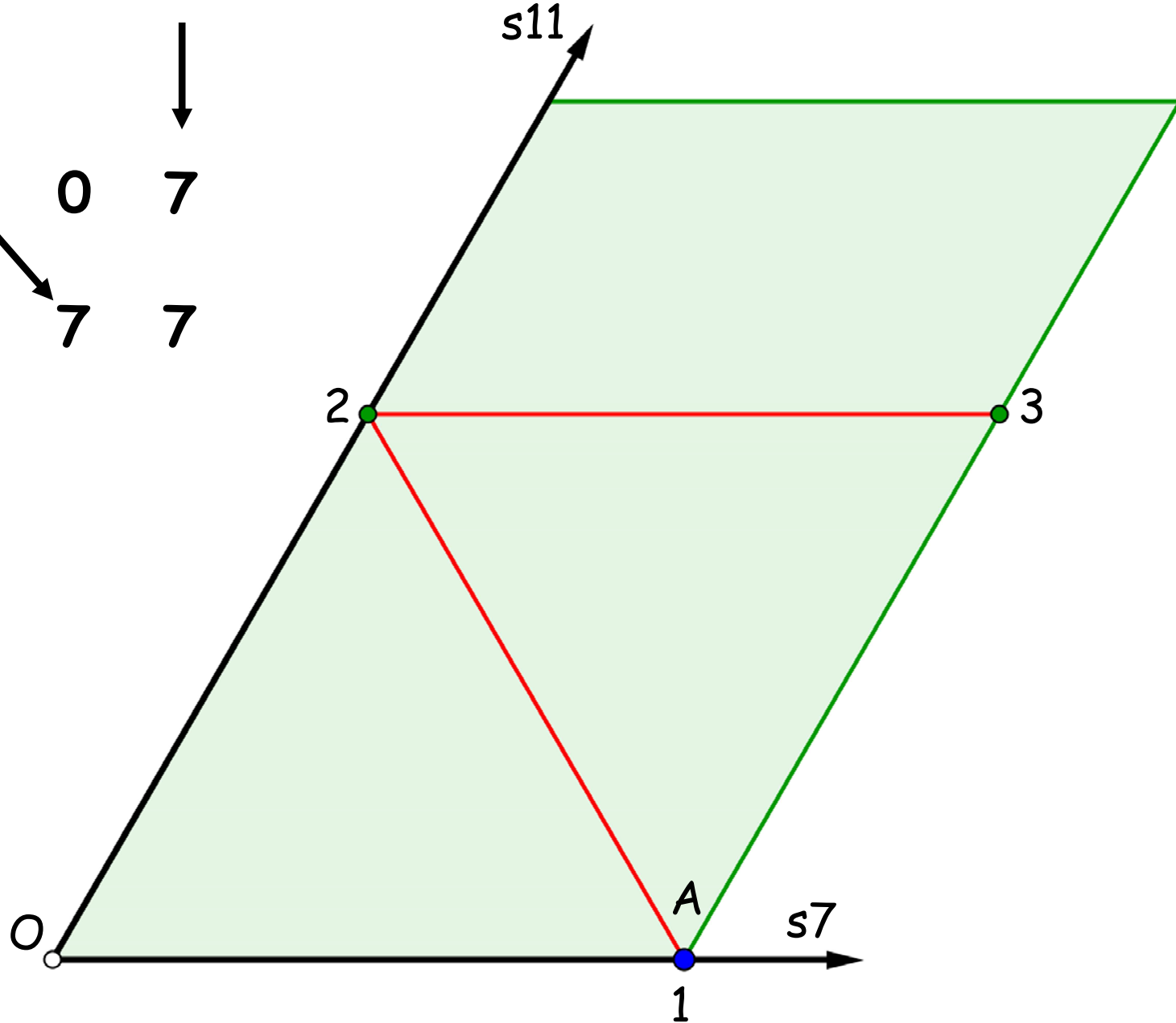
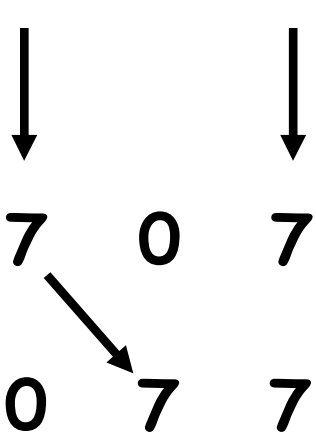
**Deux récipients de capacité 7 et 11 litres.
Comment obtenir exactement 2 litres?
(pas de marques, eau à volonté)**

Explication graphique de M.C.K. Tweedie (1939)

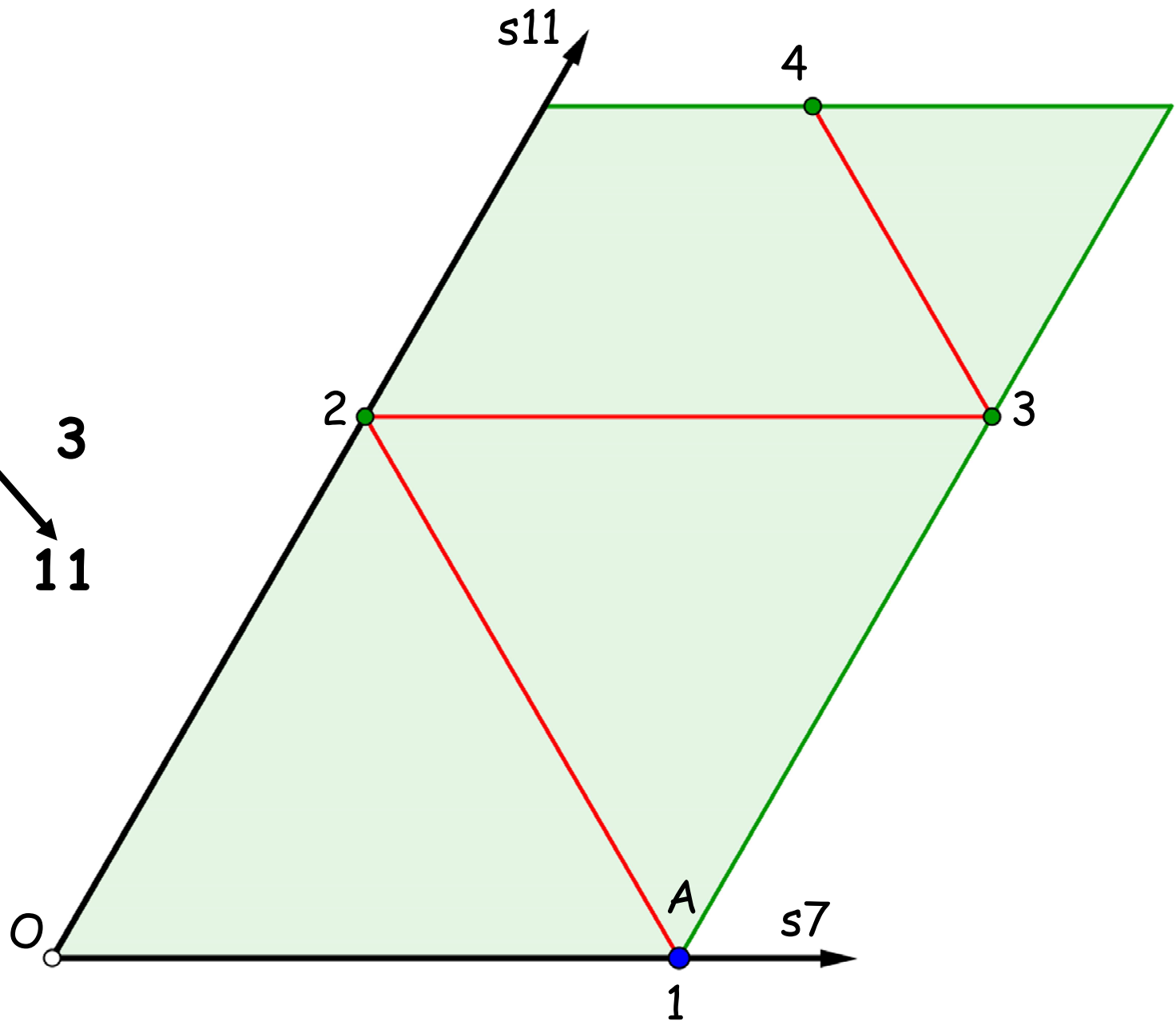
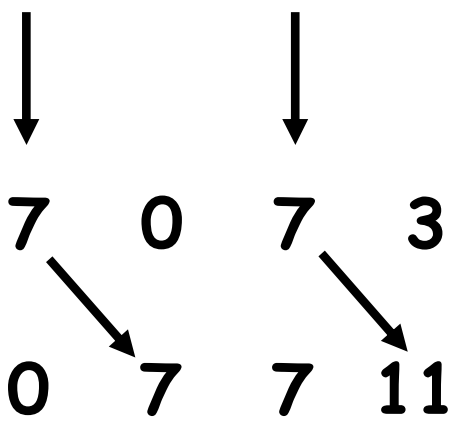
Problèmes aqueux



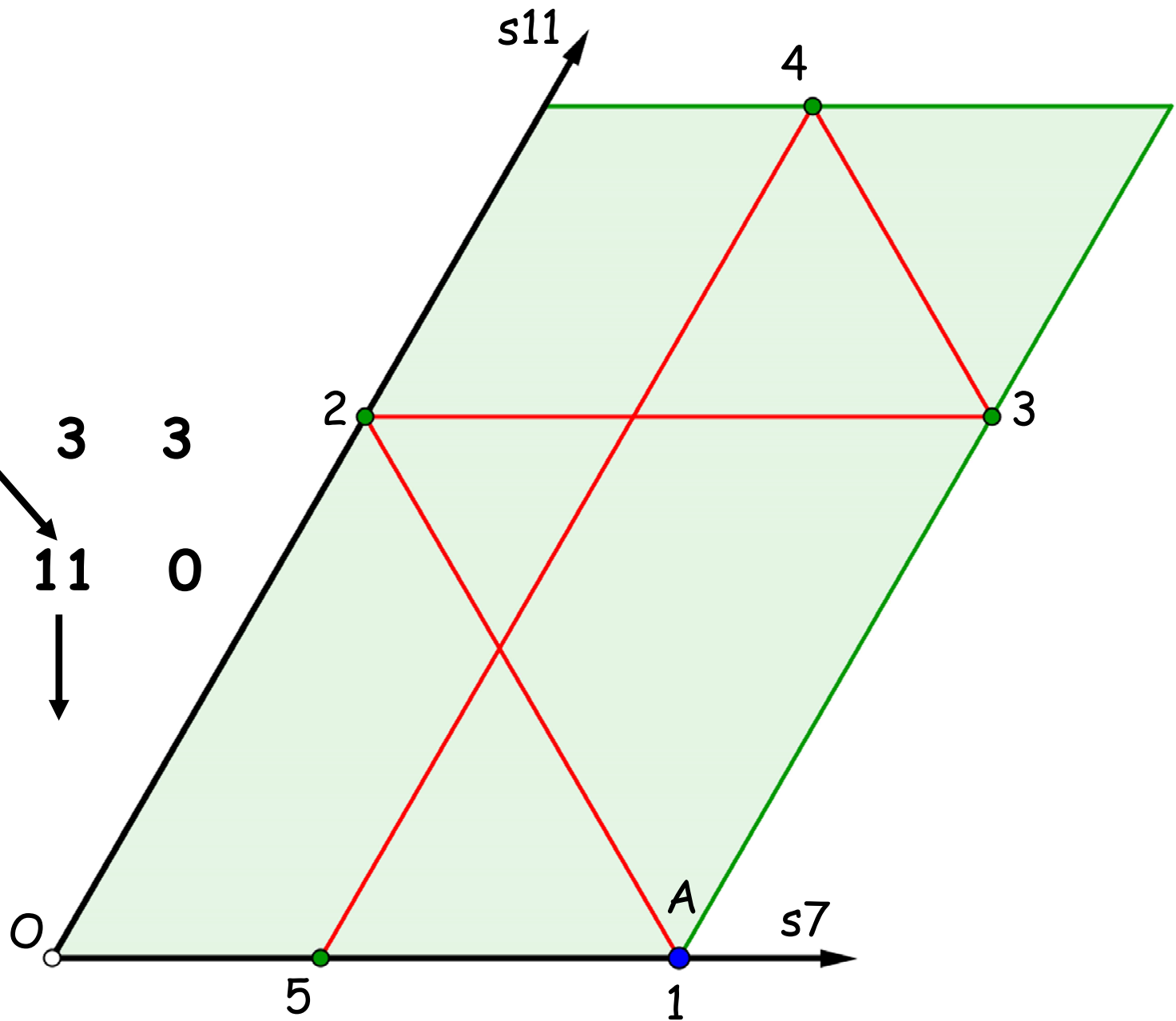
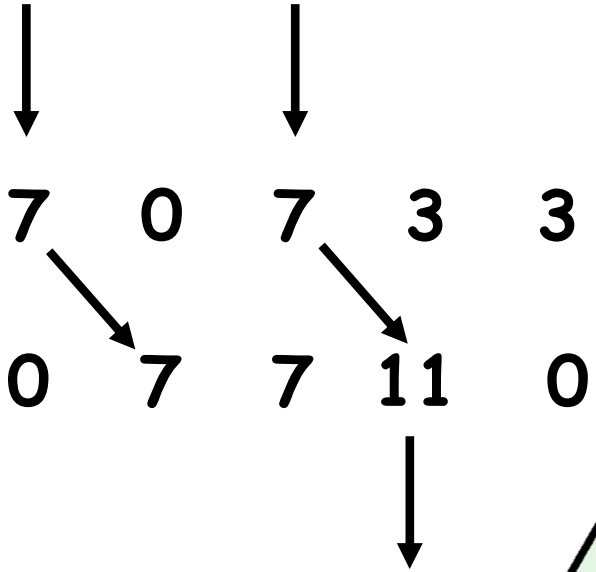
Problèmes aqueux



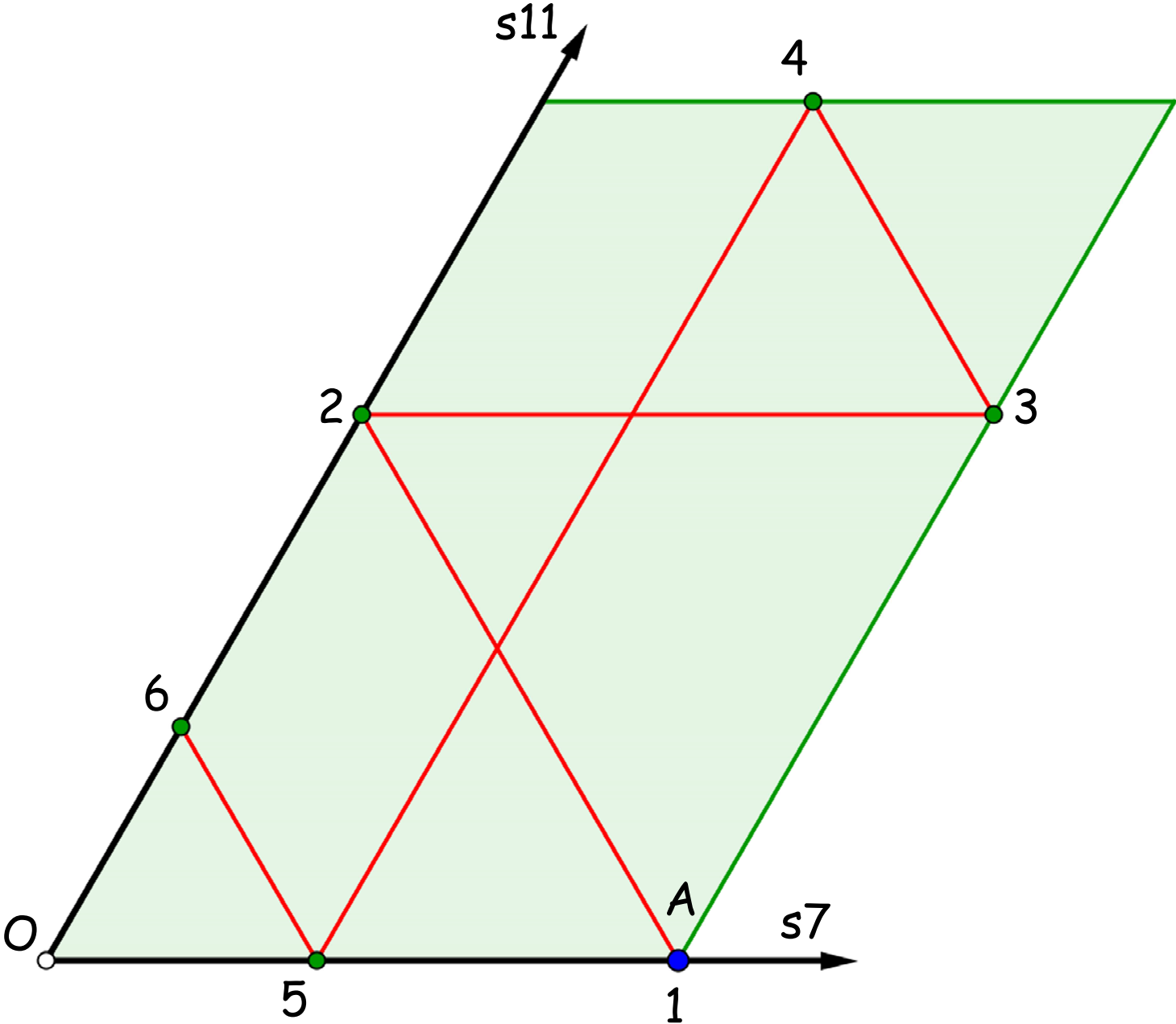
Problèmes aqueux



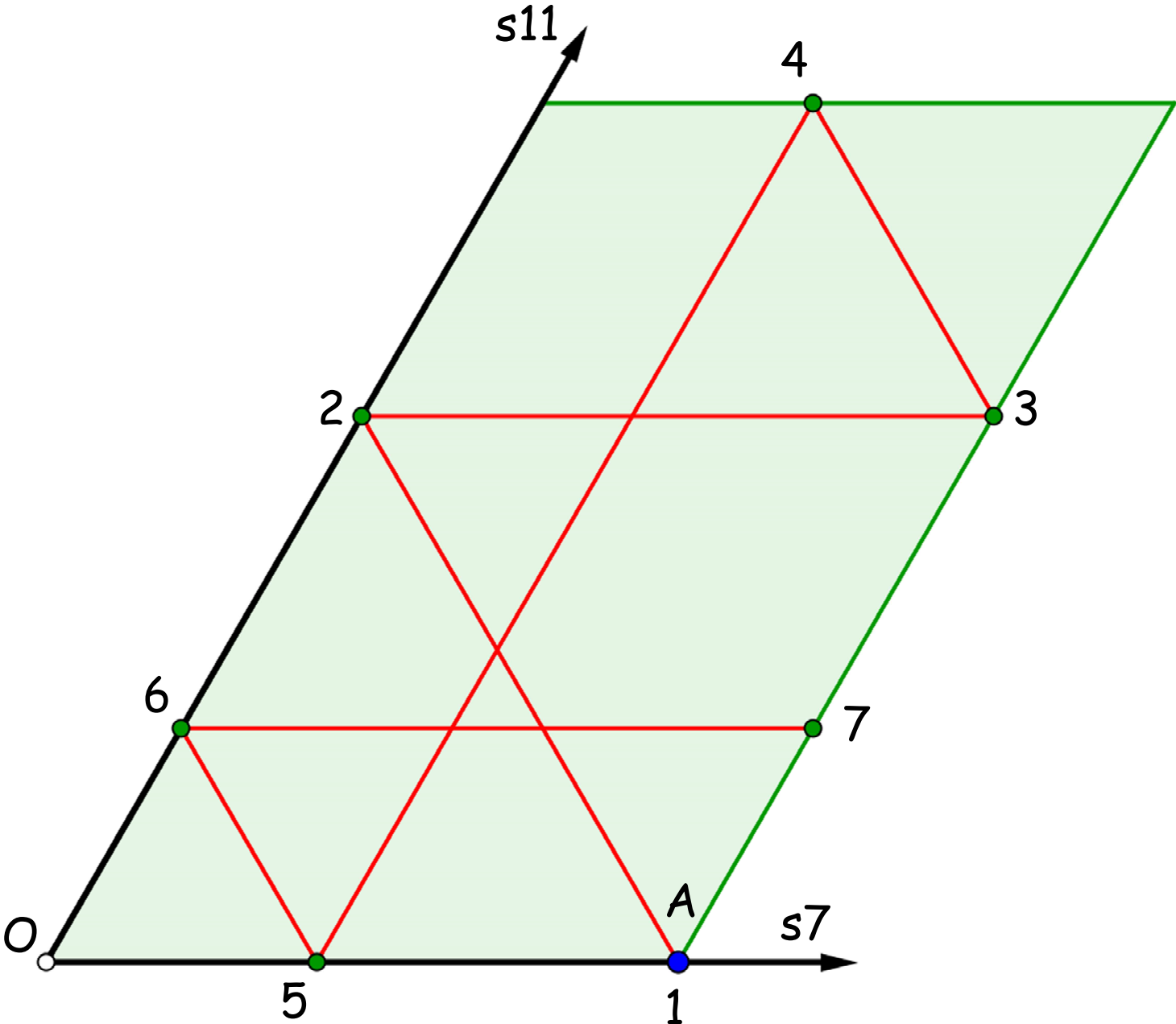
Problèmes aqueux



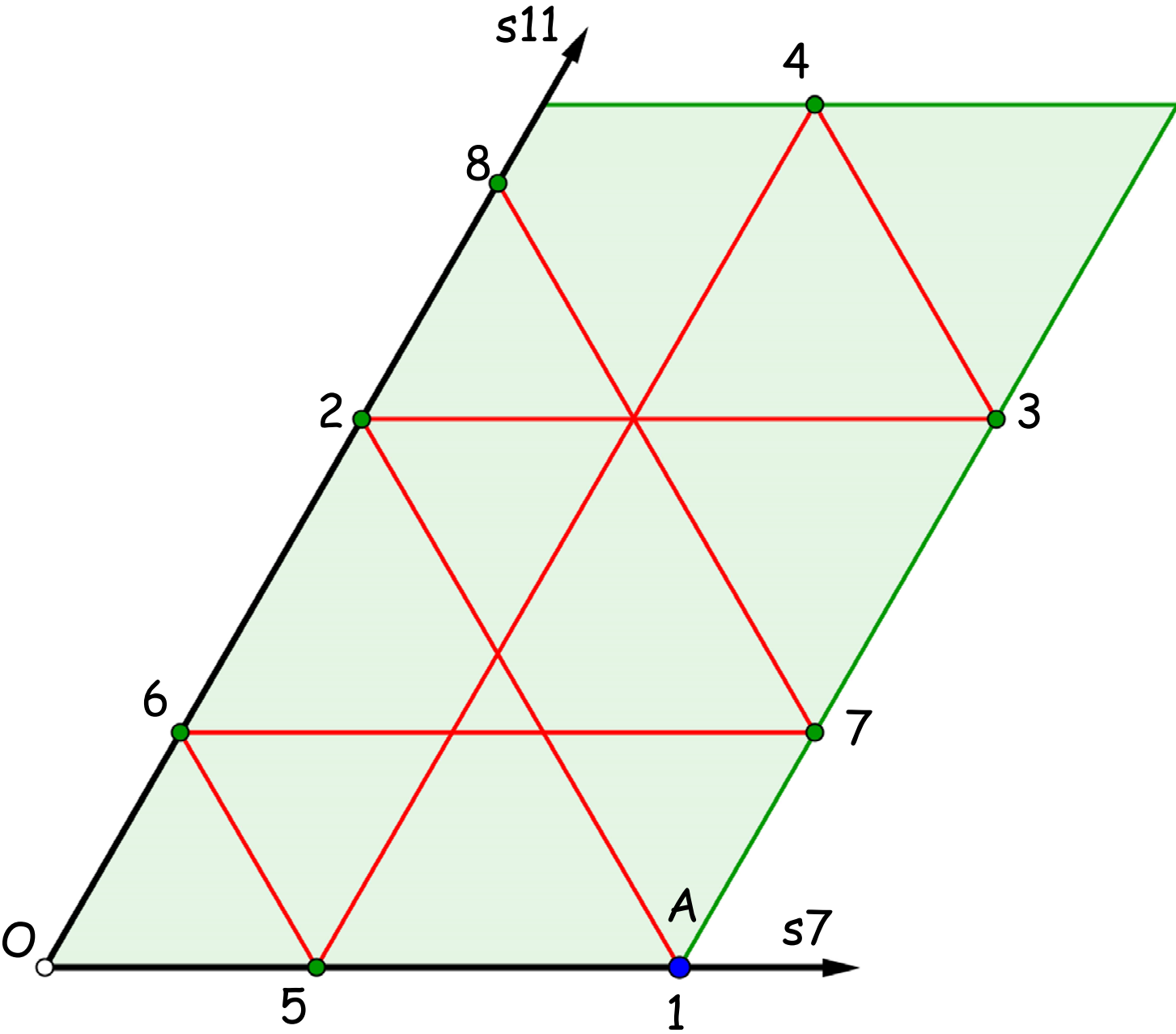
Problèmes aqueux



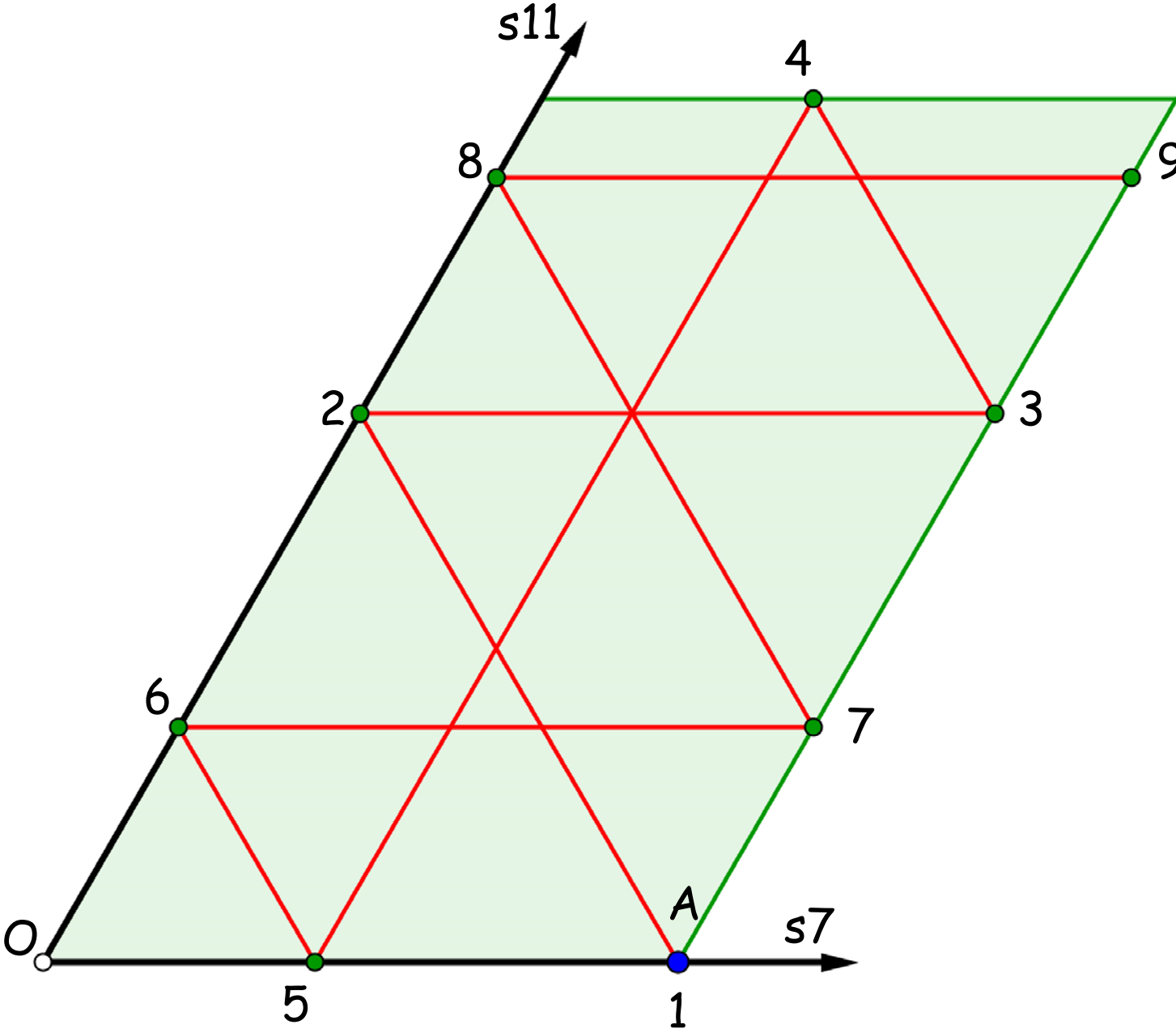
Problèmes aqueux



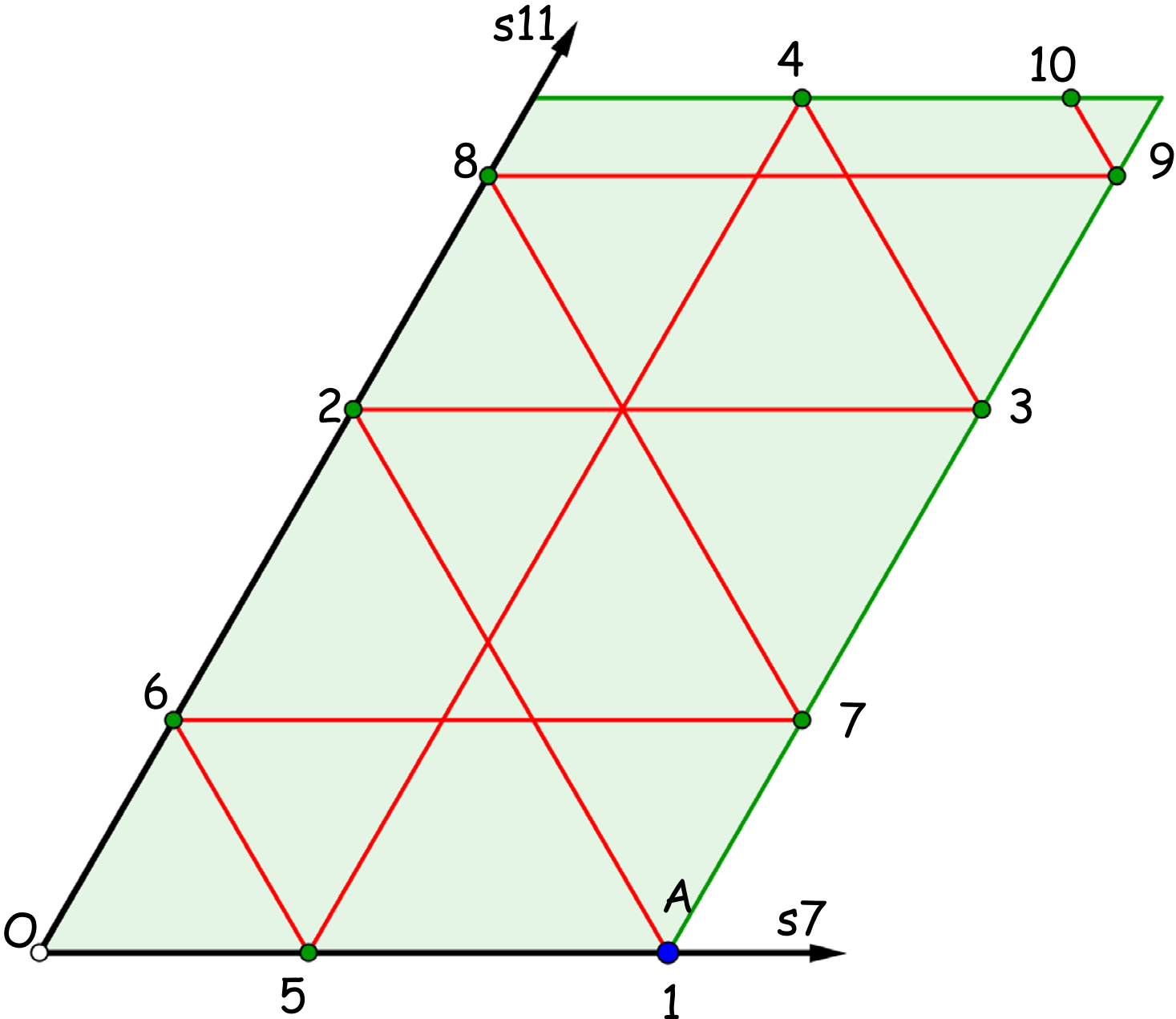
Problèmes aqueux



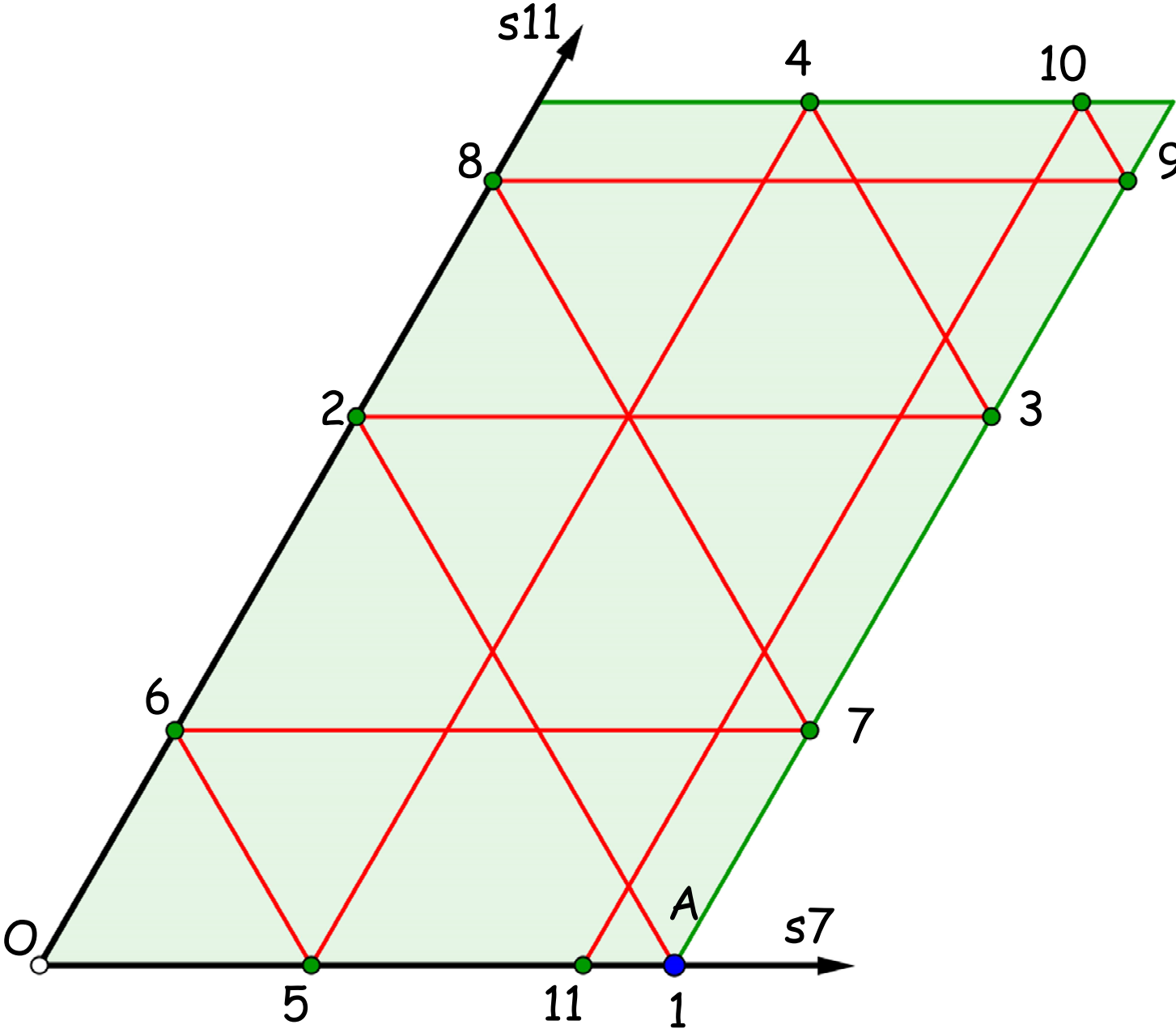
Problèmes aqueux



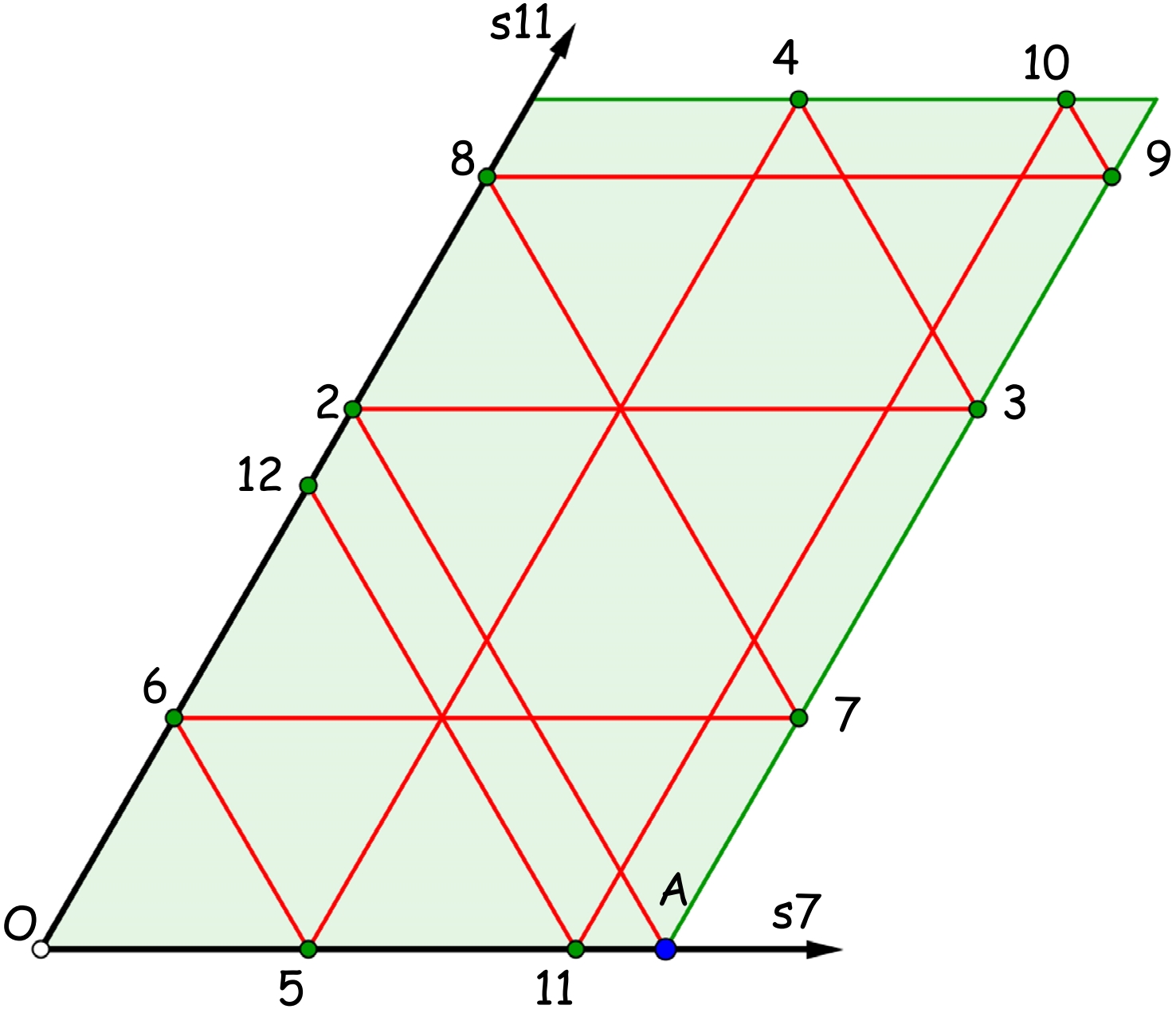
Problèmes aqueux



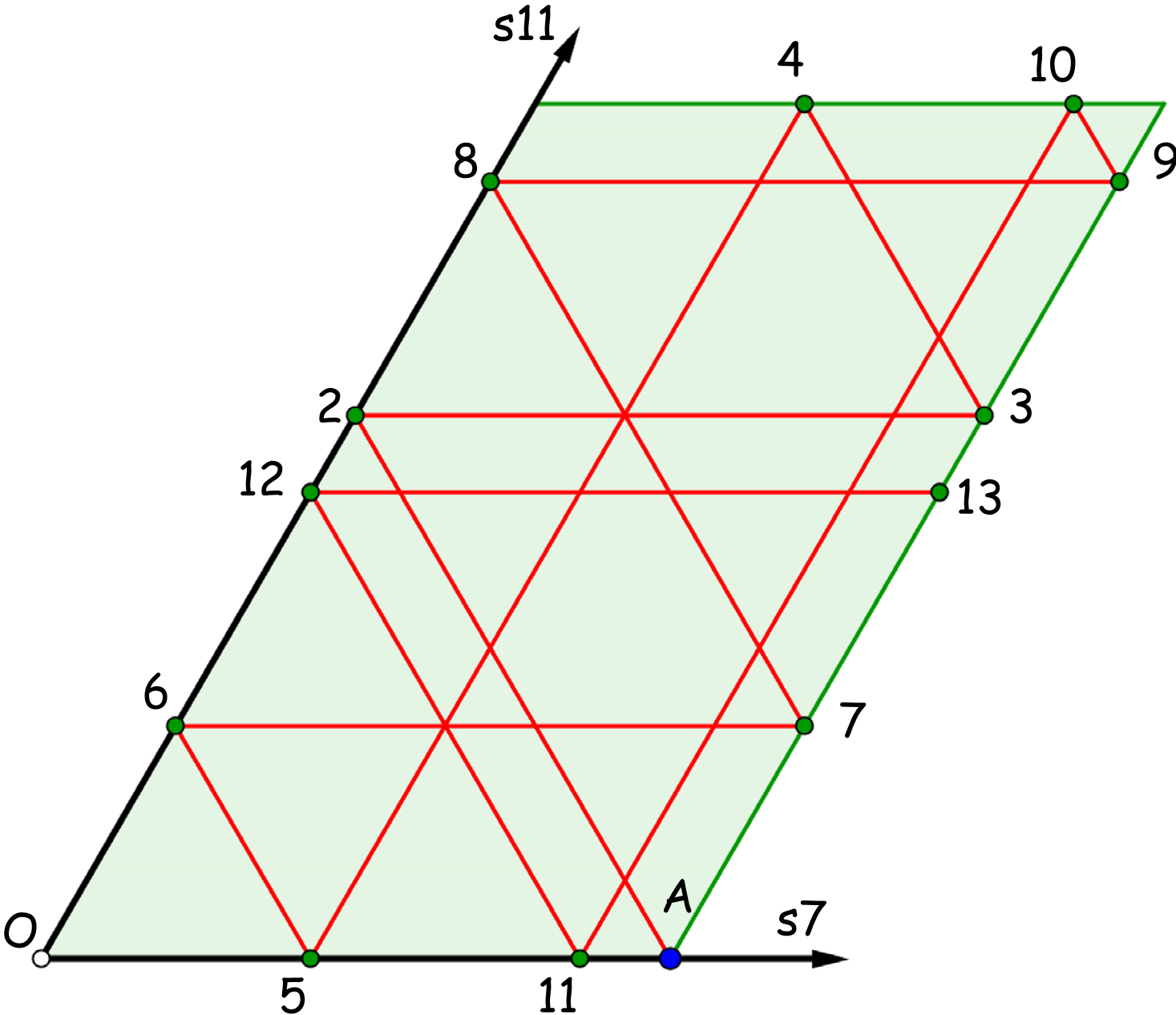
Problèmes aqueux



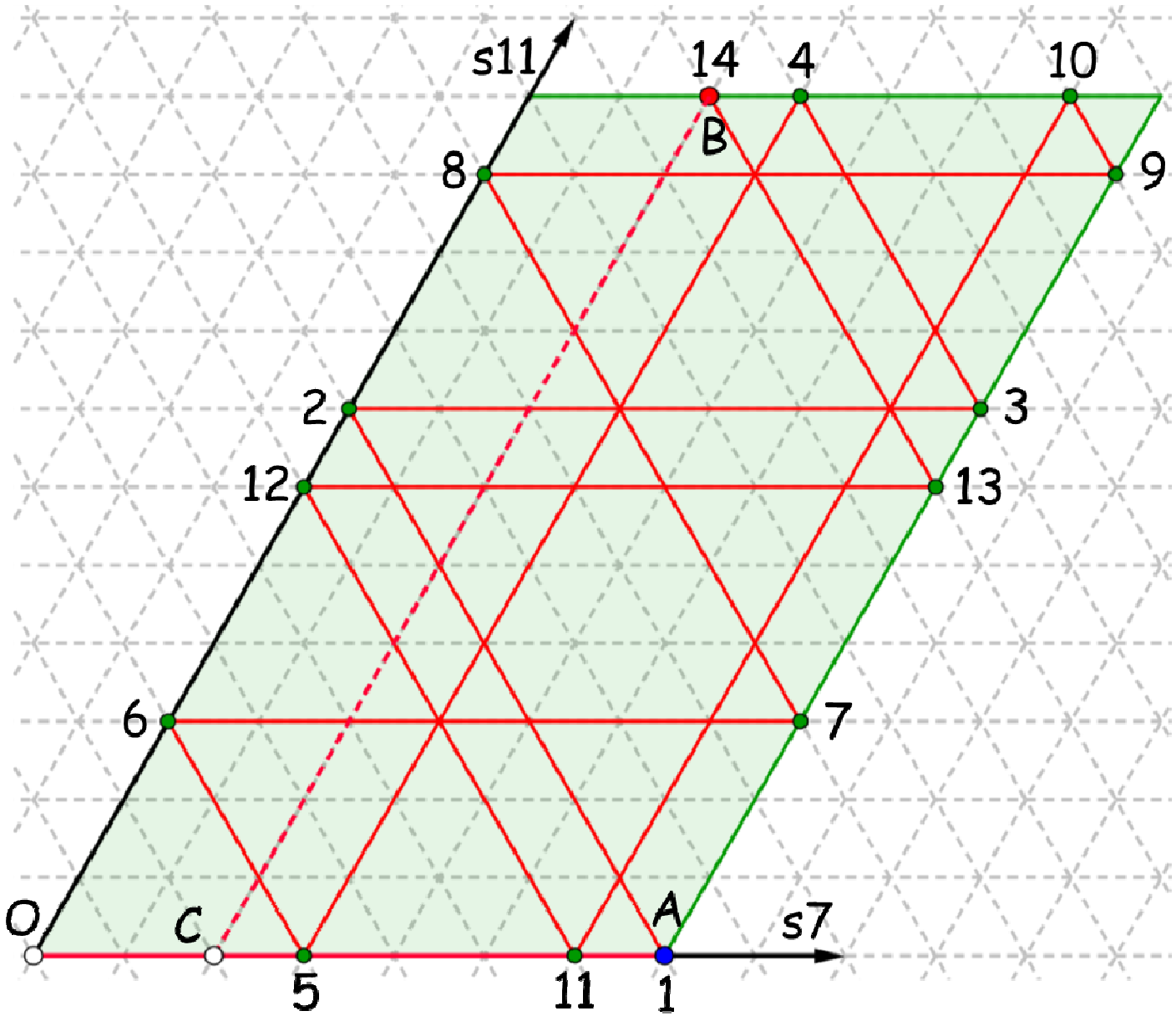
Problèmes aqueux



Problèmes aqueux



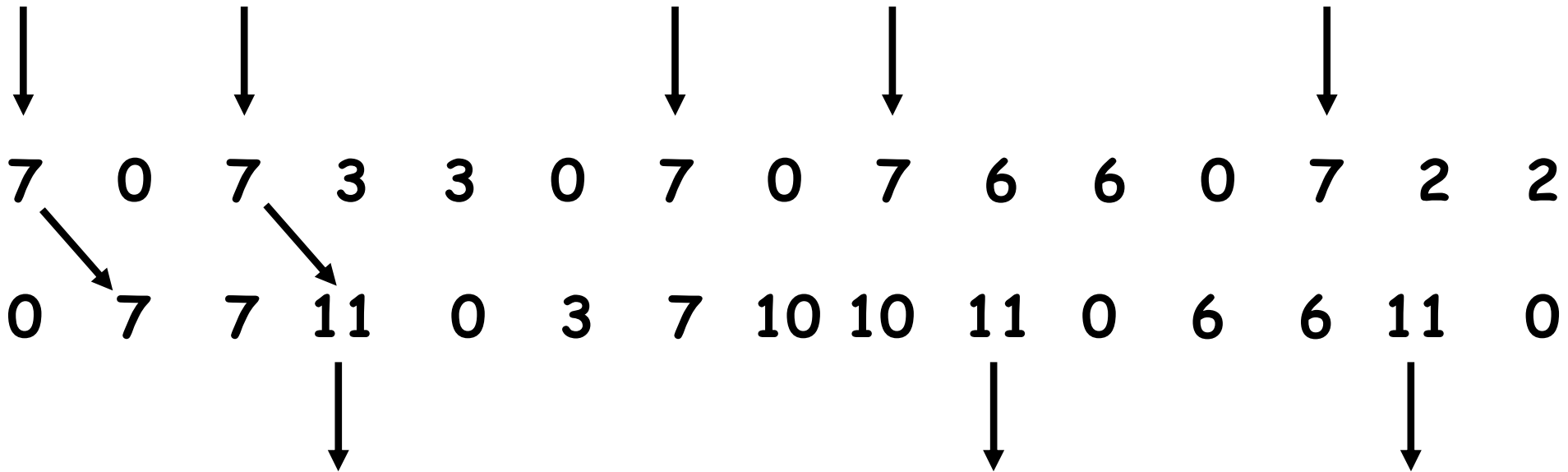
Problèmes aqueux



Problèmes aqueux



Remplissage: 5 fois 7 litres.

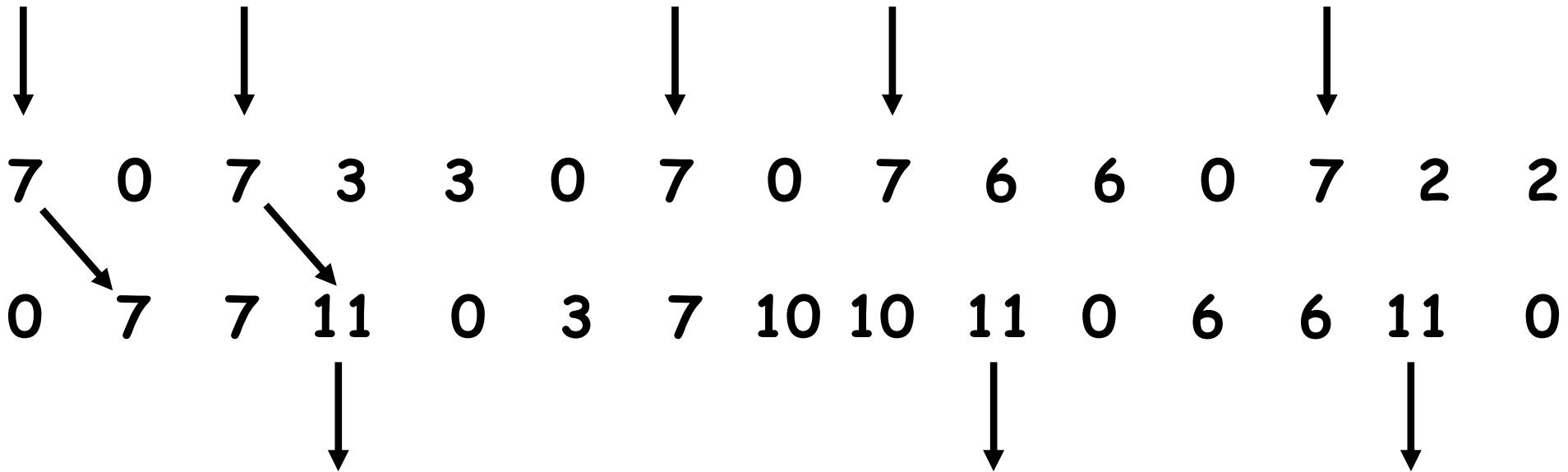


Vidange: 3 fois 11 litres.

Problèmes aqueux



Remplissage: 5 fois 7 litres.



Vidange: 3 fois 11 litres.

$$\text{Bilan: } 5 \times 7 - 3 \times 11 = 2$$

Théorème de seaux



Equation diophantienne: $7x + 11y = 2$

Théorème de Bachet de Méziriac (1581-1638)

Solution générale:

$$x = 5 - 11.k$$

$$y = -3 + 7.k$$

$$k = 0 \quad x = 5 \quad y = -3$$

$$k = 1 \quad x = -6 \quad y = 4$$

Problèmes aqueux

