



Institut
Pierre
Simon
Laplace



Les ondelettes et leur histoire

Marie Farge
CNRS-INSMI
LMD, ENS Paris, PSL

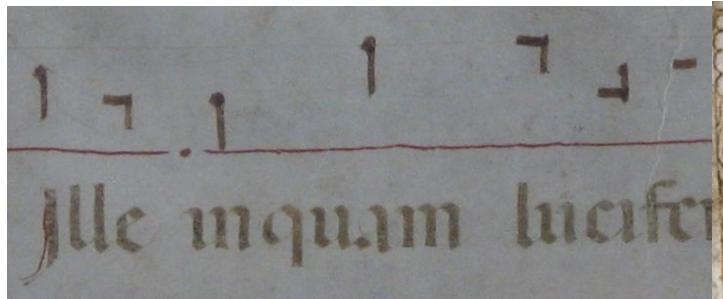
KafeMath
'La Coulée Douce', Paris
12 Octobre 2017

Choix d'une représentation adéquate

'A representation is a formal system for making explicit certain entities or types of information, together with a specification of how the system does this. For example, the Arabic, Roman and binary numerical systems are formal systems for representing numbers. [...] A representation therefore is not a foreign idea at all, we all use representations all the time. However, the notion that we can capture some aspects of reality by making a description of it using a symbol, and that to do so can be useful, seems to me a fascinating and powerful idea ... This issue is important, because how information is presented can greatly affect how easy it is to do different things with it. This is evident even from our number example : it is easy to add, to subtract and even to multiply if the Arabic or binary representation are used, but it is not at all easy to do these things, especially multiplication, with Roman numerals. This is a key reason why the Roman culture failed to develop mathematics in the way the Arabic culture had.'

David Marr, 1982
Vision
Freeman

Une représentation pour la musique



Guido d'Arrezzo
Micrologos
1025

Décomposition
et notation :

$$f_n = f_0 \cdot a^n$$
$$a = 2^{1/12}$$

7 tons

ut re mi fa sol la si
(*sa re ga ma pa da ni*)
plus 12 demi-tons

Une représentation pour la musique



Jean-Sébastien Bach
(1685-1750)



Frédéric Chopin
(1810-1849)

Choix d'une représentation adéquate

‘Si nous considérons un morceau de musique contenant plusieurs mesures et qu’une note, *la* par exemple, figure une fois dans le morceau, l’analyse harmonique nous présentera la fréquence correspondante avec une certaine amplitude et une certaine phase, sans localiser le *la* dans le temps. Or, il est évident qu’au cours du morceau il est des instants où l’on n’entend pas le *la*. La représentation est néanmoins mathématiquement correcte, parce que les phases des notes voisines du *la* sont agencées de manière à détruire cette note par interférence, lorsqu’on ne l’entend pas et à la renforcer, également par interférence, lorsqu’on l’entend; mais s’il y a dans cette conception une habileté qui honore l’analyse mathématique, il ne faut pas se dissimuler qu’il y a également une défiguration de la réalité: en effet, quand on n’entend pas le *la*, la raison véritable est que le *la* n’est pas émis’.

Jean Ville, 1948

*Théorie et applications de la notion de signal analytique
Cables et transmissions, Cours de 2ème année, n° 1*

Choix d'une représentation adéquate

'On peut avoir intérêt, en théorie des communications, à représenter un signal oscillant comme superposition d'ondelettes élémentaires, dont chacune possède à la fois une fréquence et une localisation dans le temps assez bien définies. L'information utile est en effet souvent véhiculée à la fois par les fréquences émises et par la structure temporelle du signal (l'exemple de la musique est caractéristique). La représentation d'un signal comme fonction du temps exhibe mal le spectre des fréquences en jeu, alors qu'au contraire son analyse de Fourier masque l'instant d'émission et la durée de chacun des éléments du signal. Une représentation adéquate devrait combiner les avantages de ces deux descriptions complémentaires, tout en présentant un caractère discret mieux adapté à la théorie des communications.'

Roger Balian, 1984

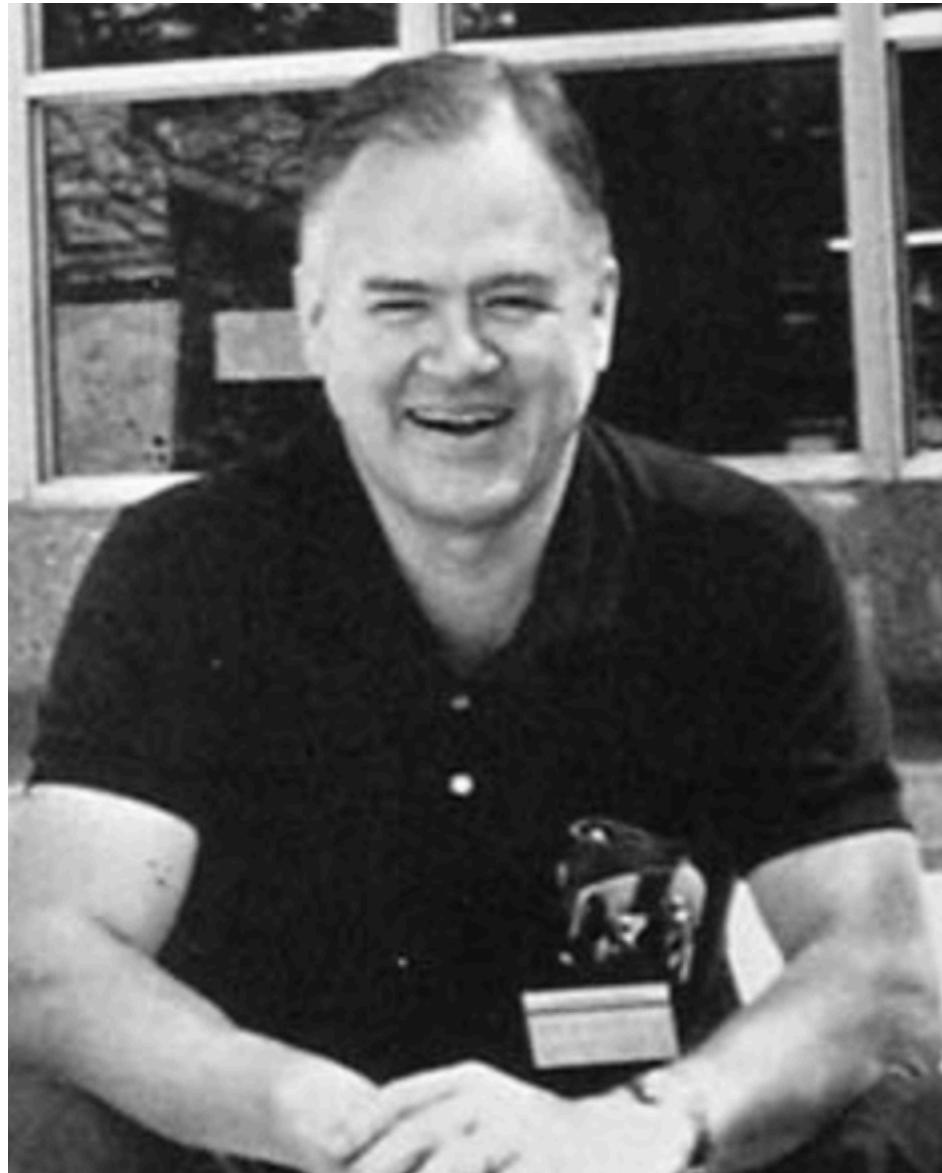
Un principe d'incertitude fort en théorie du signal

Comptes-Rendus Acad. Sci., 292(2), 1357-1361

Joseph Fourier
(1768-1830)



John Tukey
(1915-2000)



James William Cooley
(1926-2016)



Transformée de Fourier

Signal à analyser :

$$f(x) \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$$

Analyse

Coefficients de l'analyse :

$$\widehat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi k \cdot x} dx$$

Synthèse

Signal

reconstruit :

$$f(x) = \int_{-\infty}^{\infty} \widehat{f}(k) e^{i2\pi k \cdot x} dk$$

Conservation du produit scalaire

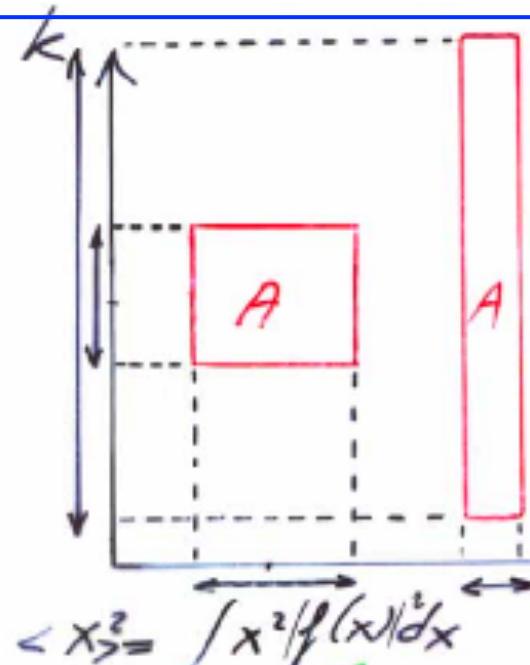
$$\int_{-\infty}^{\infty} f_1(x) \cdot f_2(x) dx = \int_{-\infty}^{\infty} \widehat{f}_1(k) \cdot \widehat{f}_2(-k) dk$$

Principe d'incertitude de Fourier

$$\langle k^2 \rangle = \int k^2 |f(k)|^2 dk$$

$$\widehat{f(x)} = \frac{1}{\sqrt{2\pi}} \int f(k) e^{ikx} dk$$

Space dilatation
↔
Wavelength contraction



f such that
 $\int f(x) dx = 1$

$$\langle x^2 \rangle - \langle k^2 \rangle \geq A$$

$$A = \frac{1}{2} \text{ or } \frac{1}{4\pi} \text{ or } \frac{1}{16\pi^2}$$

or $\frac{\hbar}{2\pi}$ with $\hbar = 6.6 \cdot 10^{-34} \text{ J s}$

Time-frequency plane,

or position-wavefunction plane,

or position-momentum ($p = \frac{\hbar k}{2\pi}$, \hbar Planck's constant),

or phase space,

or information plane.

Léon Brillouin, 1956
Science and information theory
Academic Press

Transformées intégrales

Analysis

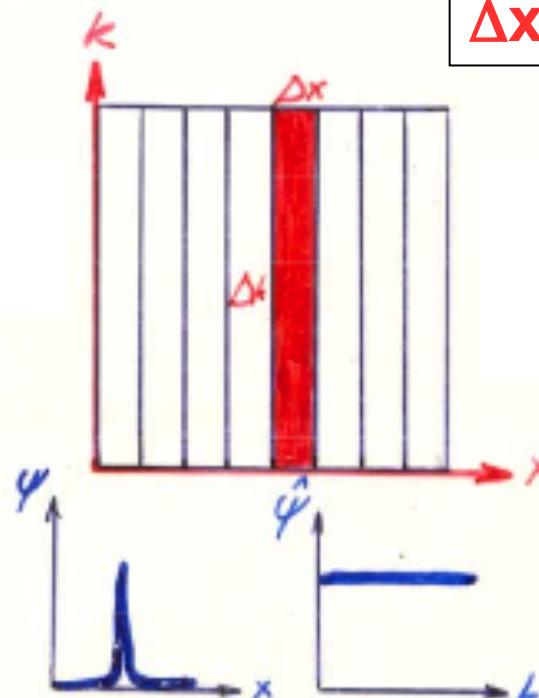
$$\mathcal{F}(k) = \int f(\vec{x}) \psi_k(\vec{x}) d\vec{x}$$

Synthesis

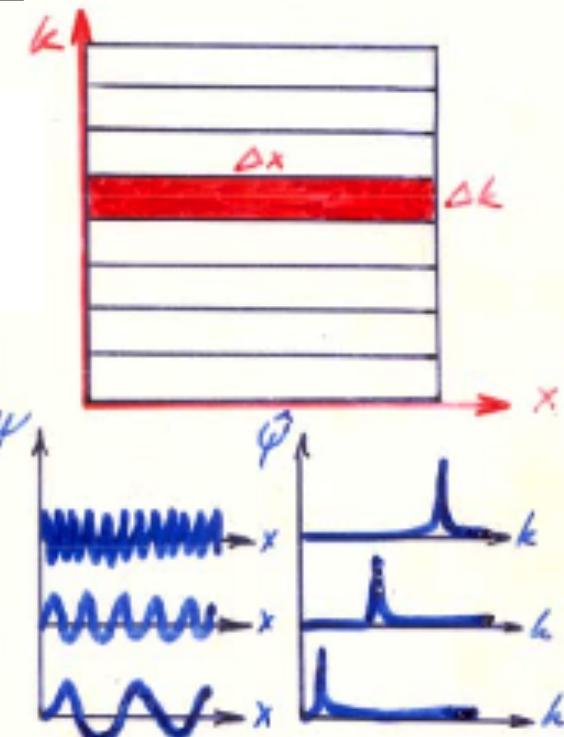
$$f(\vec{x}) = \frac{1}{c} \int \mathcal{F}(k) \psi_k(\vec{x}) dk$$

$$\Delta x \Delta k = A$$

Points de grille

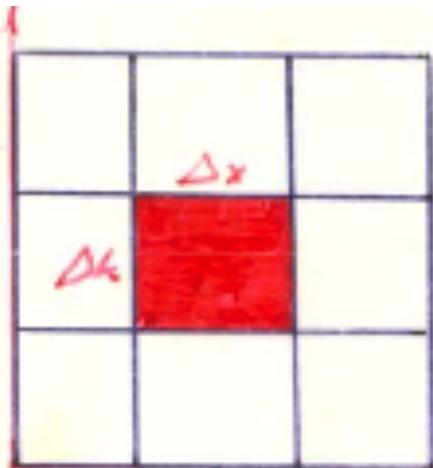


Modes de Fourier (1807)



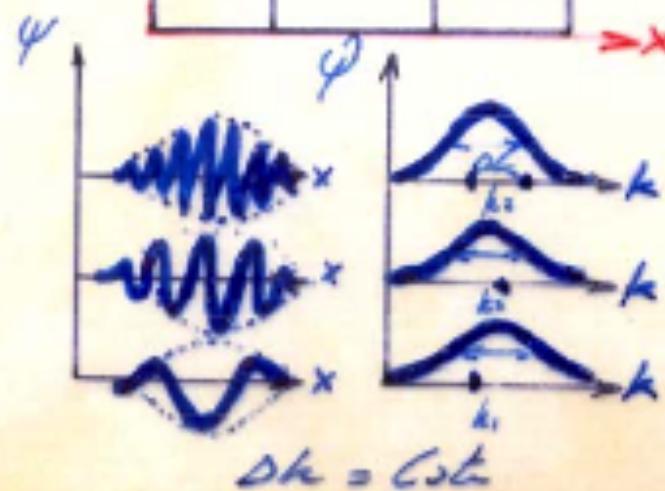
Pavage de l'espace de phase

Gabor
(1946)



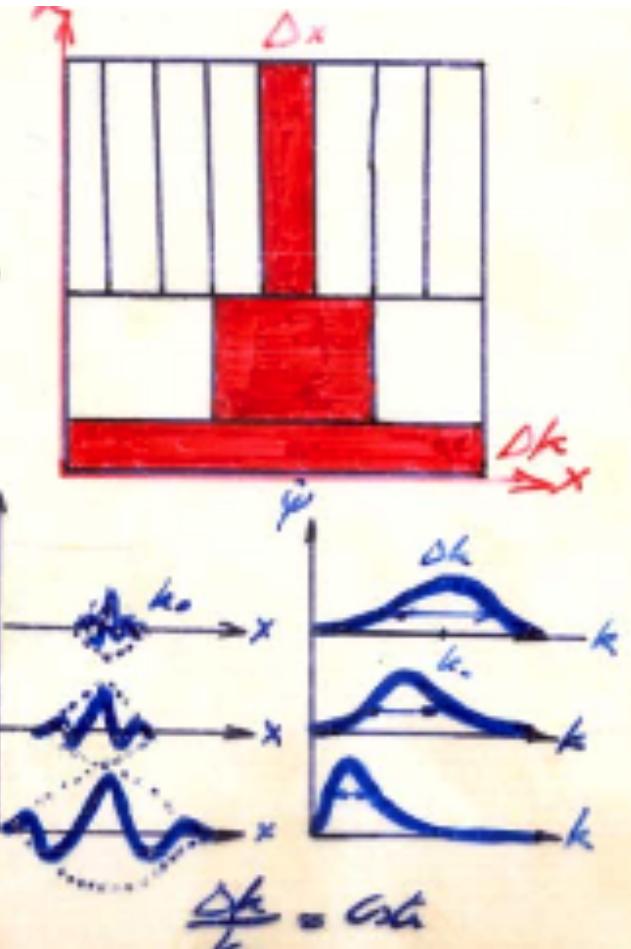
Ondelettes
(1984)

Balian's
obstruction
(1981)



Représentation en
position-nombre d'onde

$\Delta x \Delta k = A$
pavé d'information



Représentation en
position-échelle

Dennis Gabor
(1900-1979)



Eugene Wigner
(1902-1995)



Jean Morlet
(1931-2007)



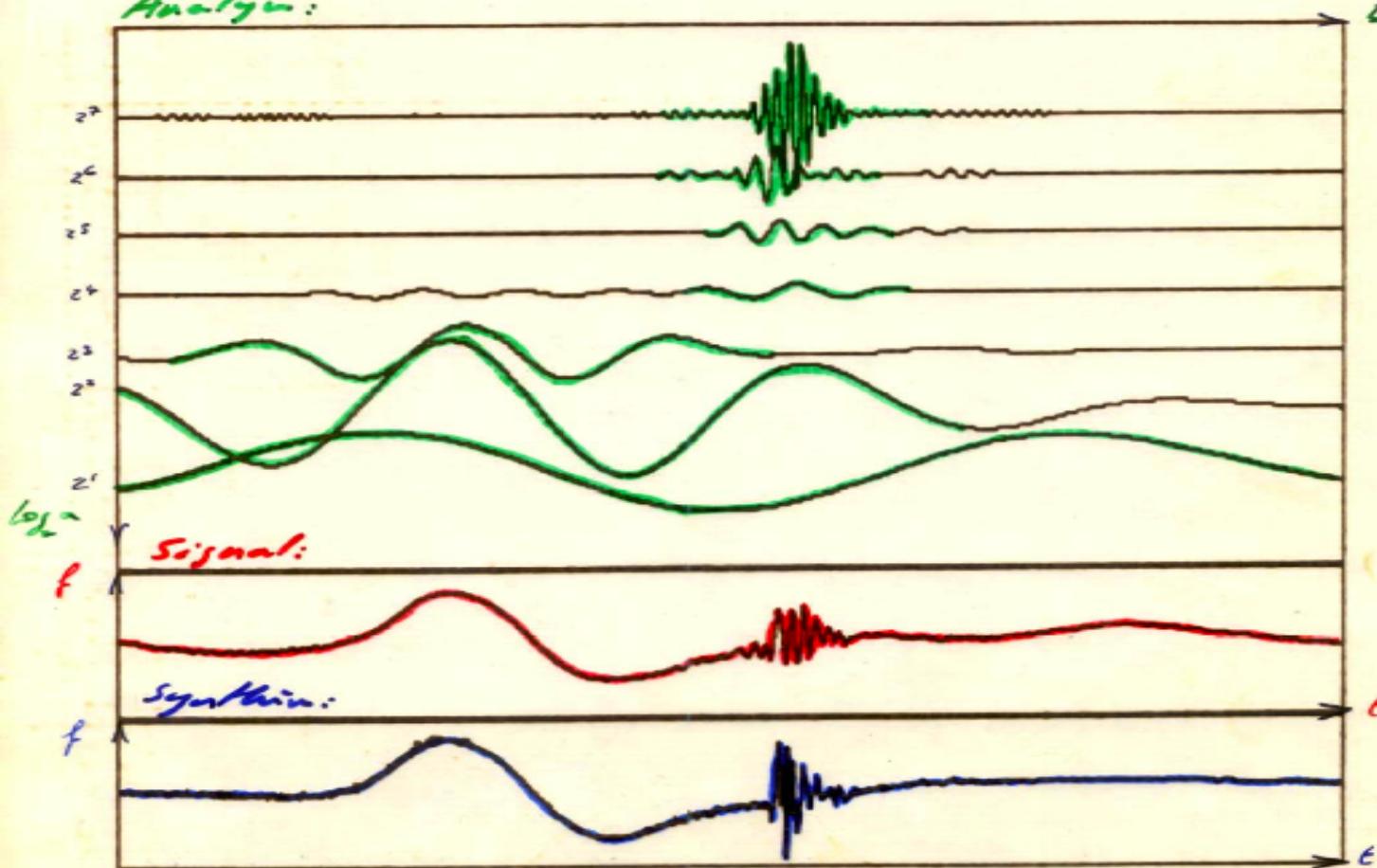
Real partParameters:

Sampling frequency: 44.1 kHz

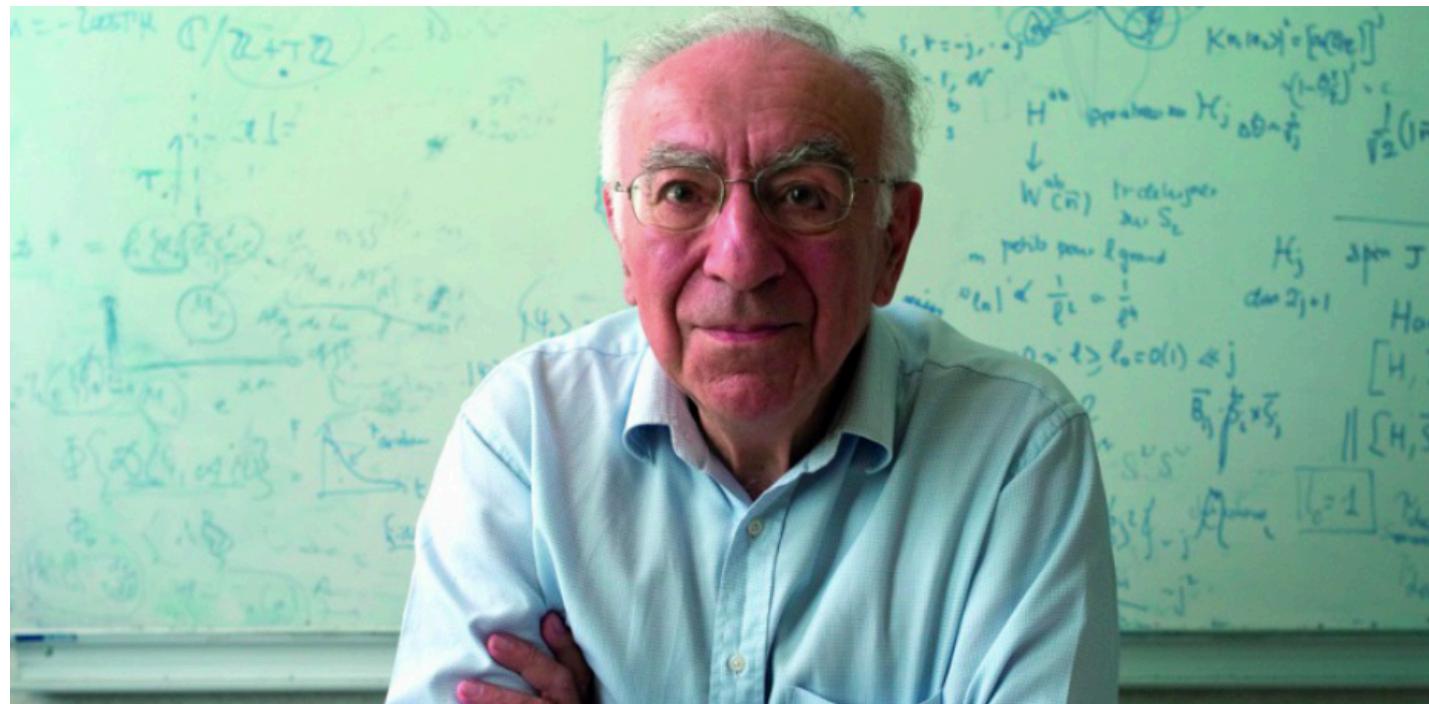
Number of voice per octave = 1

Highest voice: index = 1 frequency = 6000 Hz

Lowest voice: index = 7 frequency = 93.7 Hz

Time window = 18 ms *Signal de parole (1983)*
*Jean Morlet*Analyse:

Roger Balian (1933)

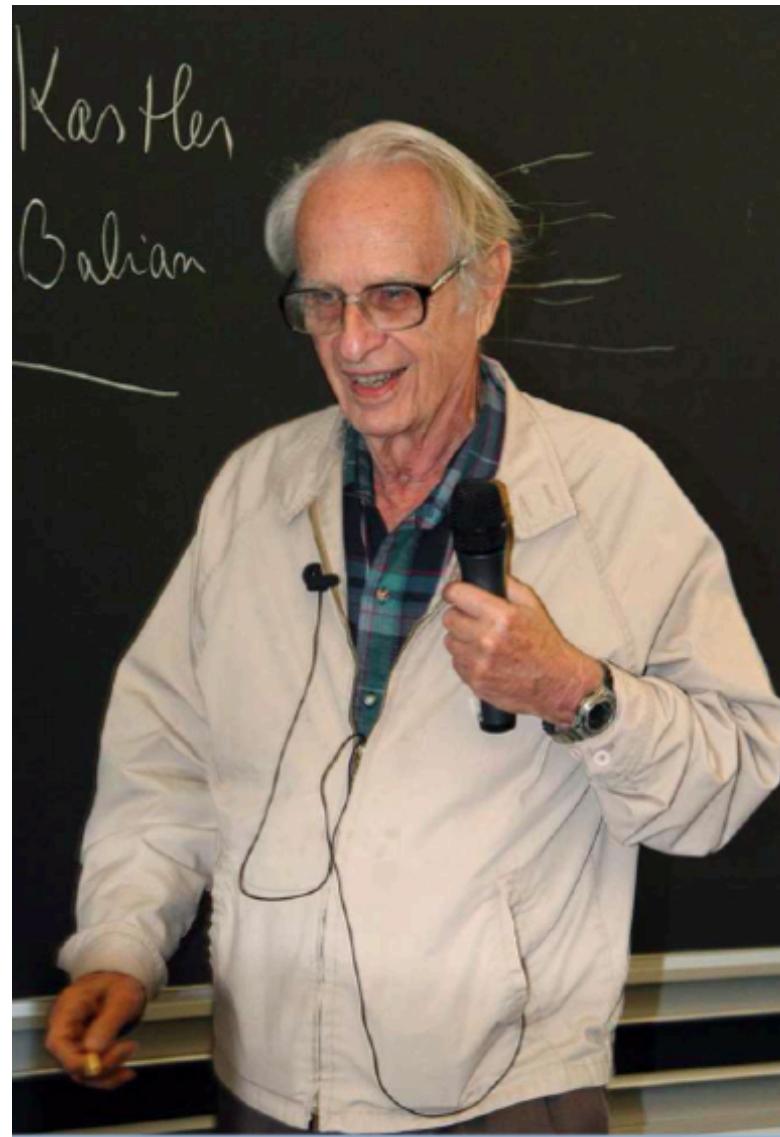


*'Je ne suis pas spécialiste de l'interprétation temps-fréquence,
mais je connais quelqu'un qui est plus spécialiste que moi'.*

Réponse de Roger Balian à Jean Morlet en 1981

Barbara Burke Hubbard, 1995
Ondes et ondelettes
Belin, collection Pour la science

*Alex Grossmann
(1930)*



Premiers articles publiés sur la CWT

Pierre Goupillaud, Alex Grossmann
and Jean Morlet, 1984

*Cycle-octave and related transforms
in seismic signal analysis*
Geoexploration, 23(1), 85-102

Pierre Goupillaud, Alex Grossmann
and Jean Morlet, 1984

*Cycle-octave representation for
Instantaneous frequency-spectra*
Geophysics, 49(5), 669

Alex Grossmann and Jean Morlet, 1984
*Decomposition of Hardy functions into square
integrable wavelets of constant shape*
SIAM J. Math. Anal., 15(4), 723-736

CIRM-SMF, Marseille-Luminy, 2008

Pierre Goupillaud



Alex Grossmann

Choix de l'ondelette ‘mère’

Condition d’admissibilité

$$C_\psi = \int_0^\infty |\widehat{\psi}(k)|^2 \frac{dk}{|k|} < \infty$$
$$\int_{-\infty}^\infty \psi(x) dx = 0 \quad \text{or} \quad \widehat{\psi}(0) = 0$$

*Génération de la famille d’ondelettes analysantes
par translation (b) et par dilatation (a)
de l’ondelette ‘mère’ choisie*

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi \left(\frac{x - b}{a} \right)$$

$$\widehat{\psi}_{a,b(k)} = \sqrt{a} \widehat{\psi}(ak) e^{-\imath 2\pi kb}$$

Transformée continue en ondelettes

Jean Morlet



Alex Grossmann



Analyse

$$\tilde{f}(a, b) = \int_{-\infty}^{\infty} f(x) \psi_{a,b}^*(x) dx$$

Synthèse

$$f(x) = \frac{1}{C_\psi} \int_0^{\infty} \int_{-\infty}^{\infty} \tilde{f}(a, b) \psi_{a,b}(x) \frac{da db}{a^2}$$

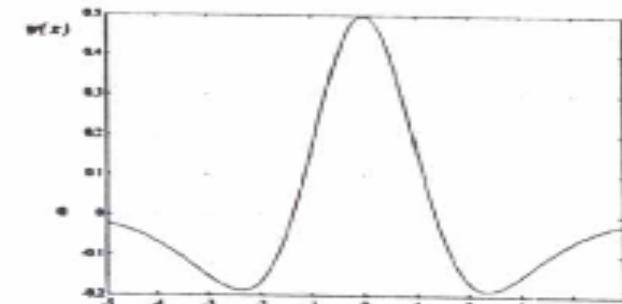
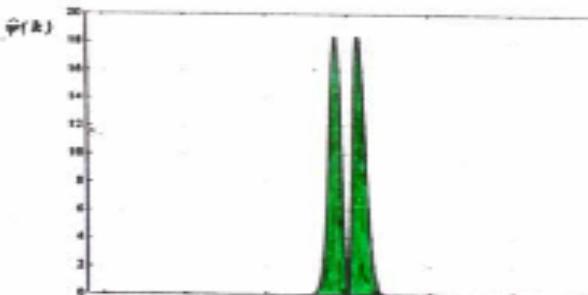
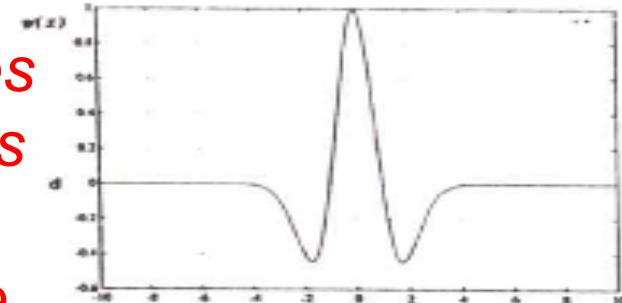
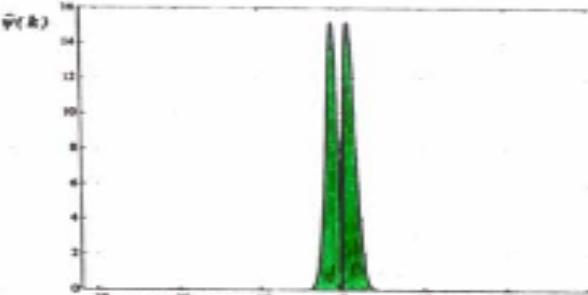
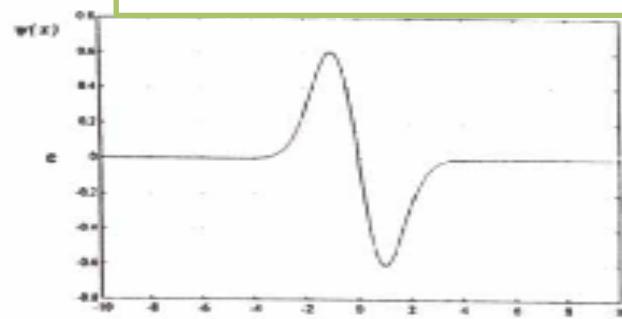
Conservation du produit scalaire

$$\langle f_1, f_2 \rangle = \int_{-\infty}^{\infty} f_1(x) f_2^*(x) dx = \frac{1}{C_\psi} \int_0^{\infty} \int_{-\infty}^{\infty} \tilde{f}_1(a, b) \tilde{f}_2^*(a, b) \frac{dadb}{a^2}$$

Ondelettes à valeurs réelles

1^{ère} et 2nd dérivées de la Gaussienne

Ondelettes vues dans l'espace physique



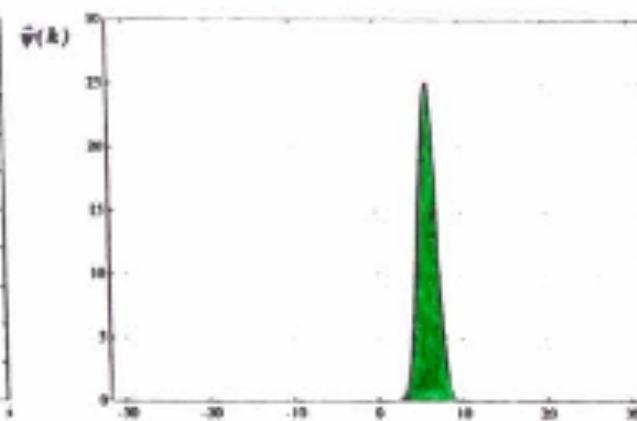
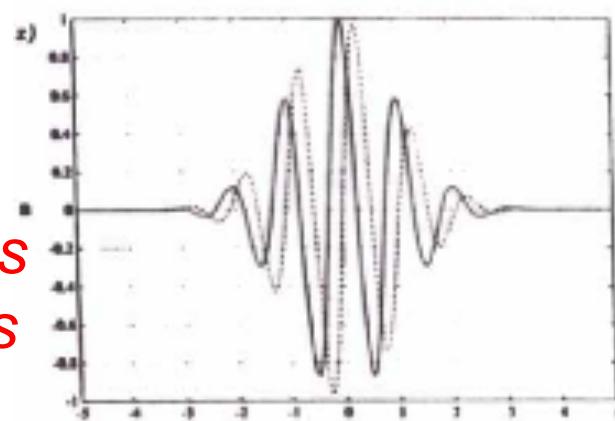
Ondelettes vues dans l'espace spectral

Difference de deux Gaussiennes

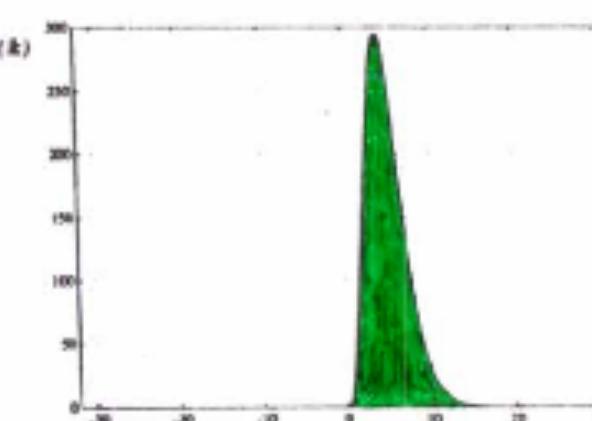
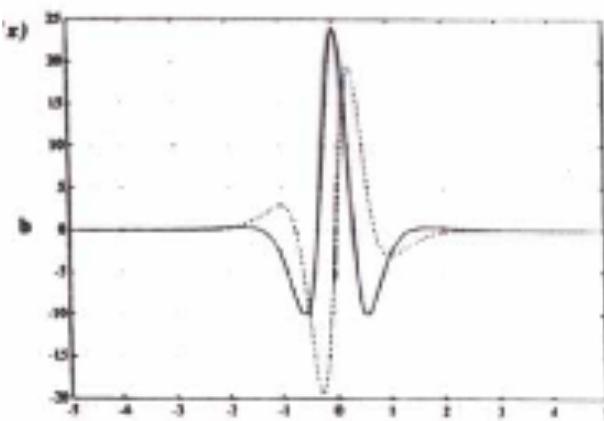
Ondelettes à valeurs complexes

Ondelette de Morlet

Ondelettes vues dans l'espace physique



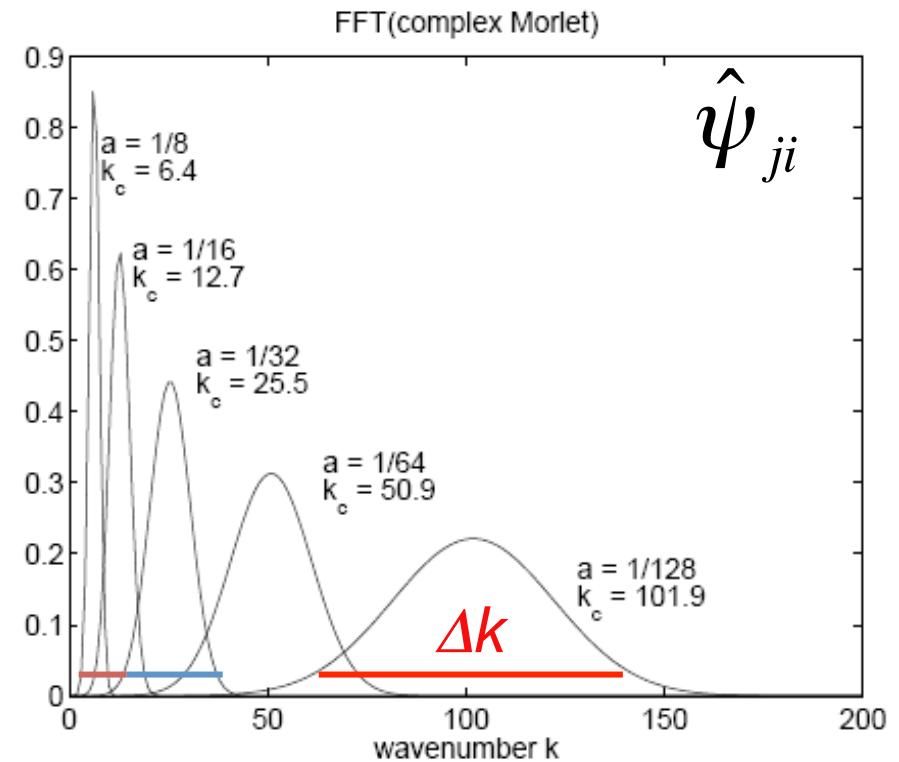
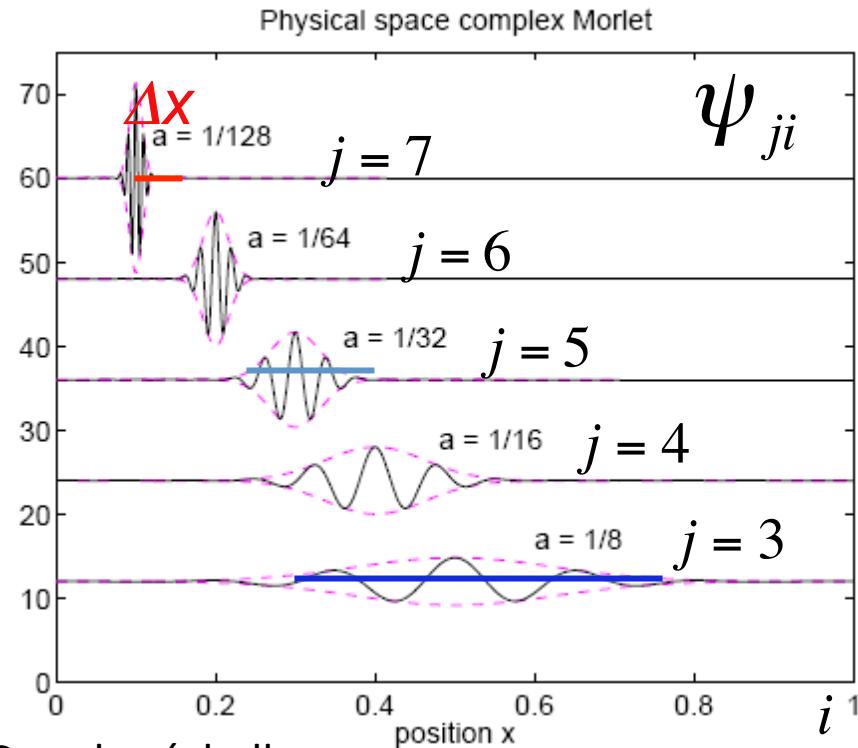
Ondelettes vues dans l'espace spectral



Ondelette analytique de Paul

Famille d'ondelettes analysantes

Petites échelles



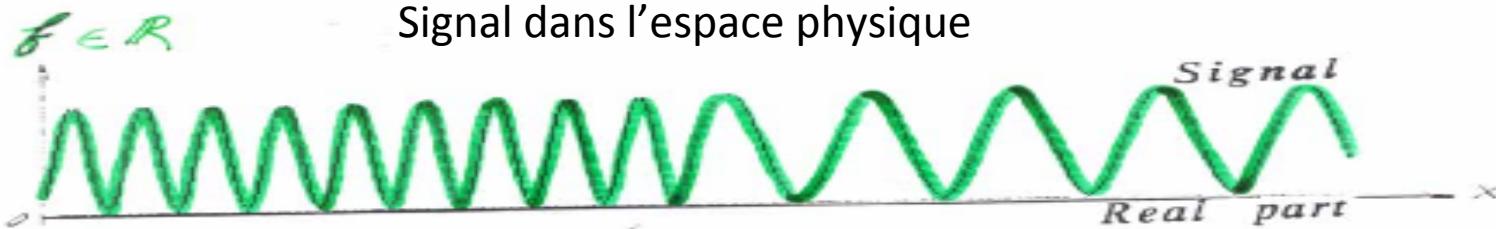
Grandes échelles

Espace physique

Espace spectral

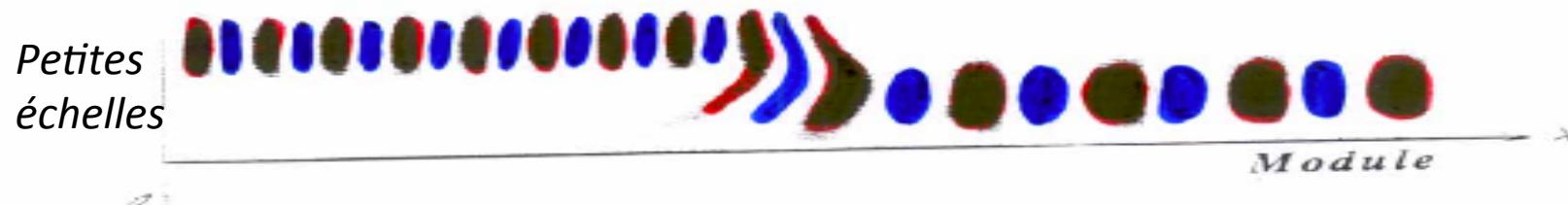
$$\Delta x \Delta k > A$$

Exemple d'analyse de signaux académiques



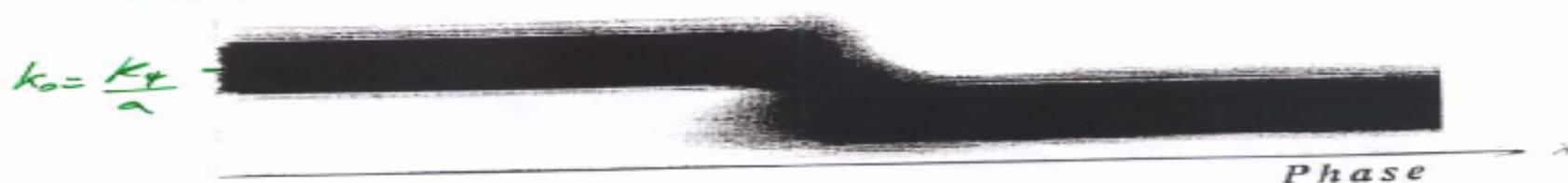
Grandes échelles $\tilde{f} \in \mathbb{R}$

Partie réelle des coefficients d'ondelettes du signal transformé



$\text{Module } (\tilde{f}) \in \mathbb{R}^+$

Module des coefficients d'ondelettes du signal transformé

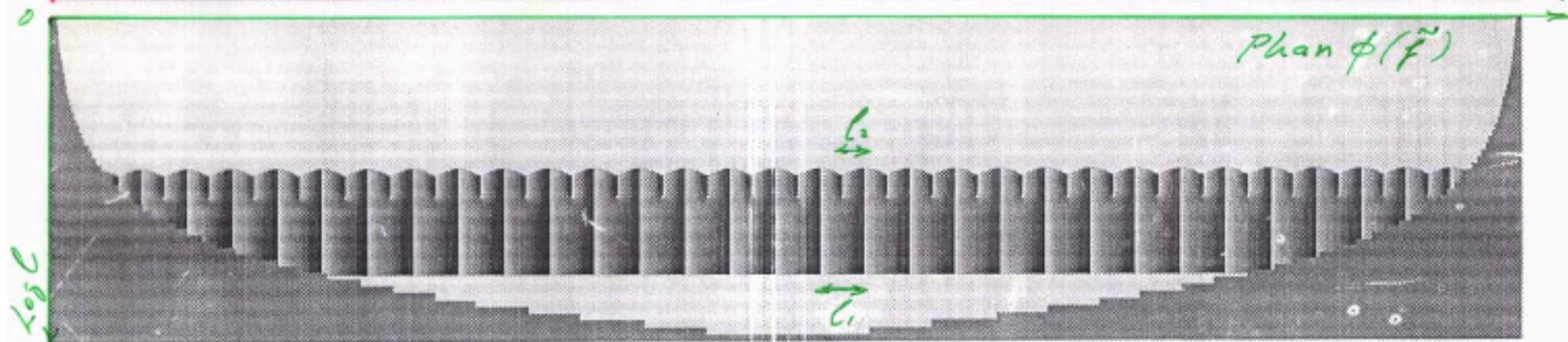
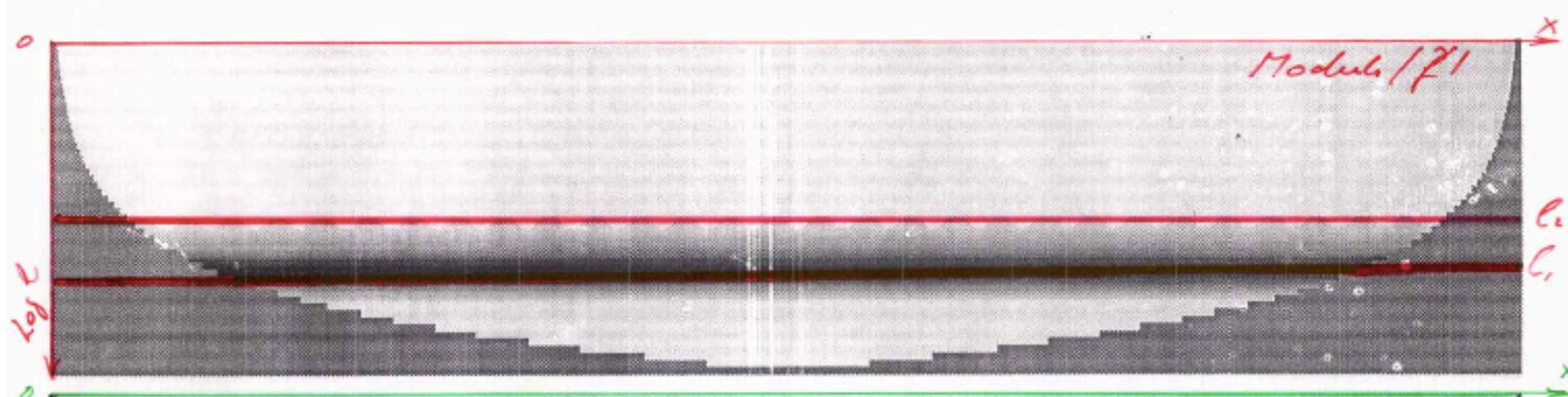
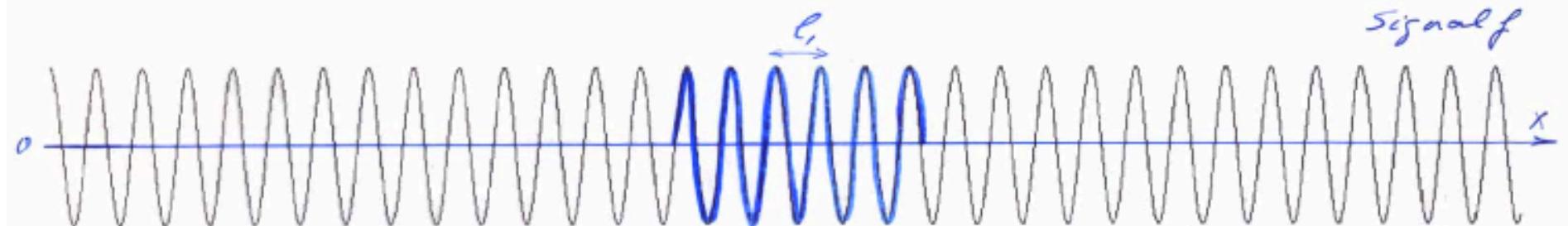


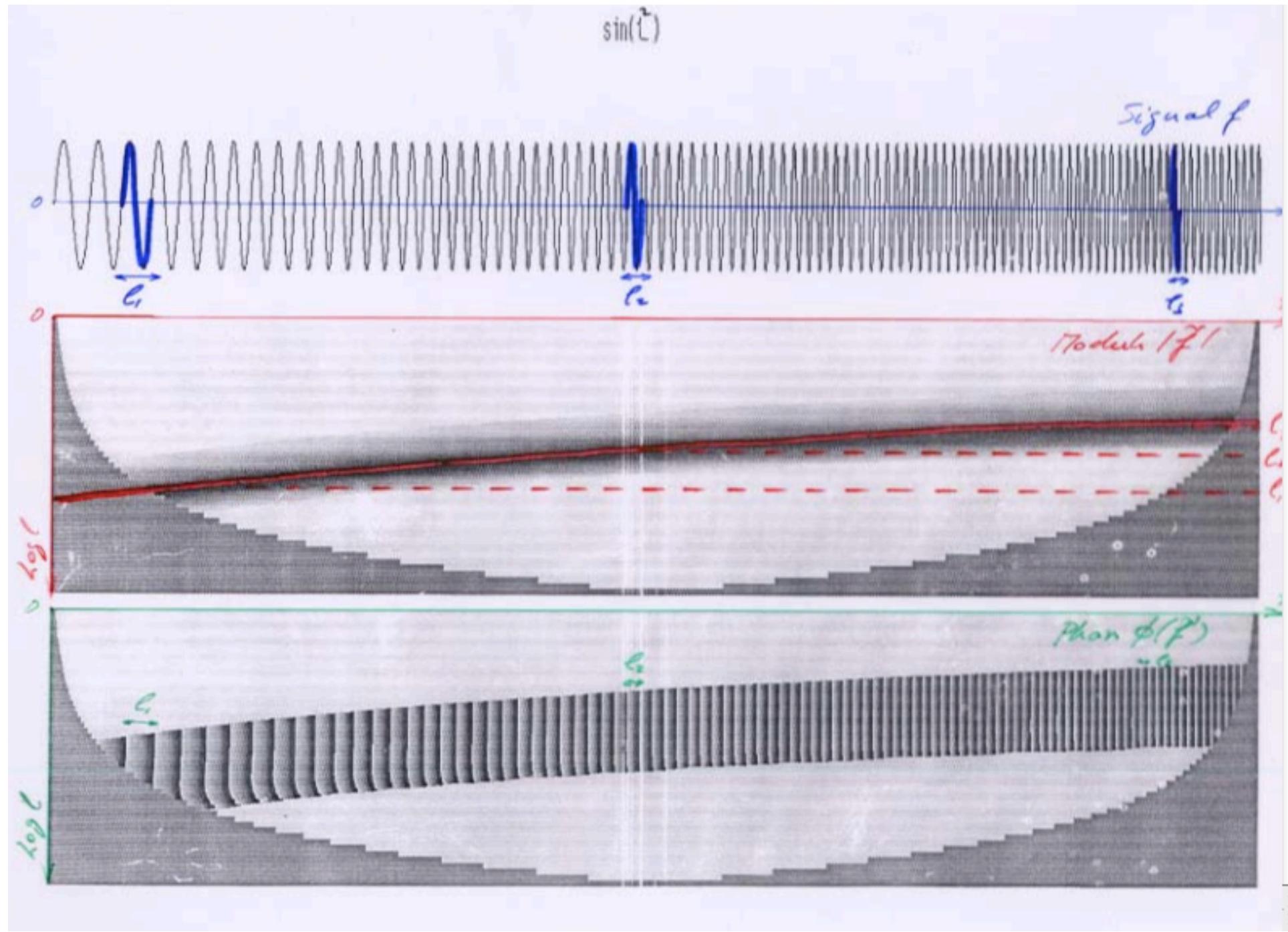
$\text{Phase } (\tilde{f}) \in \mathbb{C}^{[0, \pi]}$

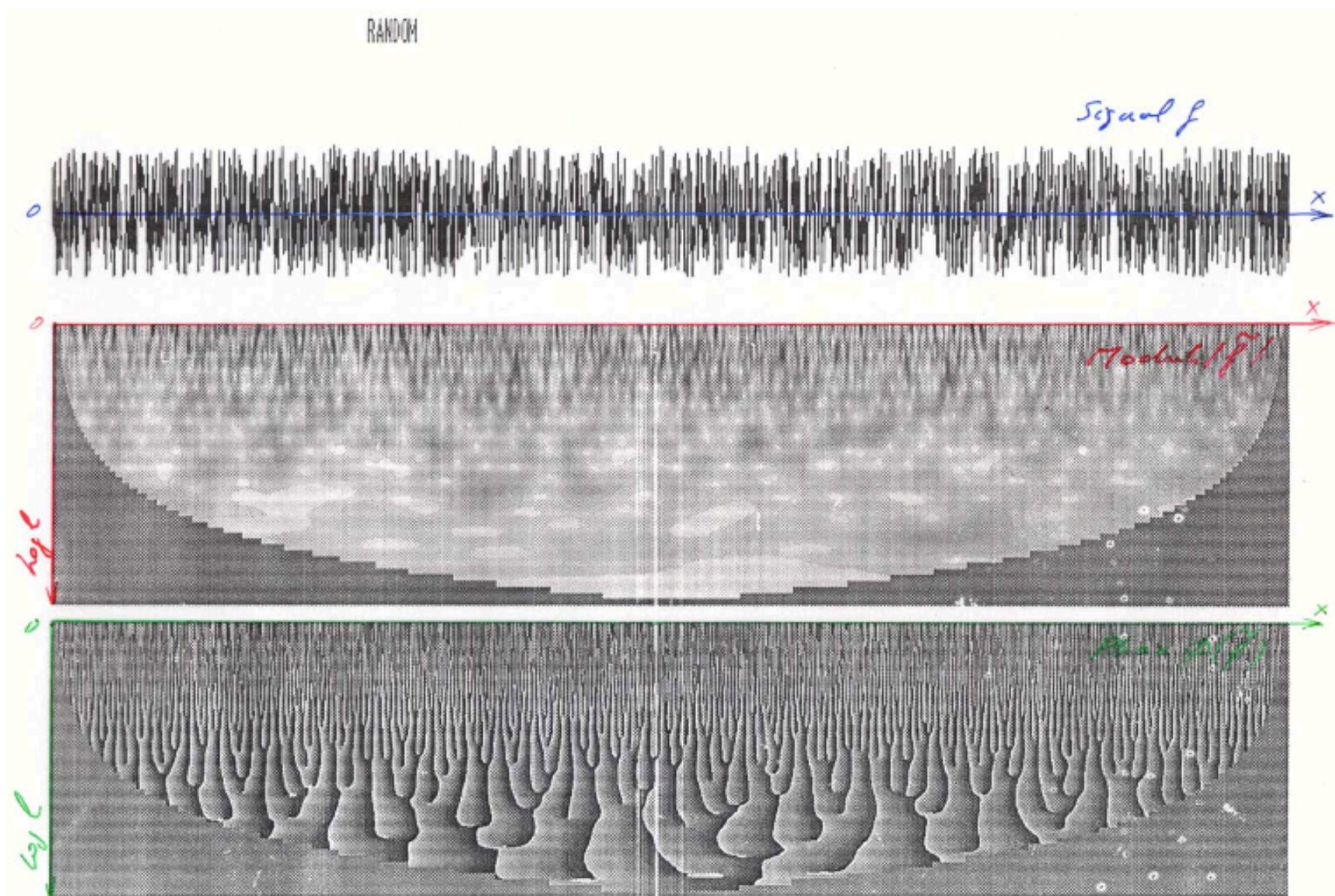
Phase des coefficients d'ondelettes du signal transformé

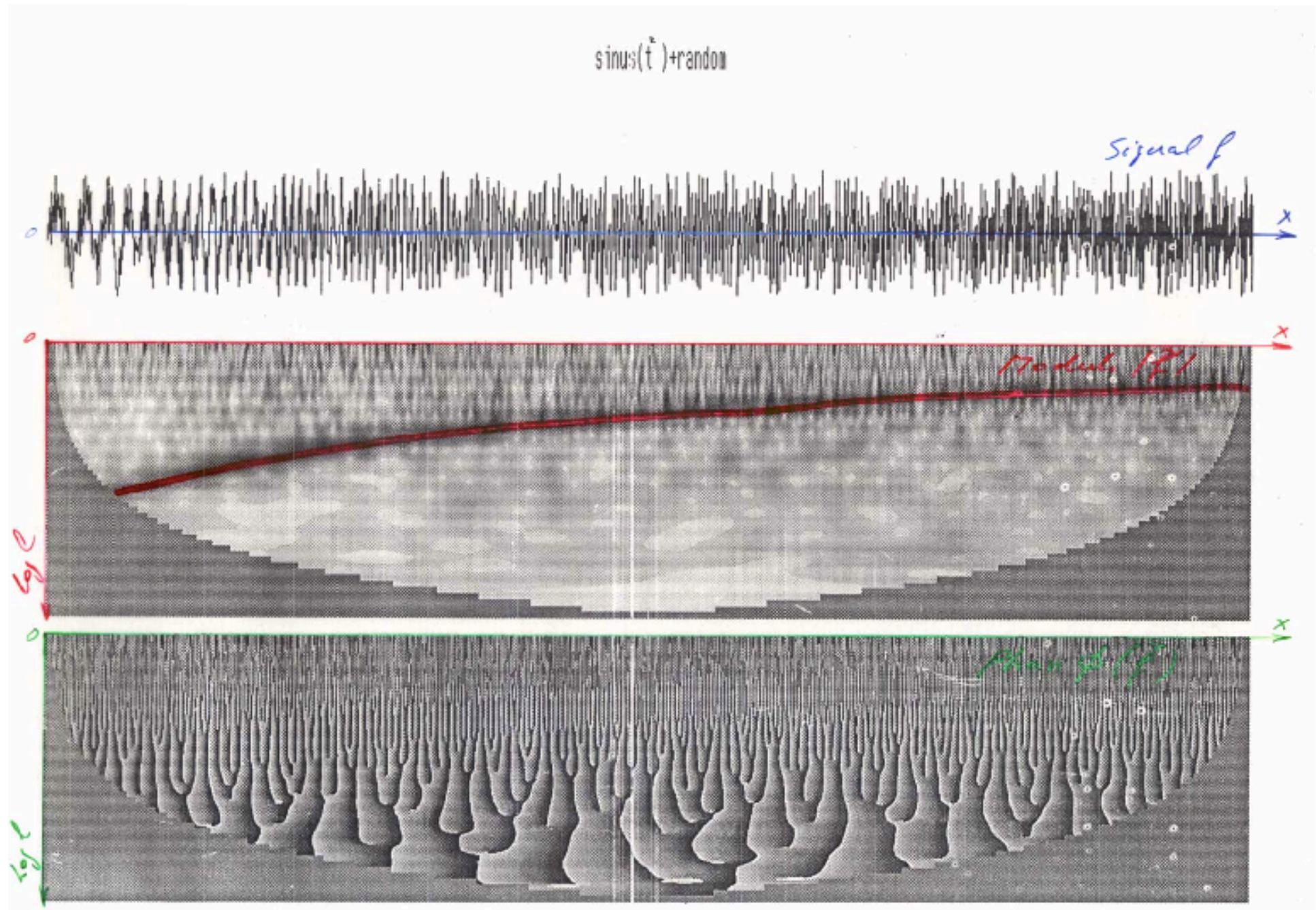


$$\cos(t) + 0.02\cos(2t)$$



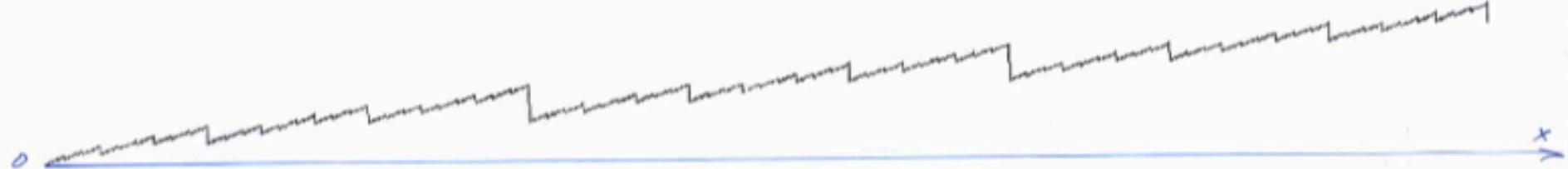




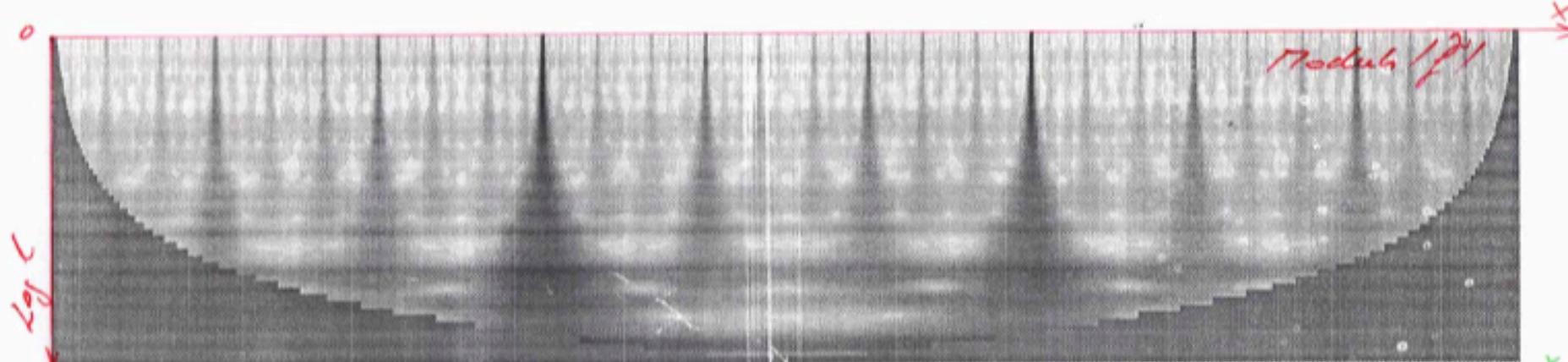


SAUTS

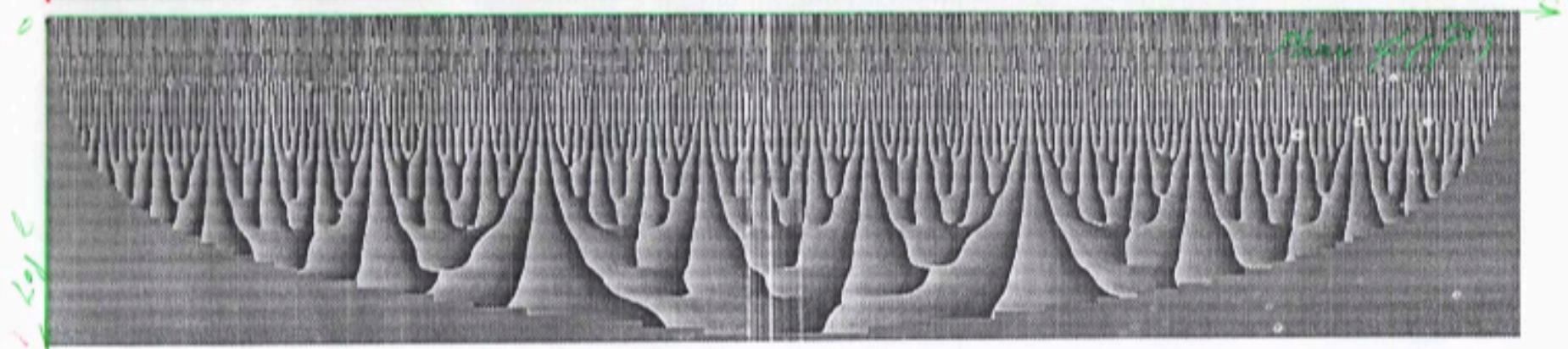
Signal f

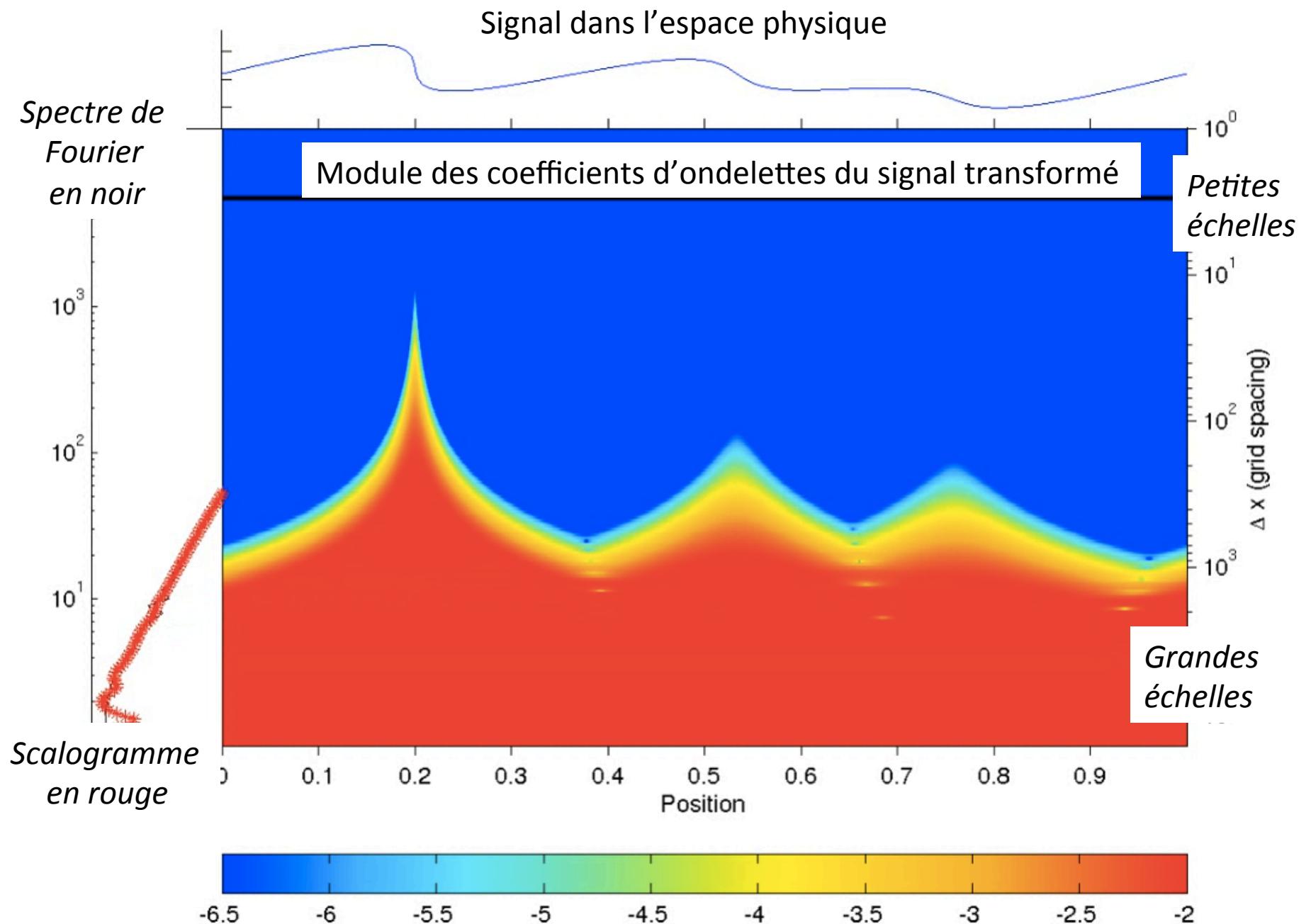


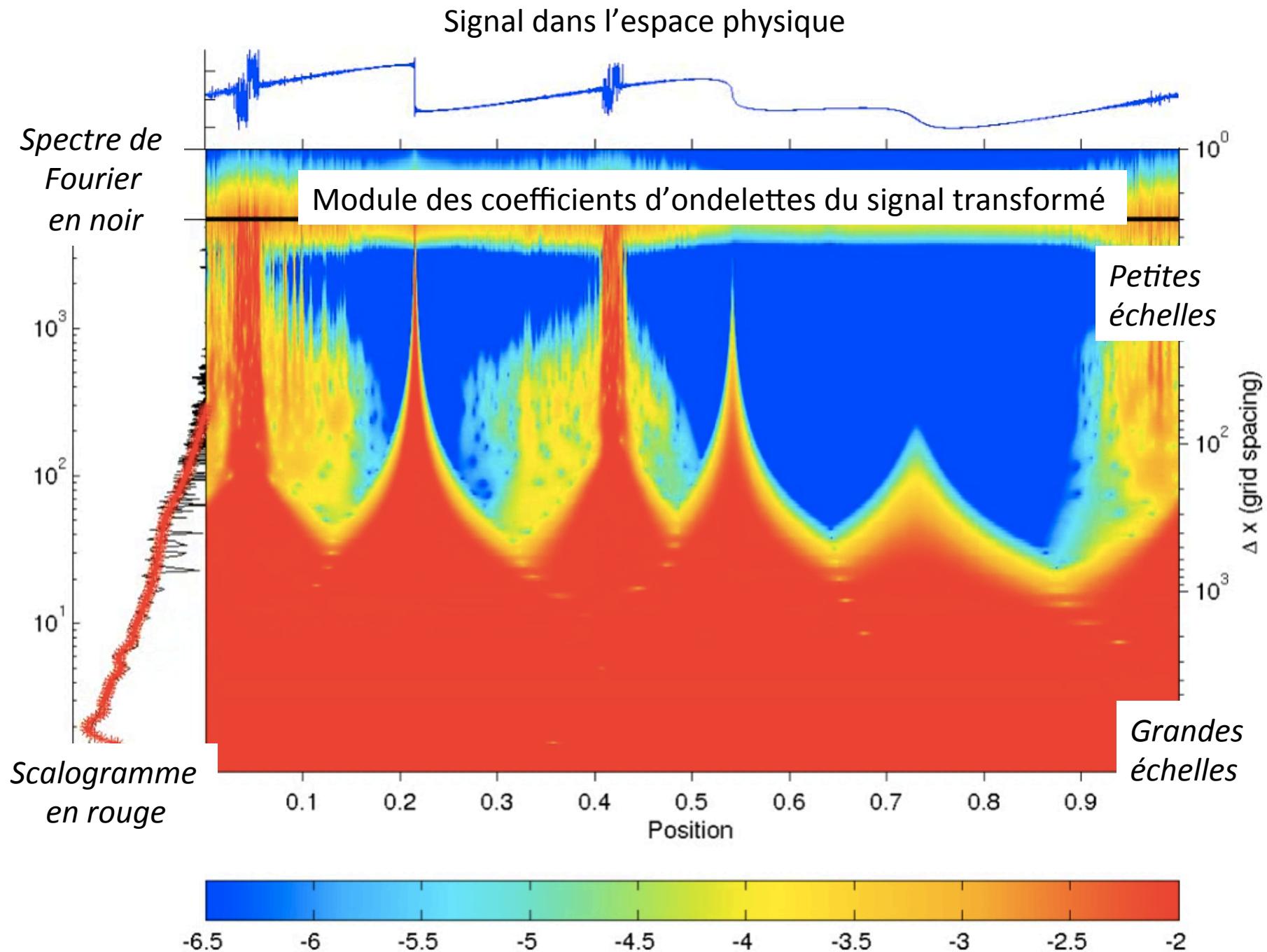
Modulus $|f''|$

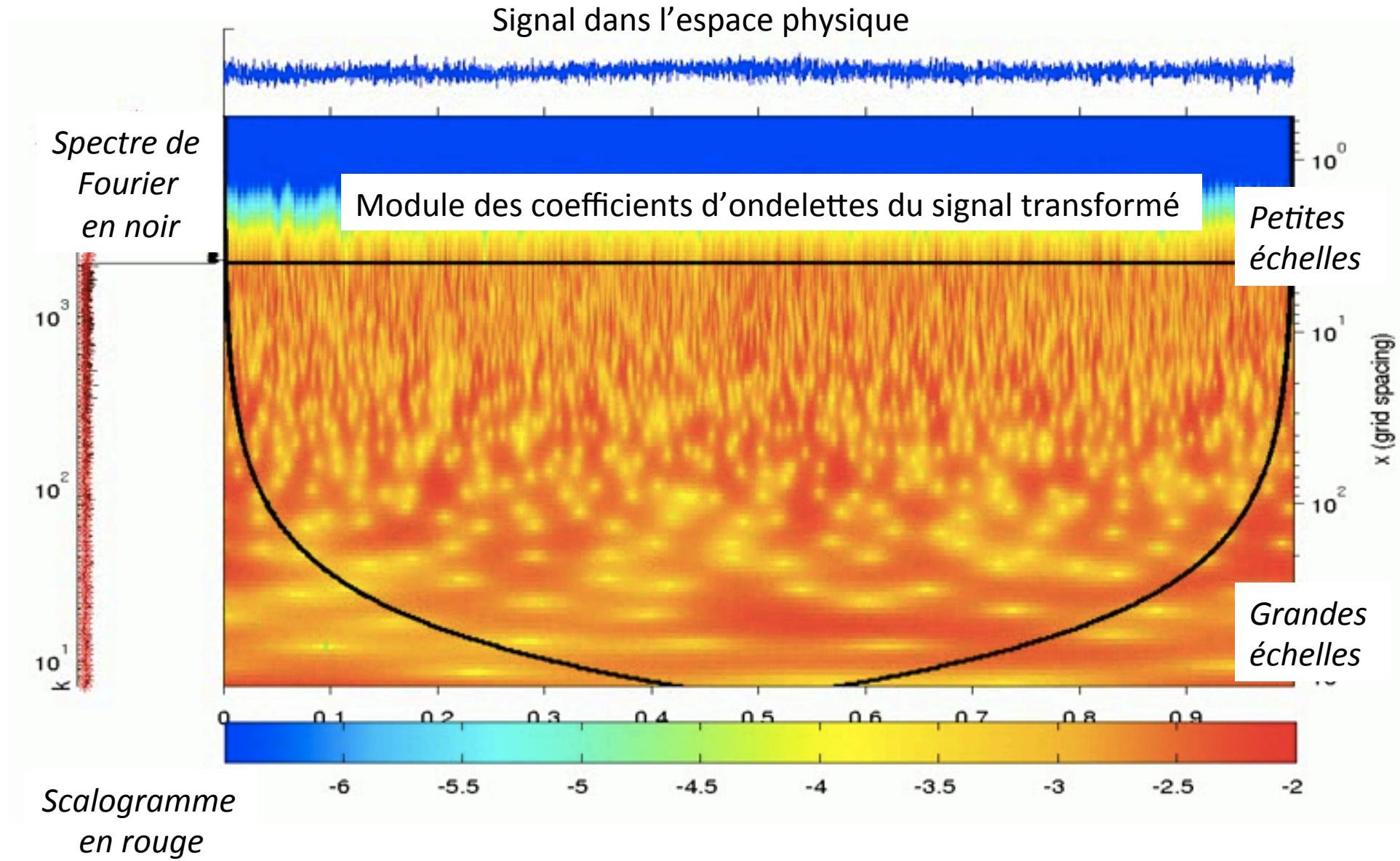


Phase $f'''(x)$



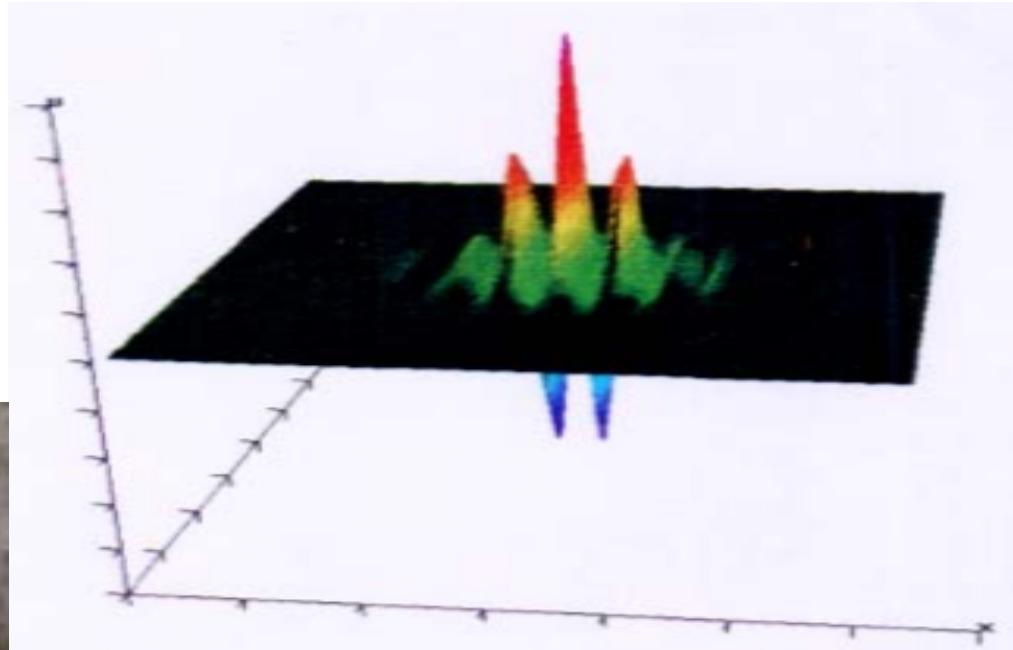






Transformée en ondelettes continues 2D

Jean-Pierre
Antoine

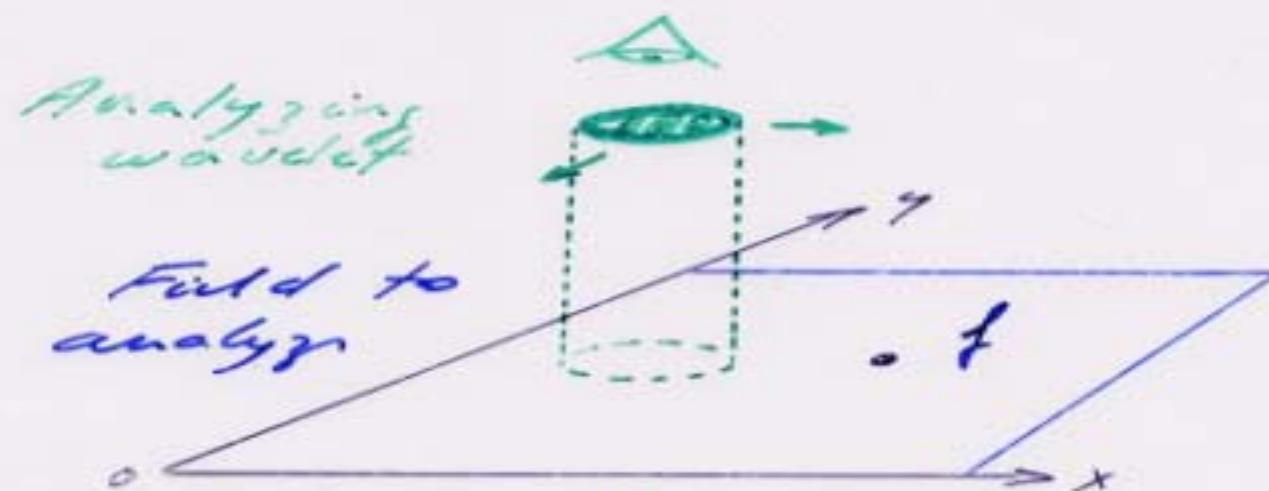


Ondelette 'mère': Morlet 2D

Romain
Murenzi



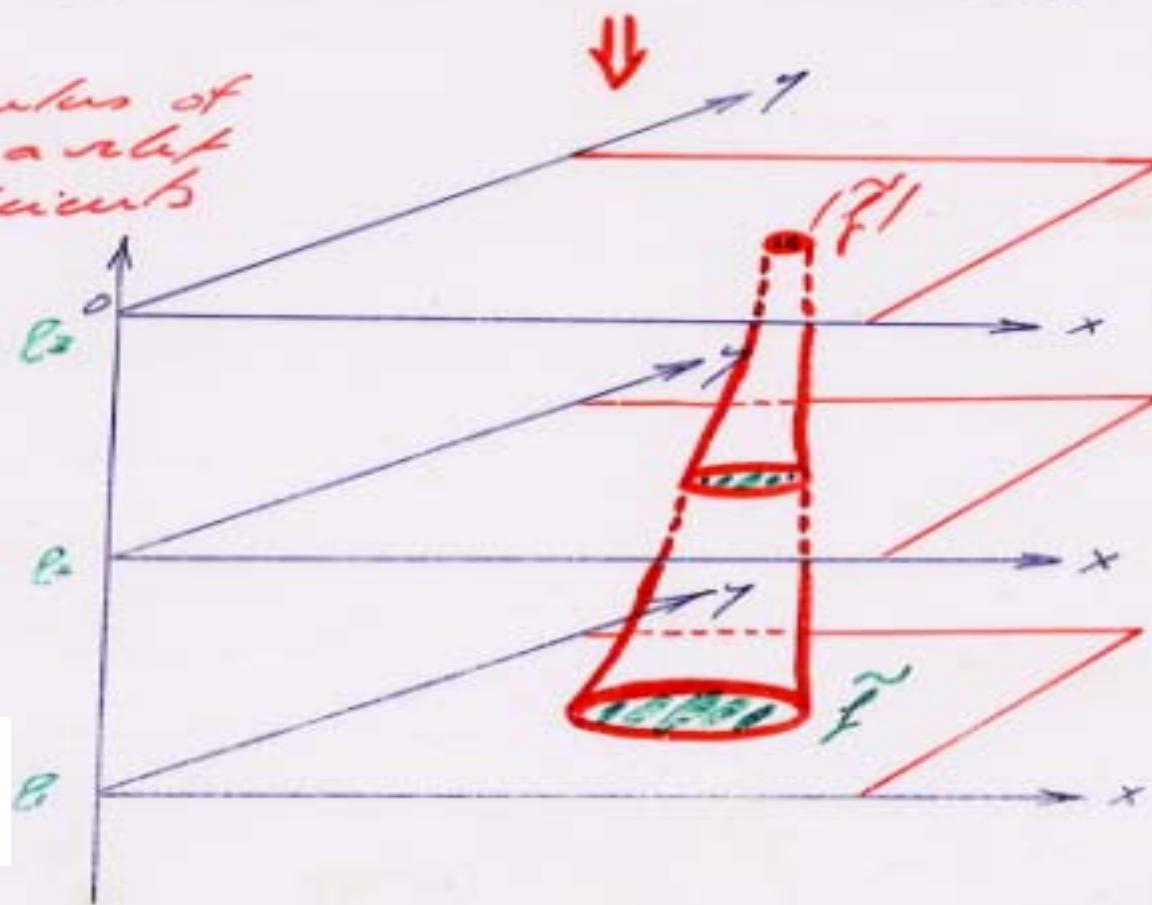
La famille d'ondelettes analysantes
est engendrée par translation,
dilatation et rotation
de l'ondelette 'mère'



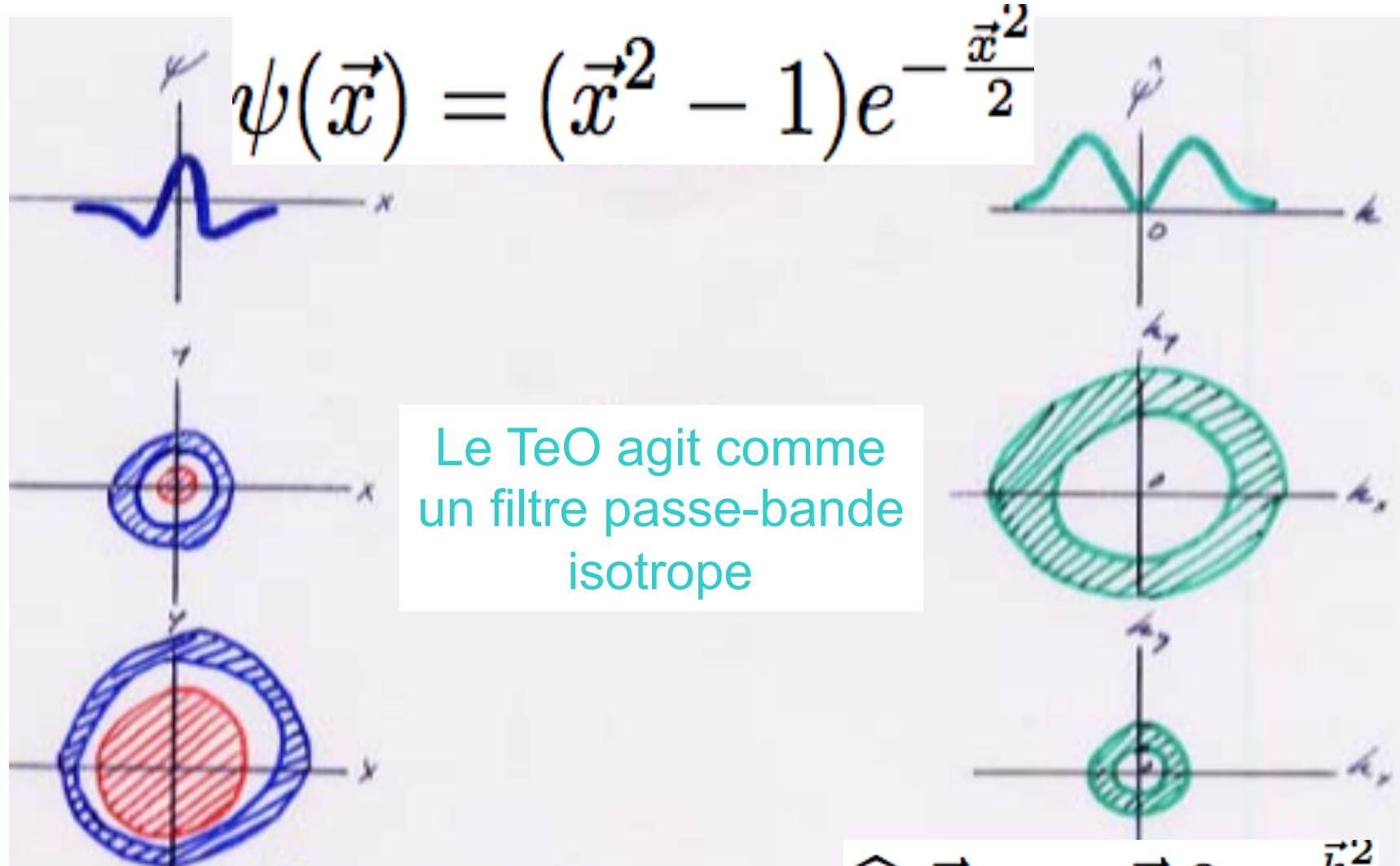
Molecules of
the wavelet
coefficients

Petites
échelles

Grandes
échelles

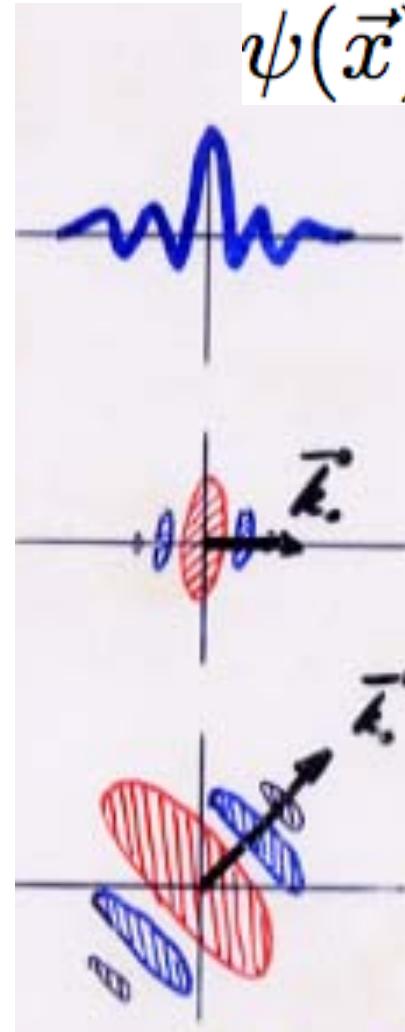


Ondelettes à valeurs réelles 2D

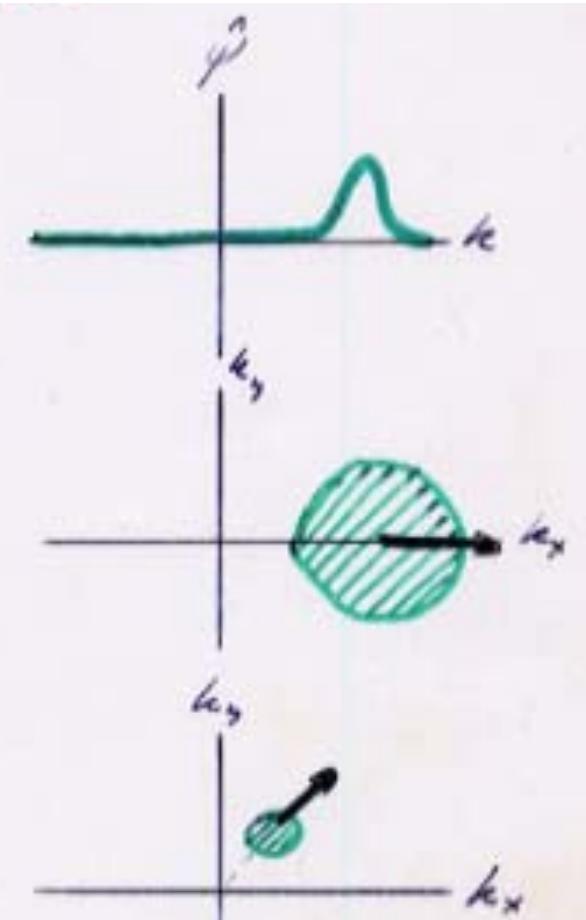


$$\hat{\psi}(\vec{k}) = |\vec{k}|^2 e^{-\frac{\vec{k}^2}{2}}$$

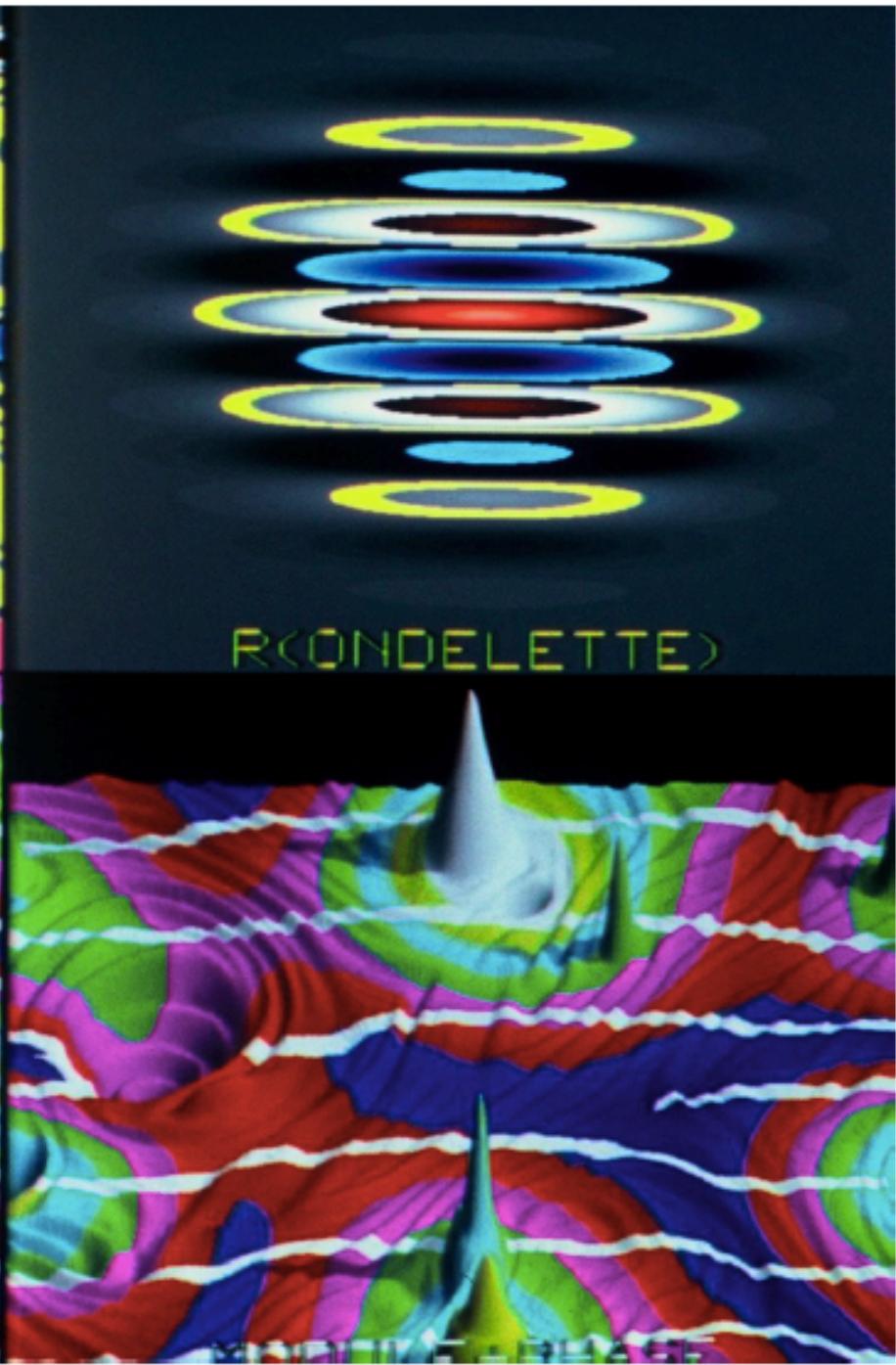
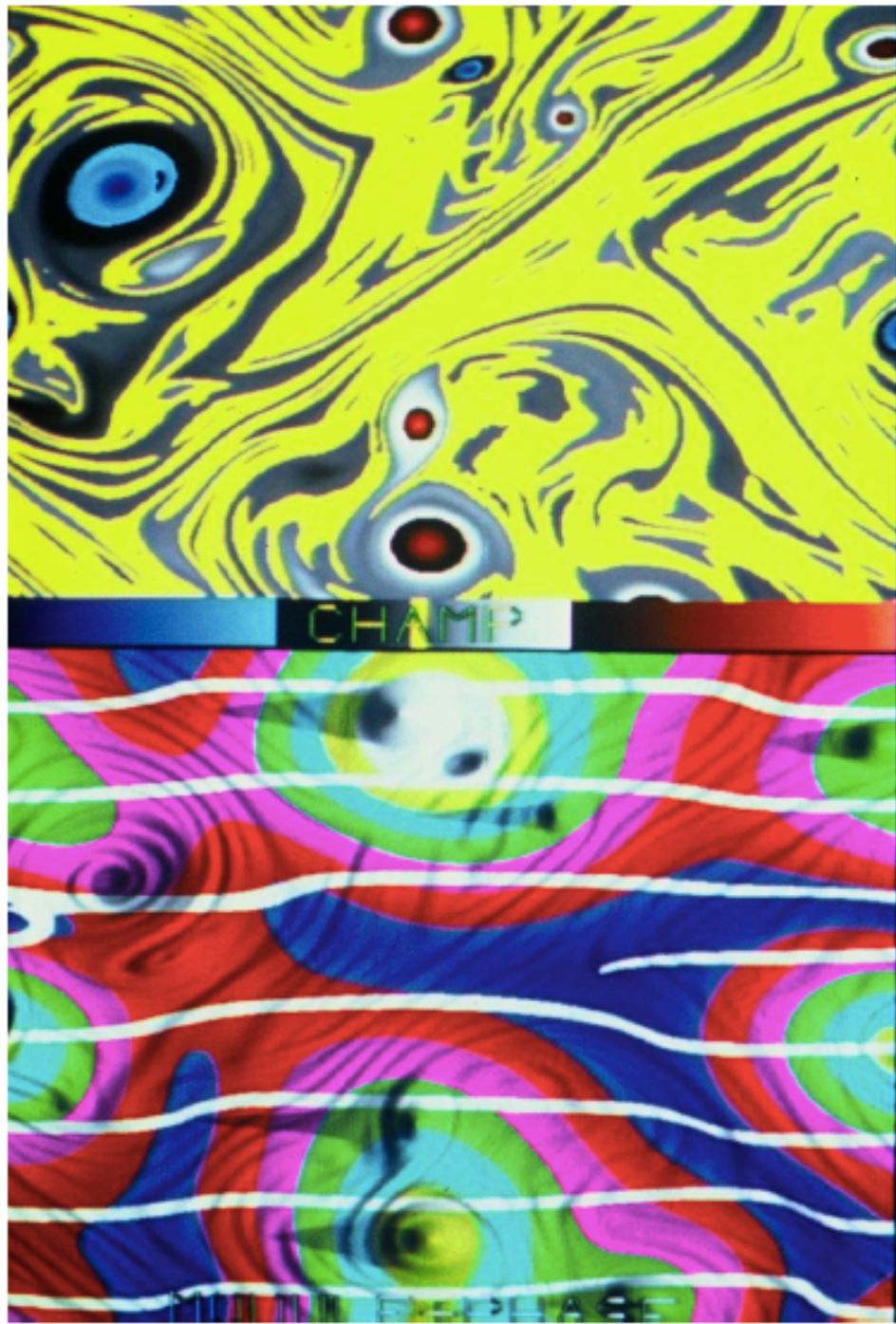
Ondelettes à valeurs complexes 2D

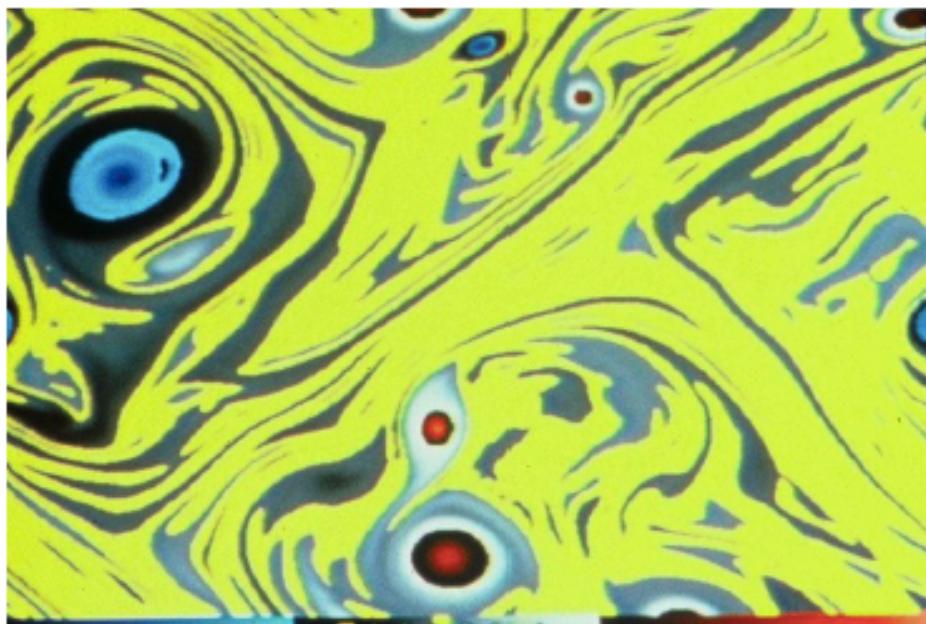


La TeO agit
comme un filtre
passe-bande
anisotrope

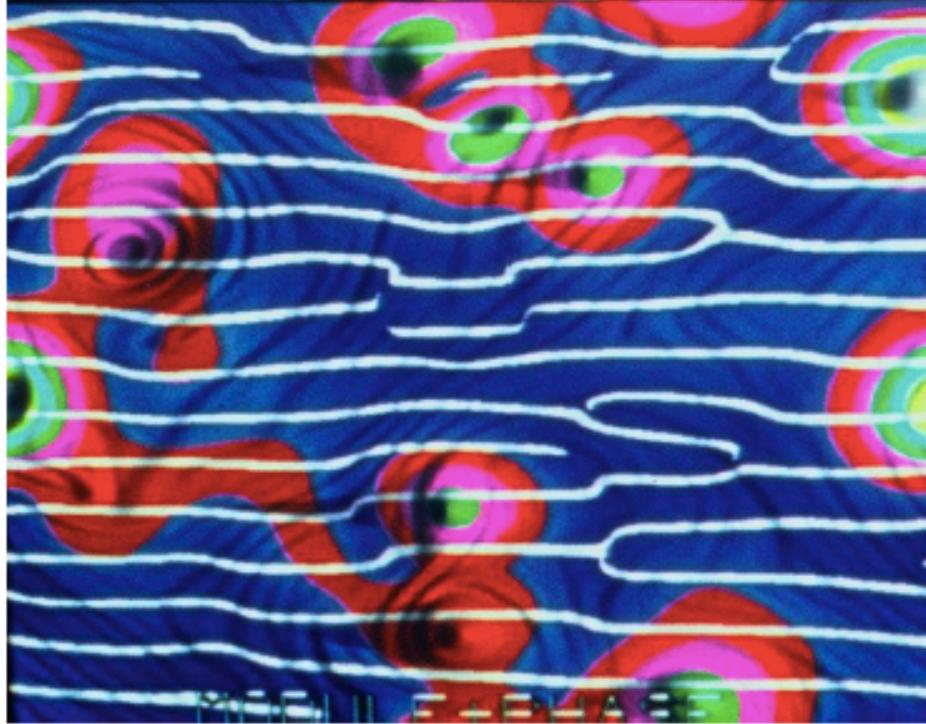


$$\hat{\psi}(\vec{k}) = e^{-\frac{(\vec{k}-\vec{k}_\psi)^2}{2}}$$

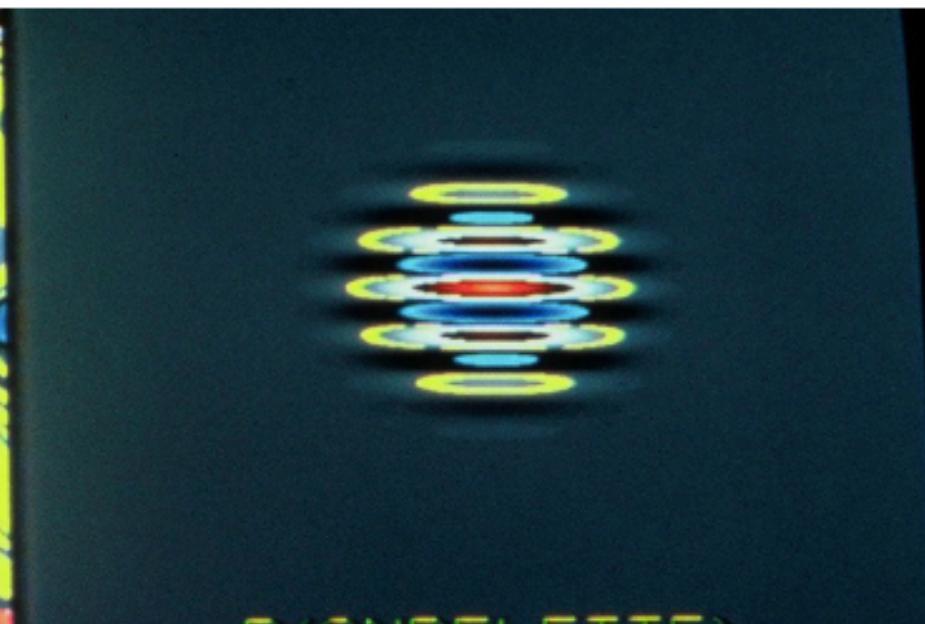




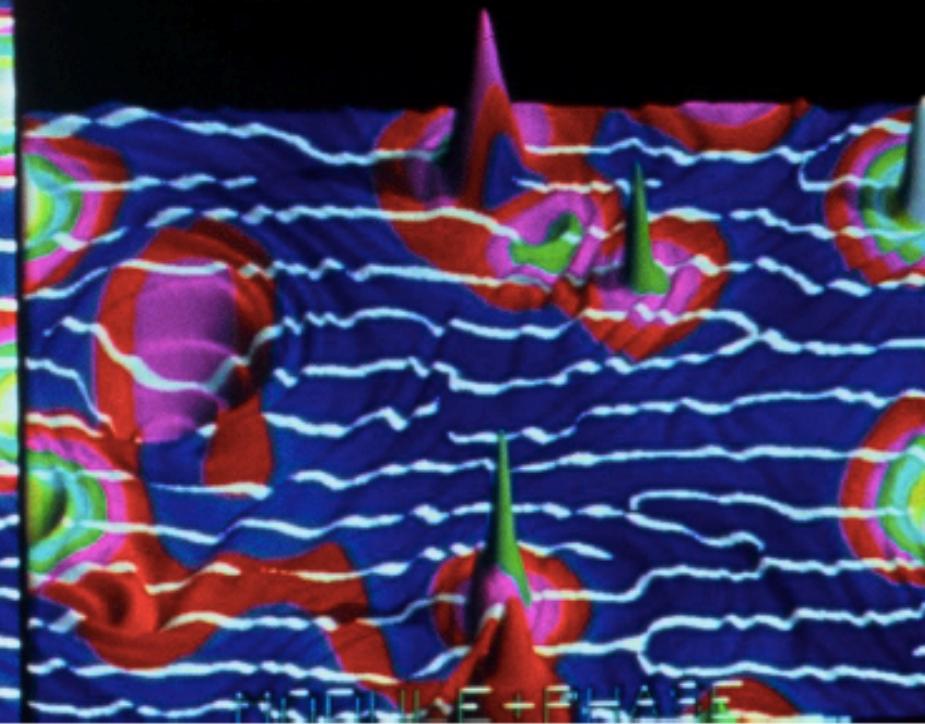
CHAMP



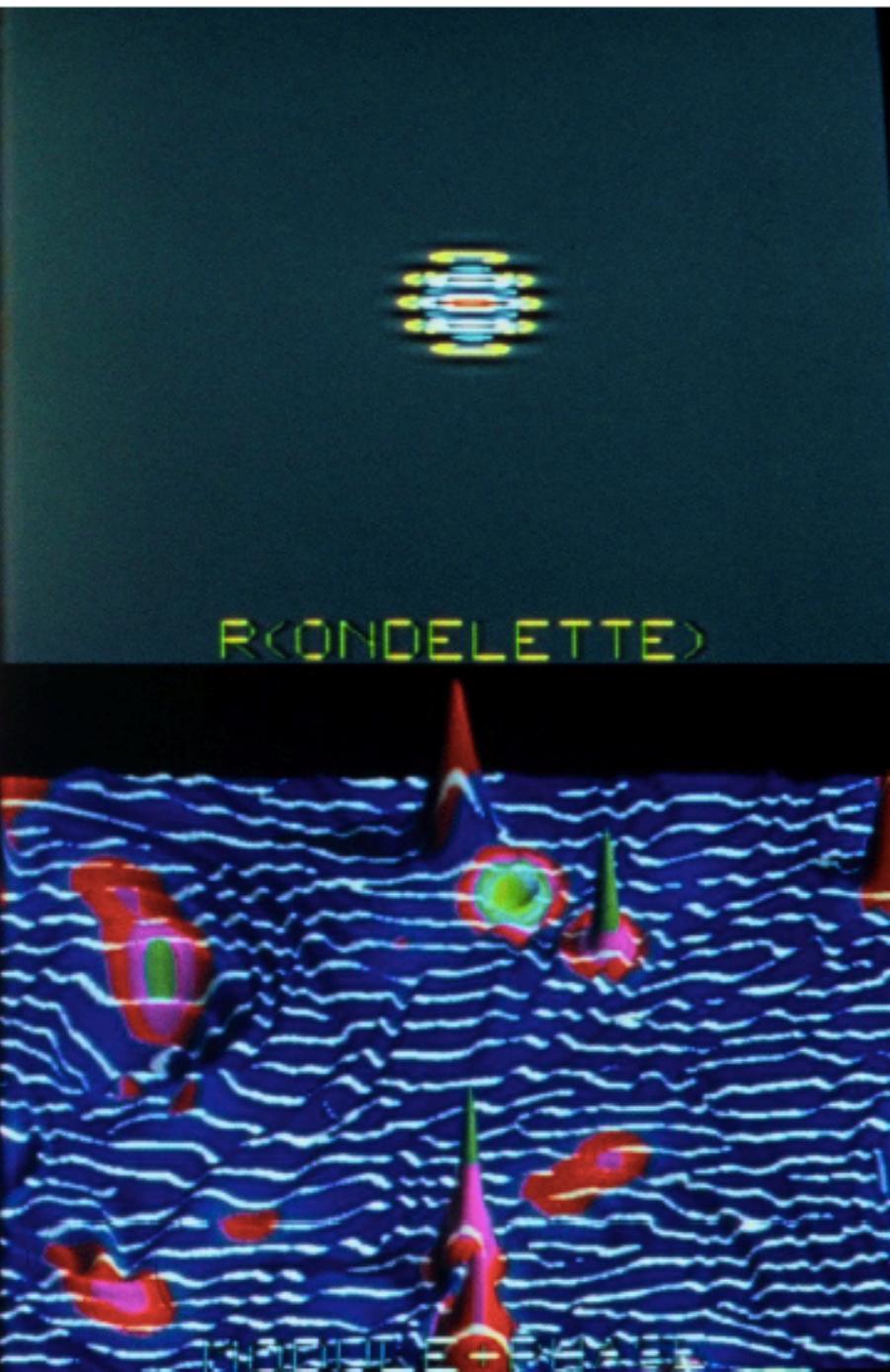
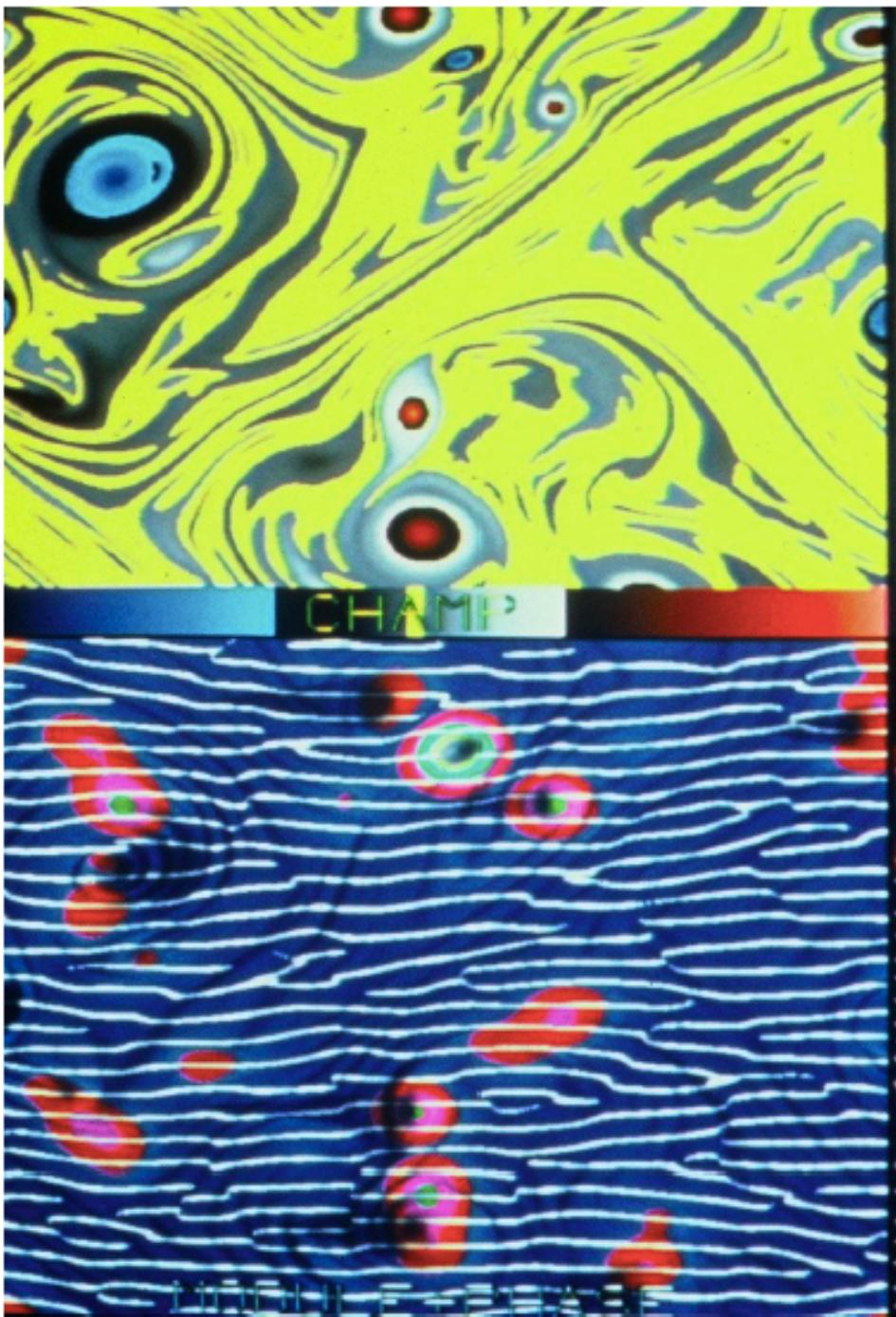
DIVERGENCE + PHASE



RONDELETTE

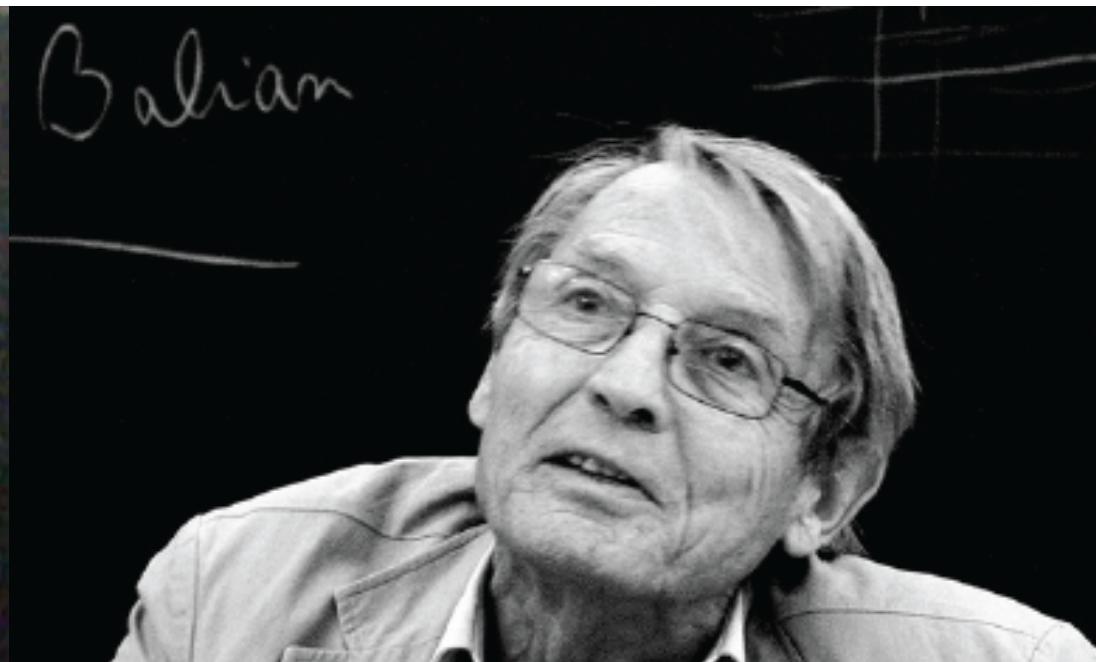
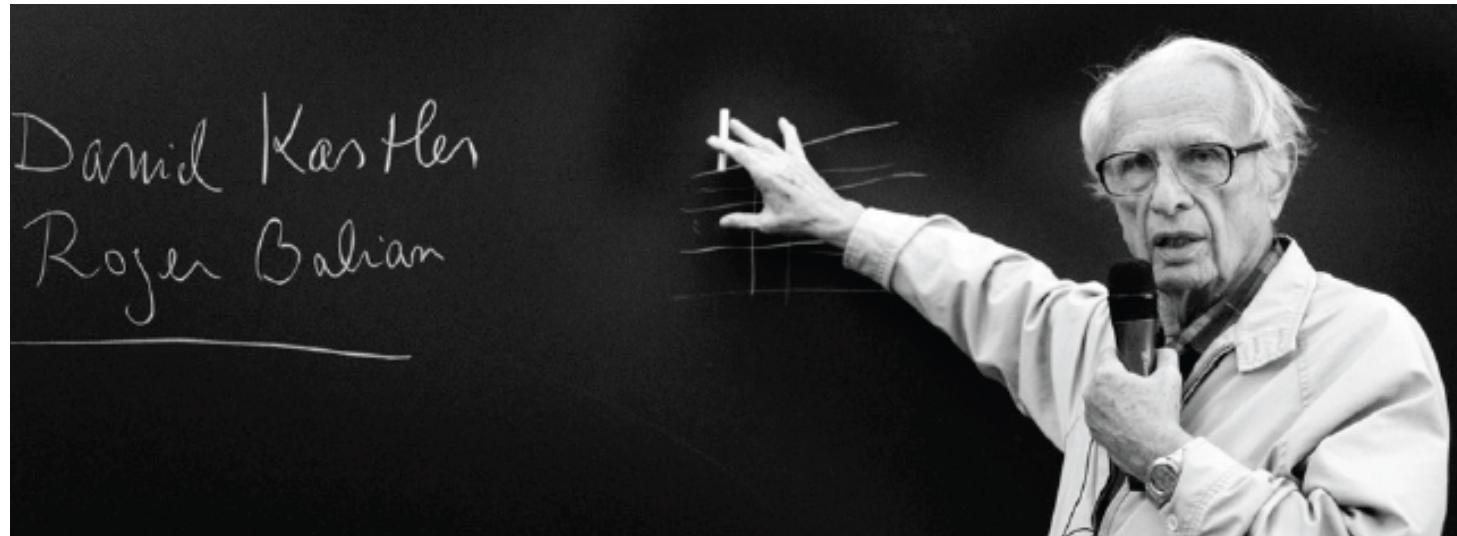


DIVERGENCE + PHASE



CIRM-SMF, Marseille-Luminy, 2011

Alex
Grossmann
Jean-Claude
Risset
Ginette
Saracco



CIRM-SMF, Marseille-Luminy, 2011

Bruno Torresani

Thierry Paul

Alex Grossmann

Richard Kronland-Martinet

Kai Schneider



CIRM-SMF, Marseille-Luminy, 2011



Jean-Pierre Kahane

Yves Meyer

L'affaire Bruno Le Roux embarrasse la gauche et la droite

► Le ministre de l'Intérieur a démissionné le 21 mars, après les révélations sur les CDD de ses filles à l'Assemblée nationale. Matthias Fekl le remplace

► Il faut être « impeccable face aux institutions et aux règles qui les régissent », a-t-il dit, après que le parquet national financier eut ouvert une enquête

► En pleine affaire Fillon, la démission de Le Roux s'imposait, alors que la droite tentait de détourner l'attention des ennuis judiciaires de son candidat

► En meeting à Courbevoie, Fillon n'a commenté ni la démission du socialiste ni les progrès de l'enquête qui le met en cause

PAGE 8 ET 10, ENTRETIEN PAGE 28

LE MATHÉMATICIEN YVES MEYER REÇOIT LE PRIX ABEL

► La « théorie des ondelettes » du scientifique a révolutionné la compression des données audiovisuelles

► Il est le quatrième Français à se voir décerner depuis 2003 l'une des plus prestigieuses récompenses de sa discipline

PAGE 10



Société Les troublants contrats de Fillon

La société de conseil du candidat de la droite a perçu 45 000 euros entre juin 2015 et juin 2016 pour avoir aidé le milliardaire libanais Fouad Makhlouf à développer ses réseaux en France, mais aussi en Russie. Un contrat de lobbying qui pourrait intéresser les enquêteurs, dont le champ d'investigation centre sur les possibles emplois fictifs de son épouse et de ses enfants a été étendu à des soupçons de « trafic d'influence »

PAGE 10

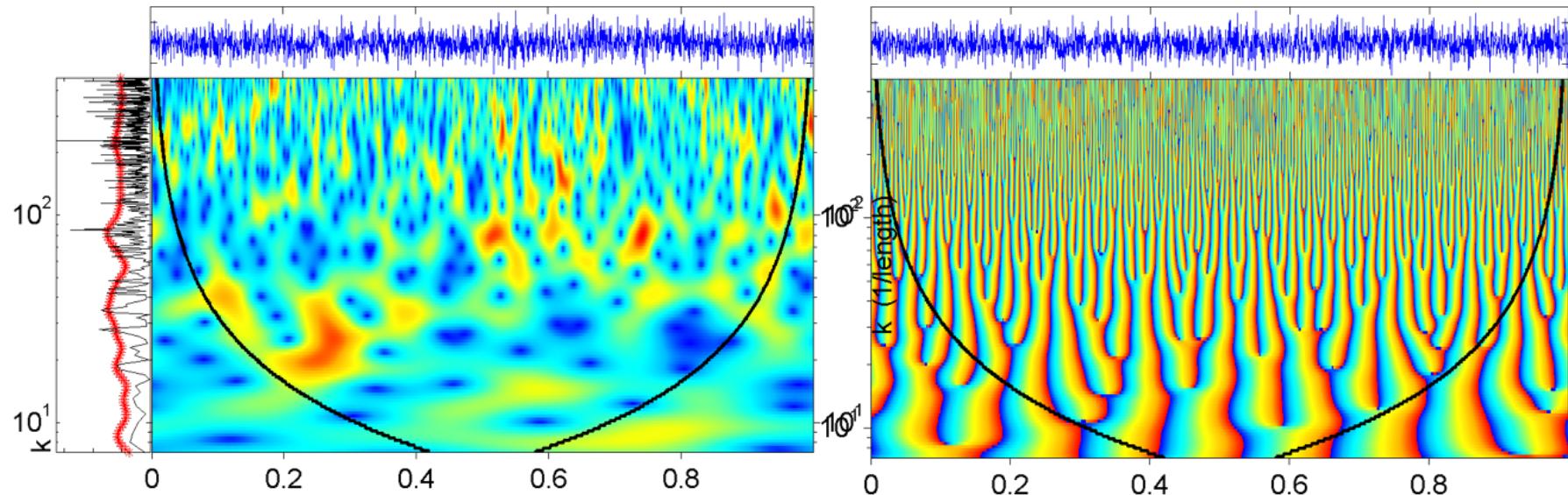
Commerce La Fnac et le Printemps se mettent au travail du dimanche

Au terme d'intenses négociations entre syndicats et direction, la Fnac ouvrira ses magasins parisiens, à compter du dimanche 26 mars, le septième jour de la semaine. Le Printemps fera de même début juillet. Près de deux ans après le vote de la loi Macron, les enseignes sont de plus en plus nombreuses à se convertir au travail dominical.

CARREFOUR ECO - PAGES 2 ET 3

Comment discréteriser la représentation

La TeO d'un bruit blanc Gaussien
revèle le noyau reproduisant de l'ondelette

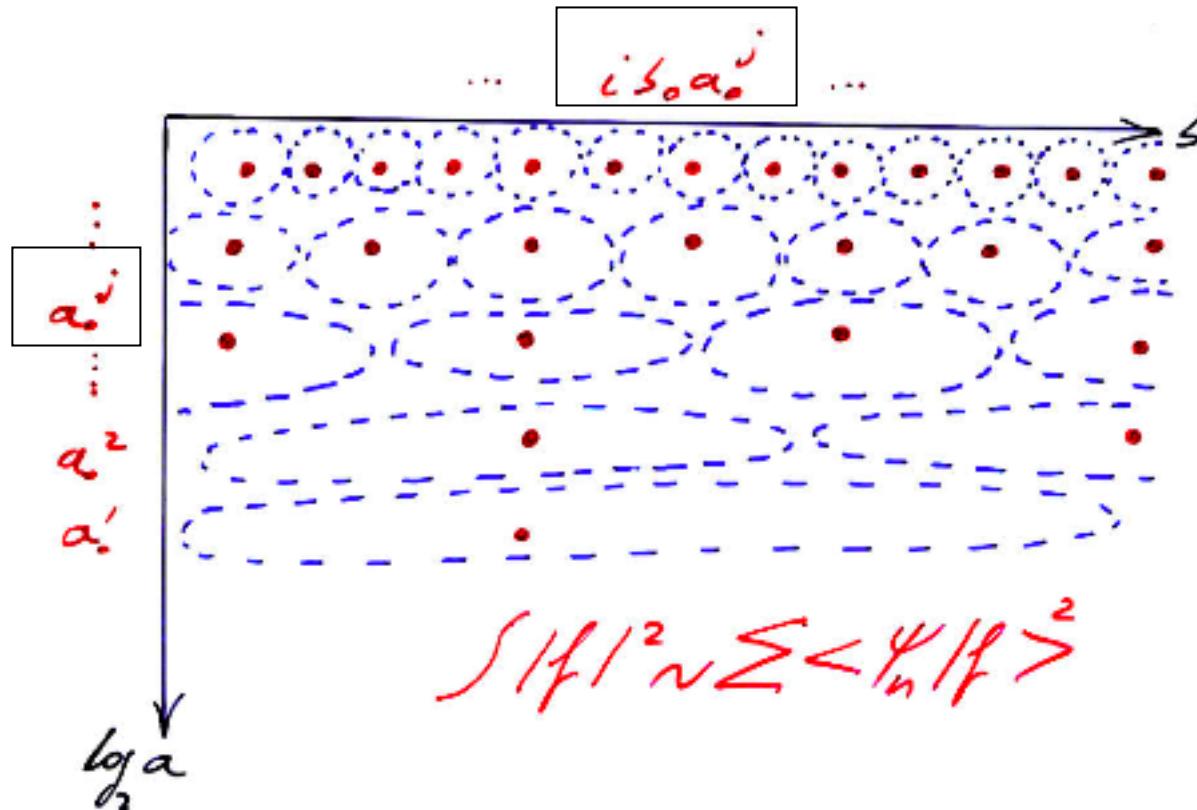


C'est la **corrélation** entre toutes les ondelettes qui correspond à **la redondance** entre les coefficients d'ondelettes

$$K(b', a', b, a) = \langle \psi_{b'a'} | \psi_{ba} \rangle$$

Wavelet frame

We can then select a finite number of wavelets restricted to a discrete grid optimally chosen such that the wavelet family associated to this grid constitutes a quasi-orthogonal basis \Rightarrow a wavelet frame



For example
for Marr wavelet
we need
 $a_0 = 2^{1/2}$
 $b_0 = 1/2$

Wavelet frame

A wavelet frame is a discrete subset of independent wavelet coefficients such that energy is quasi-conserved

$$A|f^2| \leq \sum_{ji} |\langle \psi_{ji} \rangle|^2 \leq B|f|^2$$

⇒

we can reconstruct any function of Hilbert space from the discrete subset of wavelet coefficients.

*I. Daubechies, A. Grossmann and Y. Meyer
Painless non orthogonal expansion
J. Math. Phys. 27(5), 1986*

If the frame bounds $A=B$ we have an tight frame and if $A=B=1$ we have an orthogonal basis.



한국일보

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2014.08.14 목요일 | 제20665호 | 41판

아픔·분단·순교의 땅에서 치유의 손길 내민다

프란치스코 교황 오늘 방한
세월호 유족·생존자 위로하고
위안부 피해자·해고노동자 등 만나
트위터에 "여정 시작" 한글로 인사



프란치스코 교황이 이번
방한에서 치유의 손길을 내
밀 이들은 세월호 참사 희
생자 가족과 생존 학생이
다. 교황은 돈을 우상으로
삼기는 세태를 비판하고 사
람 중심 사회를 강조해 왔
다. 세월호 참사는 회고 가

치인 인간의 존엄이 철저히 무시된 사회 구조가 빚
은 비극이다. 교황은 초기경 시절 아르헨티나의 크
로마농 화재 참사 추모 미사에서 "부에노스 아이레
스는 더울 필요가 있다. 더는 여기에 없는 우리 아이
들을 위해 충분히 울지 않았다"고 목소리를 높인 적
이 있다. 그가 세월호의 눈물을 어떻게 어루만질까.
거리의 아픔과 함께 해 온 그는 방한 기간 중 집전

하는 시복식과 미사를 통해 상처를 간직한 우리 사회
의 또 다른 이웃과도 마주한다. 일본군 성노녀에 피해
자 할머니, 이주노동자, 세부적인 일상 송년집 건설 반
대 주민, 제주 경쟁력을 주민, 생용자동차 해고노동
자, 용산참사 희생자 가족이 그들이다. 한반도 역시
치유의 기적이 필요한 꽁이다. 세계 유일의 분단국가
인 남북의 평화와 화해를 위해 교황은 기도할 것이다.

신앙의 신념을 지키다 침행을 당한 한국의 순교자
124인을 교황이 직접 복자·복녀로 선포하는 시복식
도 의미가 크다. 조선시대 천주교는 특히 하대반연
천민, 폐성, 여성의 자신의 존엄을 찾기가는 과정이
었다. 교황의 시복식 집전은 그들의 피를 영광으로
승화시키는 의미가 있다.

물론 교황이 만능 해결사는 아니다. 되어 우리 사회에 여러 질문들을 던지고 떠나지도 모른다. 그의

방한을 한 번의 이벤트가 아닌 의미 있는 전환의 계
기로 만드는 건 남은 자들의 몫이다. 기록적 신자이며
역사학자인 조광 고려대 명예교수는 "한국 사회
의 주인은 우리"라며 "우리가 할 일은 교황이 전하고
자 하는 좋은 가치를 찾아내고 행동으로 실천하는
것"이라고 말했다.

지난해 7월 브라질, 올해 5월 중동지역에 이어 한
국을 세 번째 순방지로 떠난 교황은 14일부터 4박 5
일간 한국에 머문 뒤 18일 바티칸으로 돌아간다.

한편 프란치스코 교황은 13일 트위터에 "한국으
로의 여장을 시작하여, 한국과 아시아 전역을 위한
저의 기도에 동참해주시기를 부탁 드립니다"라는 인
사글을 한글로 올렸다. 이 글은 2시간이 만에 수천
건 리트윗되고 1,000여건 정도 관심 글로 지정되는
등 큰 주목을 받았다. 김자은기자 luna@hk.co.kr

필즈상 사상 첫 여성수상자 탄생

서울세계수학자대회 개막
이란 출신 美 미르자카니 영예
제3세계 수상자도 첫 배출

'수학계의 노벨상'이라고 불리는 필즈상을 사상
최초로 여성 수상자가 13일 탄생했다. 역사적 주인
공은 마리암 미르자카니(37) 미국 스텐퍼드대 교수.
이란에서 태어나 학부 교육까지 이란에서 마치고 미
국 하버드대에서 박사학위를 받은 미르자카니 교수
는 단승에 전 세계 여성 수학자들의 불모대로 떠올
랐다.

★관련기사 4면
미르자카니 교수의 수상으로 13일 필즈상이 발표
된 서울 강남구 삼성동 코엑스 '2014 서울세계수학
자대회' 개막식에서는 수상자와 수여자, 주최자가
모두 여성으로 구성되는 진풍경이 연출됐다. 개최국
국가 원수가 시상하는 필즈상의 80년 전통에 따라
이날 수여자는 백금에 대롱이 있고, 대회를 주최하
는 세계수학연맹의 첫 여성 회장인 잉그리드 도브시
미국 듀크대 석좌교수도 시상식 무대에 섰다. 대회에
참석한 국내외 수학자들은 "수학사에서 당분간
다시 보지 못할 장면"이라며 여성 리더들에게 아낌
없는 박수를 보냈다.

미르자카니 교수와 함께 ▲아르투르 아빌라(35)
프랑스 파리 대학 교수 ▲만줄 바르가비(40) 미국
프린스턴대 석좌교수 ▲미란 해어리(38) 영국 외대
교수 등 3명도 이날 제18회 필즈상 수상자로 지
명됐다. 수상자 중 가장 어린 아빌라 교수는 고국 브
라질에서 박사학위까지 마친 '토종' 수학자로 미주
와 유럽 이외 지역에서 박사학위를 받은 최초의 필
즈상 수상자로 기록됐다. 바르가비, 해어리 교수는
각각 캐나다와 오스트리아 출생이다.

올해 필즈상은 최초 기록이 둘이나 나왔지만 이번
은 없었다. 4명 모두 국제학계에서 수상자 후보로 이
름이 오르내리며 실력을 인정받아 왔다. 박 대통령
은 이날 축사에서 "첫 여성 수상자는 도전과 열정에
박수를 보낸다"며 "수학이 젊은이들과 대중들이 친



수상·수여·주최자 모두 여성- 수학사 명장면

13일 개막한 '2014 서울세계수학자대회'에서 첫 여성 필즈상 수상자인 마리암 미르자카니(가운데) 미국 스텐퍼드대 교수가 박근혜 대통령과 잉그리드 도브시 국제수학연맹 회장과 대사진기자단

Orthogonal wavelets

There exist special wavelets

$$\psi_{ji} = 2^{j/2} \psi(2^j x - i)$$

orthogonal to all their translates (indexed i)
and to all their dilates (indexed by j).

Since wavelets have zero mean, they correspond to pass-band filters sensitive to details but insensitive to the mean and one should complement them with a scaling function which corresponds to a low-pass filter.



The wavelet family ψ_{ji} plus the scaling function ϕ_i give a complete representation in $L^2(\mathbb{R})$ and they constitute an orthogonal basis of $L^2(\mathbb{R})$.

Choice of the mother wavelet

1. Locality :

Ψ should have rapid decay at infinity.

2. Smoothness :

Ψ should have $(r-1)$ continuous derivatives and a bounded derivative of order r .

3. Orthogonality of the translates and dilates :

by discrete steps $x=2^{-j} i$ in space

and discrete steps $l=2^{-j}$ in scale

corresponding to a dyadic tiling of wavelet space.

$$\psi_{ji} = 2^{j/2} \psi(2^j x - i)$$

Compactly supported wavelets

In 1909 Alfred Haar constructed an orthogonal basis of translated and dilated functions generated from a compactly supported function which is not smooth.

'About 80 years were needed until Ingrid Daubechies proved that for $r > 0$ one can construct $\psi(x)$ of class C^r with compact support satisfying the above conditions. If $\psi(x)$ is compactly supported Pierre-Gilles Lemarié has proven that there exists multiresolution analysis behind this orthogonal basis.'

Yves Meyer

Based on those works Stéphane Mallat has proposed a fast wavelet algorithm which is used for most orthogonal wavelet applications.

Orthogonal wavelet transform

Wavelet analysis :

$$\tilde{f}_{ji} = \langle \psi_{ji} | f \rangle \quad \text{with} \quad \psi_{ji} = 2^{j/2} \psi(2^j x - i)$$

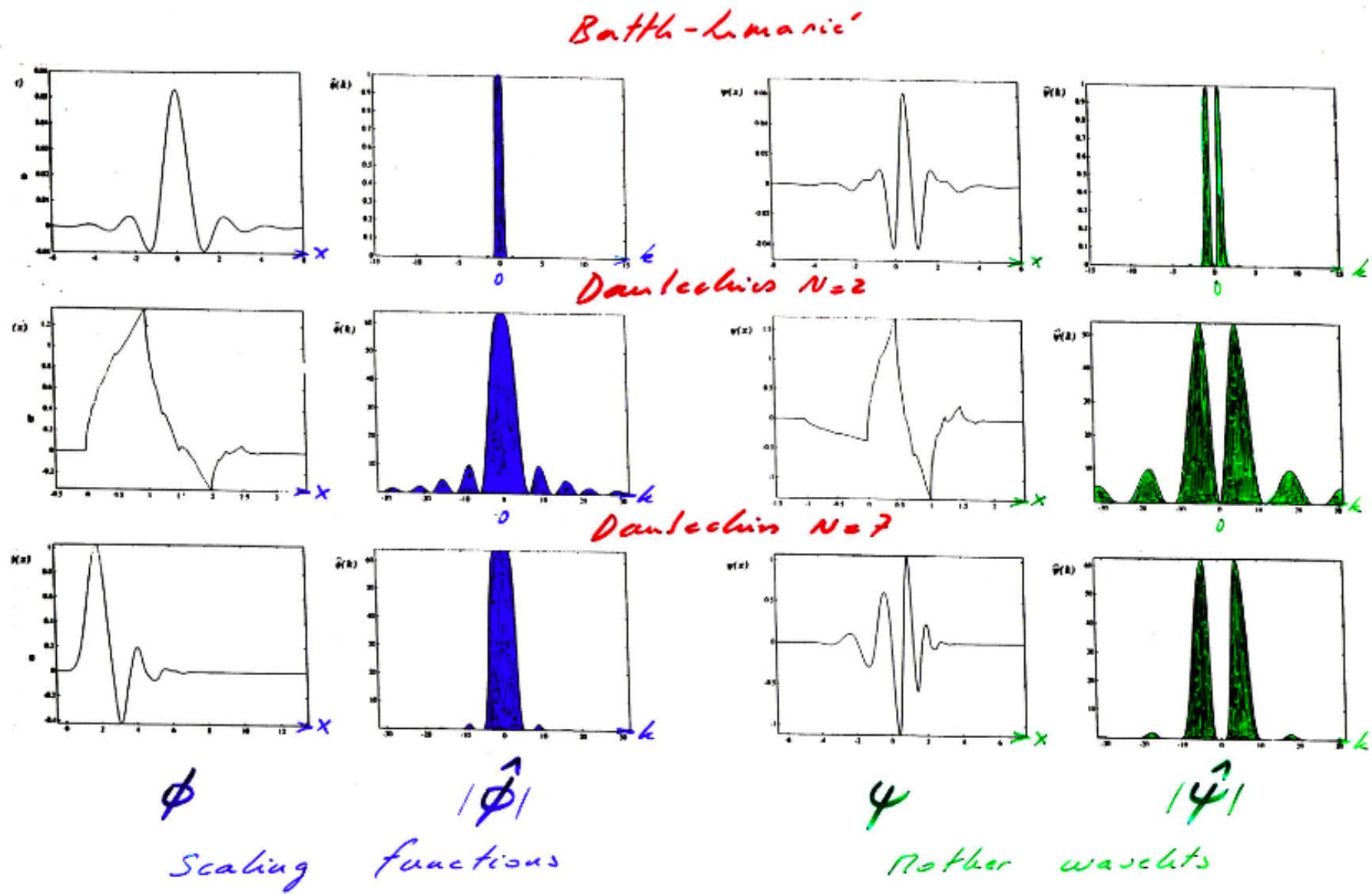
$a_0=2$ $b_0=1$

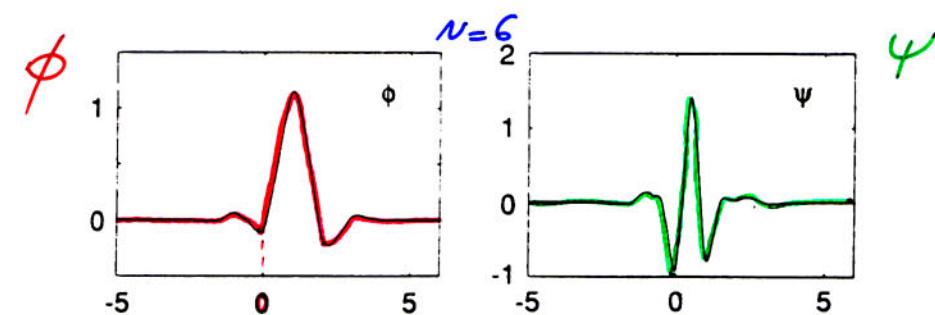
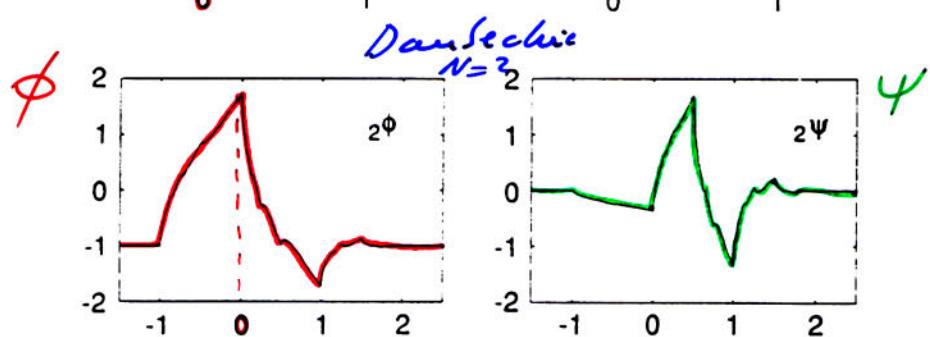
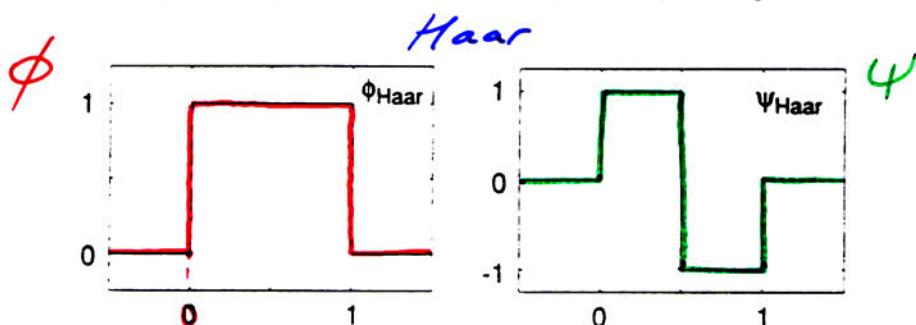
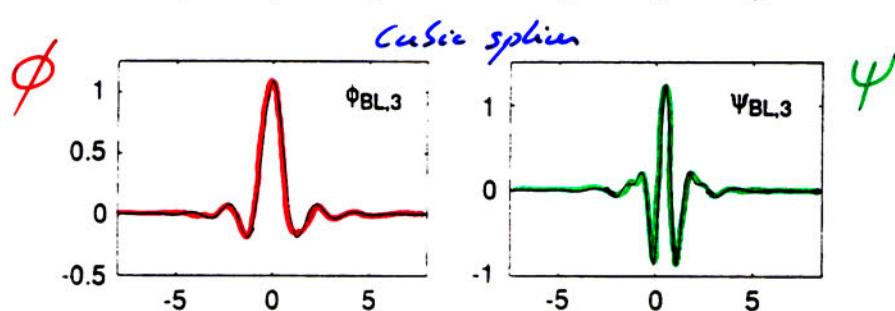
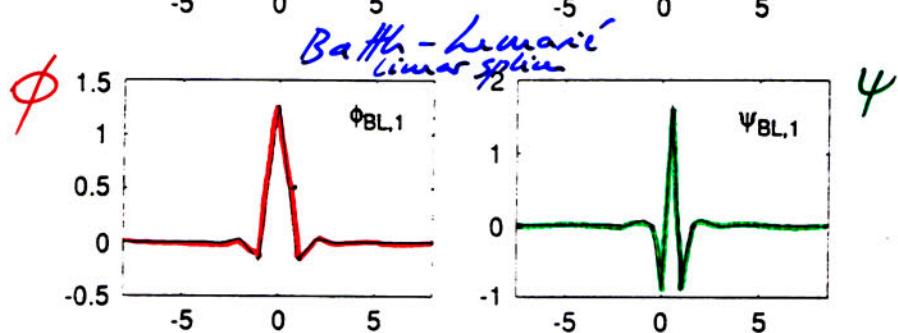
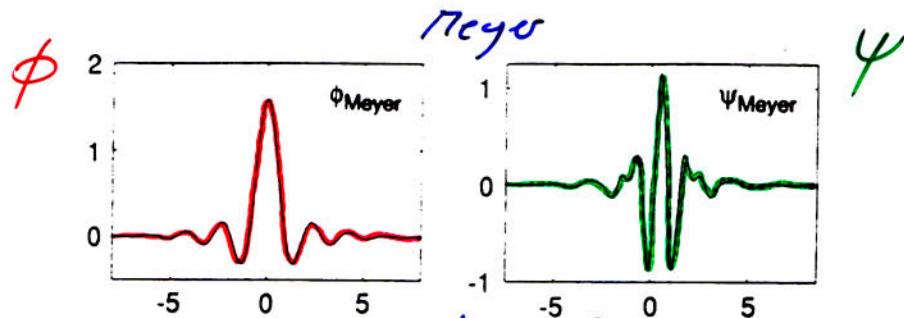
Wavelet synthesis :

$$f = \sum_{ji} \langle \psi_{ji} | f \rangle \psi_{ji}$$

A signal sampled on N points is
wavelet analyzed and synthetized in CN operations
if one uses compactly-supported wavelets
computed from a quadratic mirror filter of length M.

Examples of orthogonal wavelets





Orthogonal Wavelets

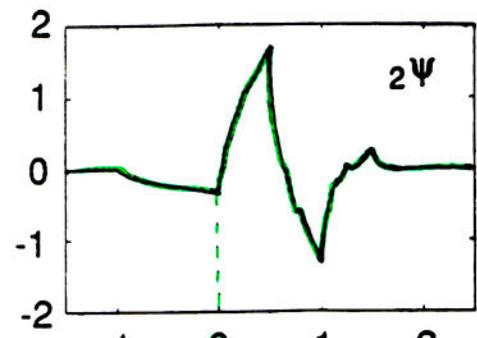
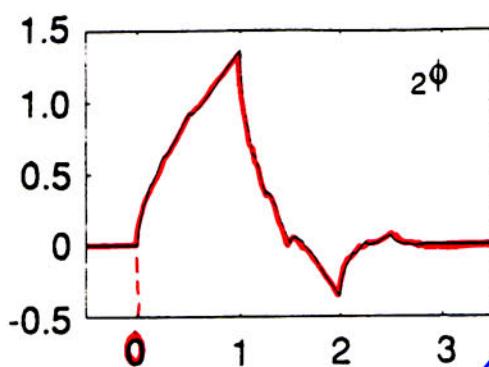
$$\phi$$

$$F_\phi(0) = \frac{1+\sqrt{3}}{4\sqrt{2}}$$

$$F_\phi(1) = \frac{3+\sqrt{3}}{4\sqrt{2}}$$

$$F_\phi(2) = \frac{3-\sqrt{3}}{4\sqrt{2}}$$

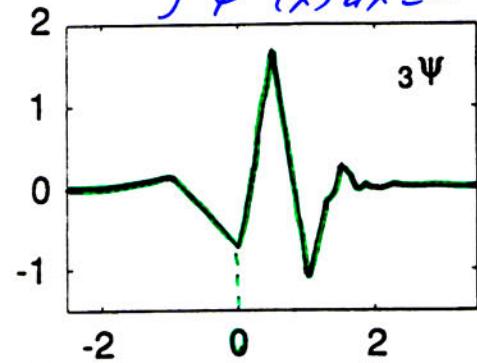
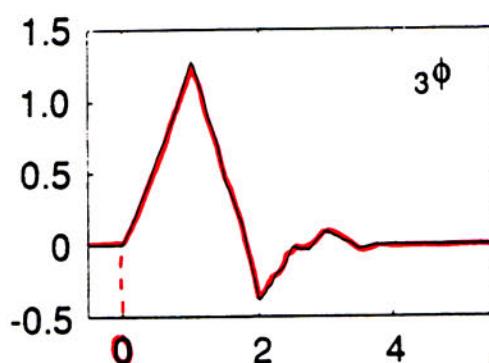
$$F_\phi(3) = \frac{1-\sqrt{3}}{4\sqrt{2}}$$



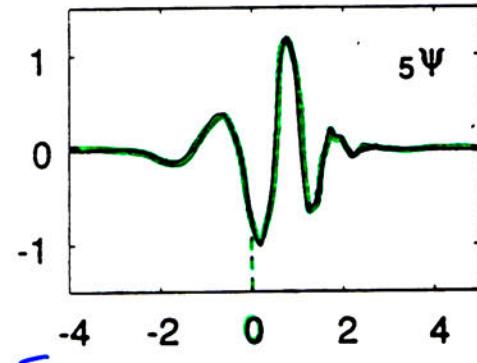
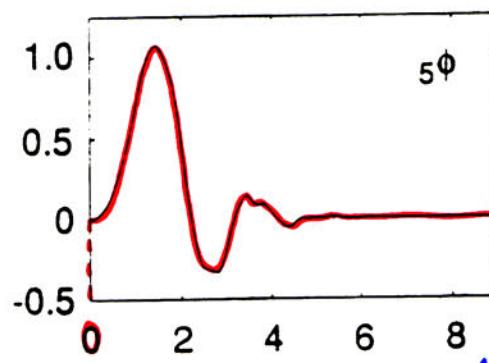
4

 $N=2$

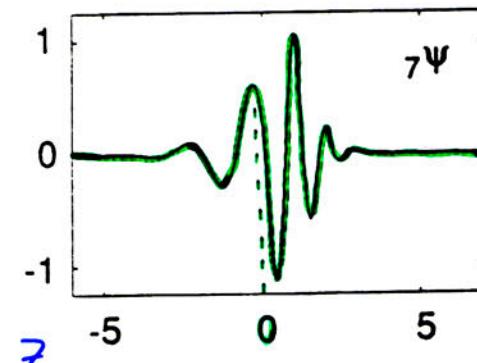
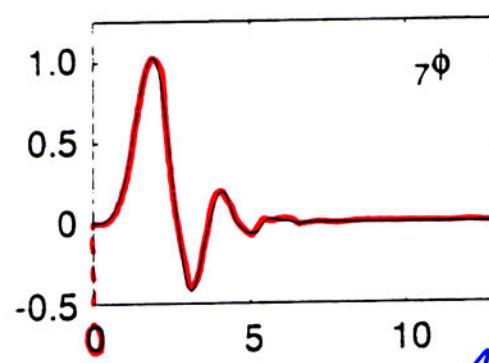
$\int \psi^n(x) dx = 0$ for $n < N$

 ϕ 

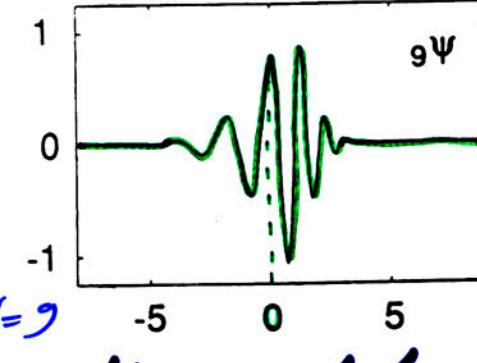
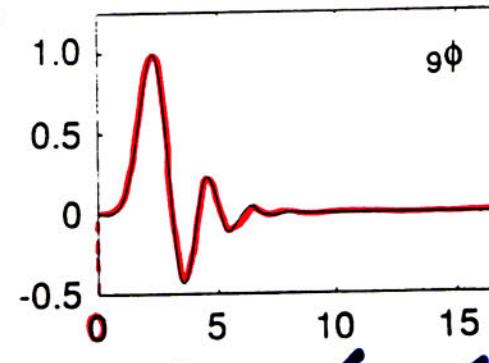
4

 $N=3$ ϕ 

4

 $N=5$ ϕ 

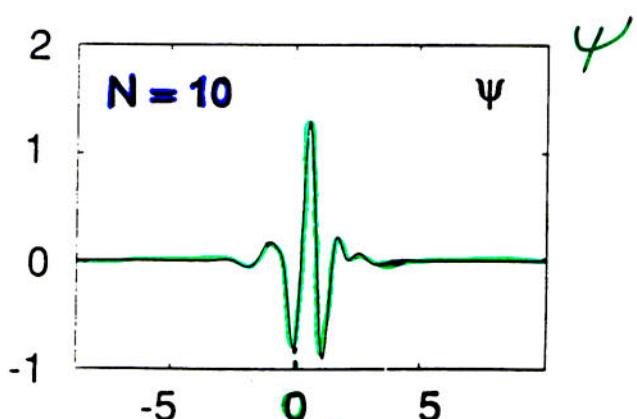
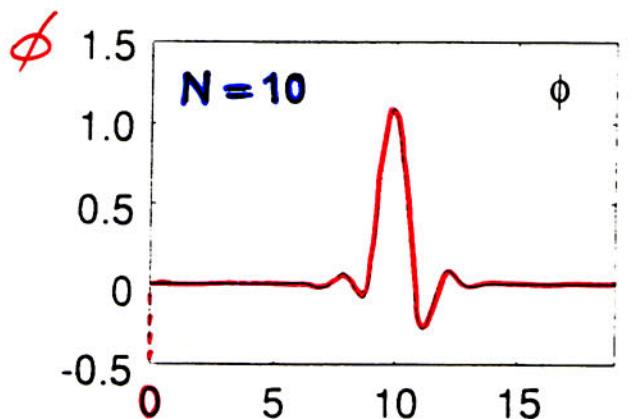
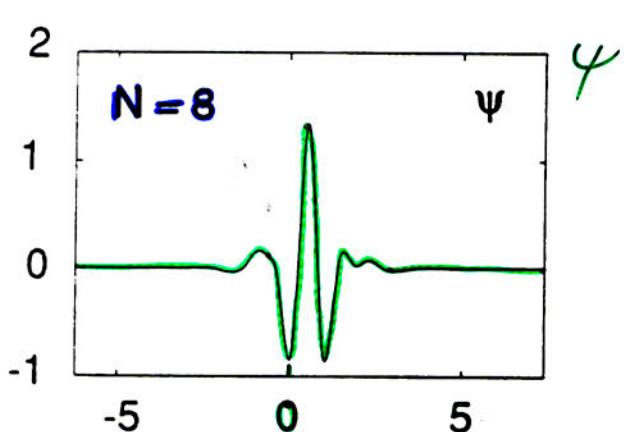
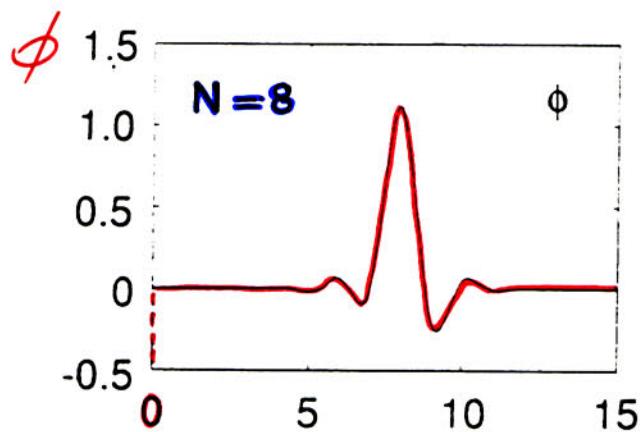
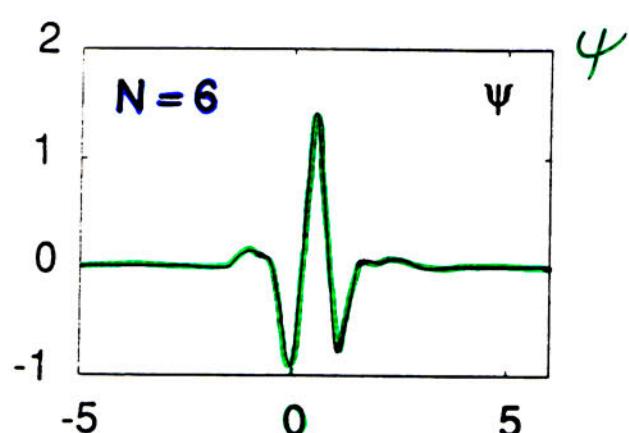
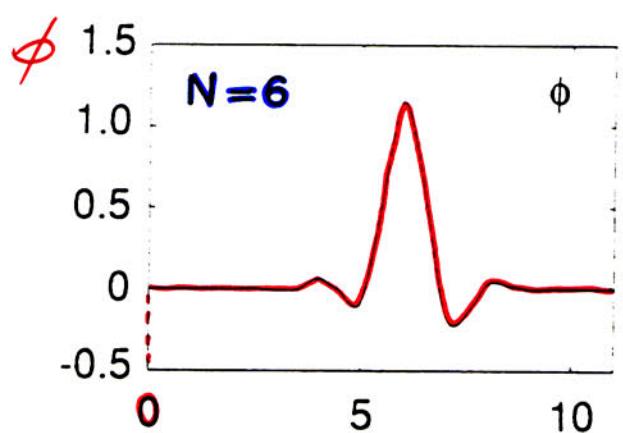
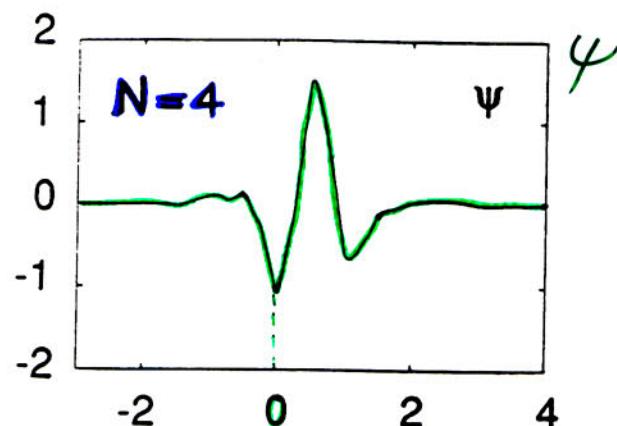
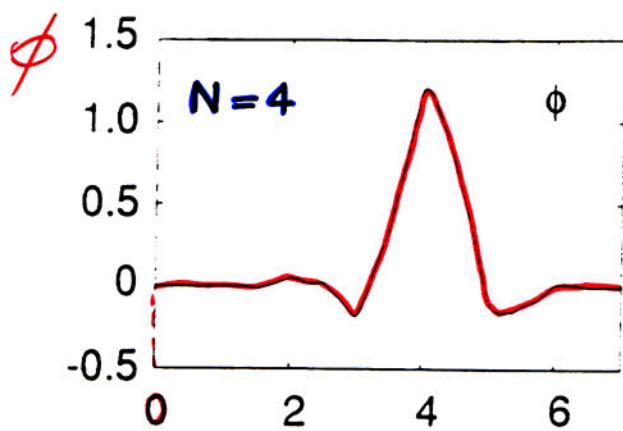
4

 $N=7$ ϕ 

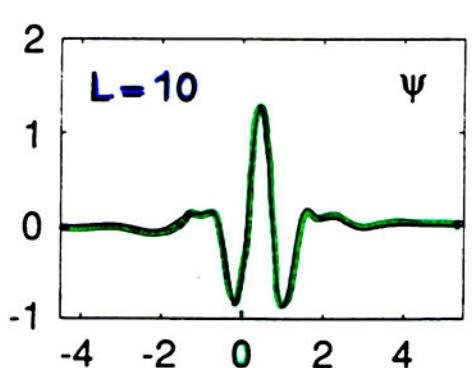
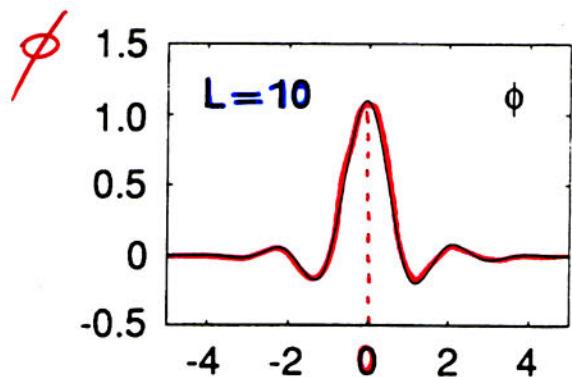
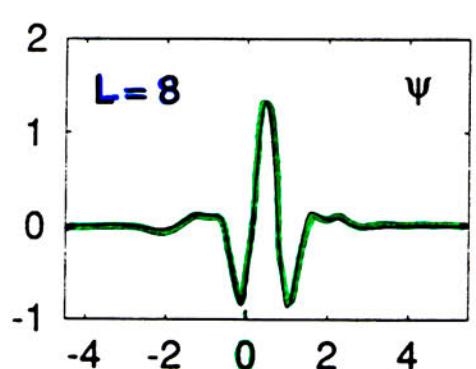
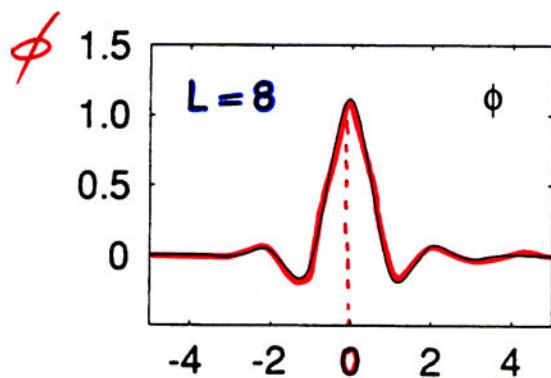
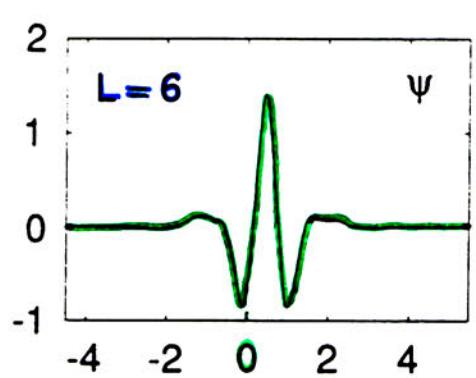
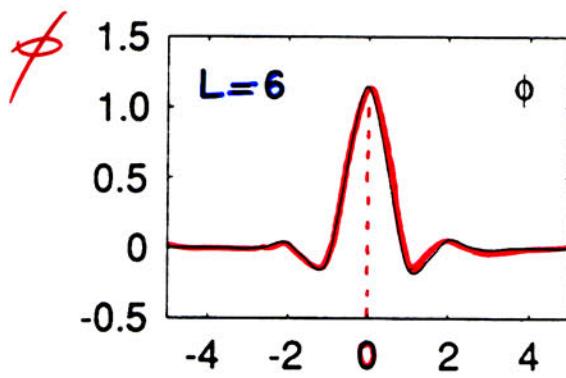
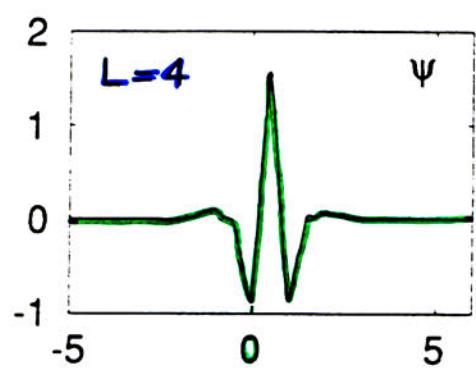
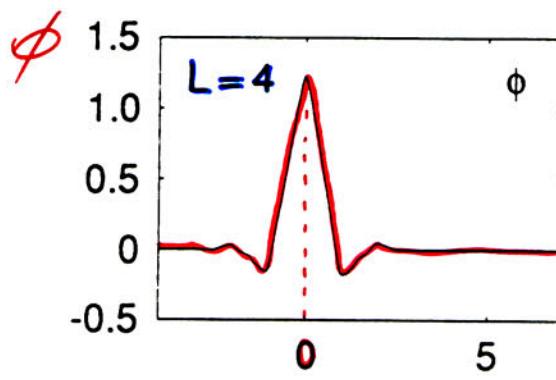
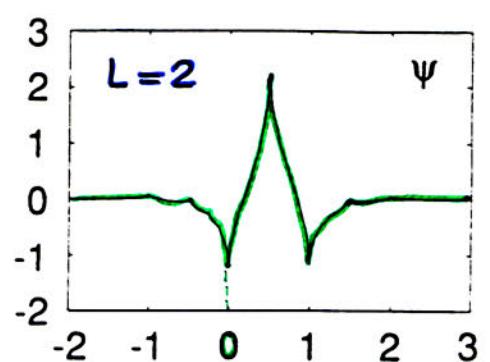
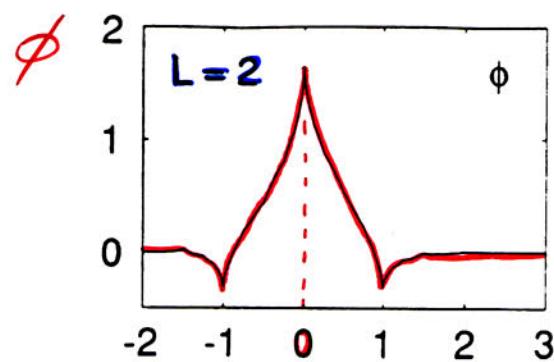
4

 $N=9$

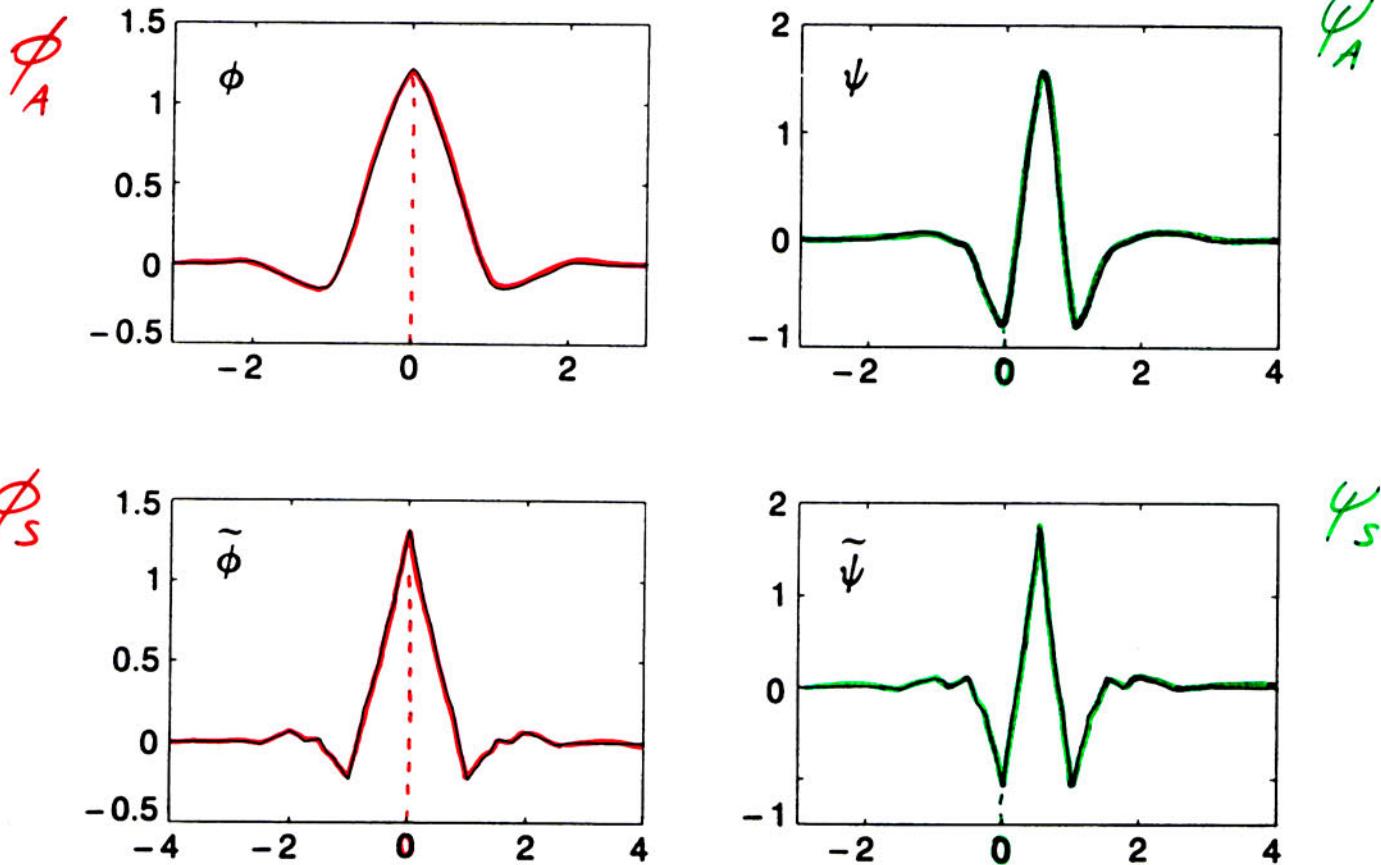
Daubchies Wavelets
Filter length $L = 2N$



*least asymmetric
compactly supported wavelets*
 $L = 3N$

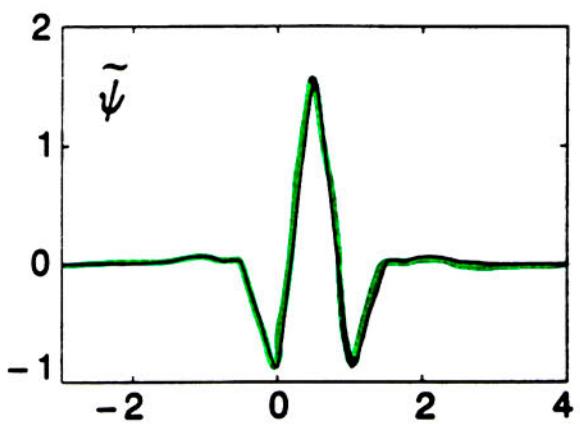
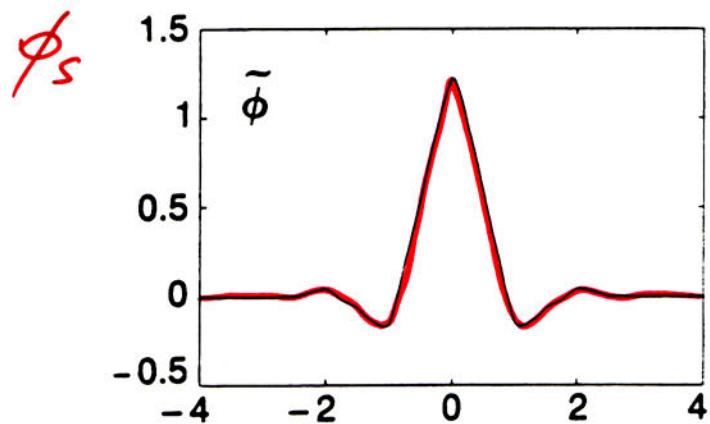
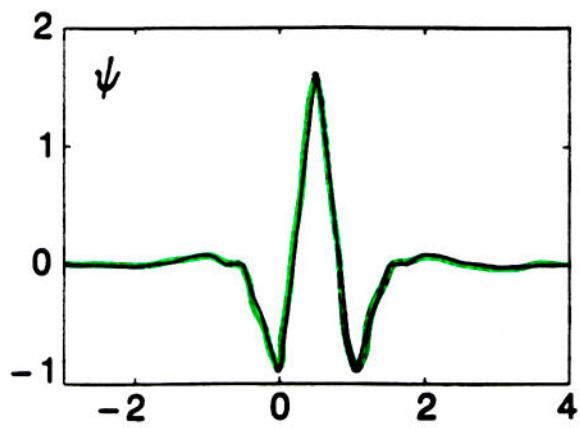
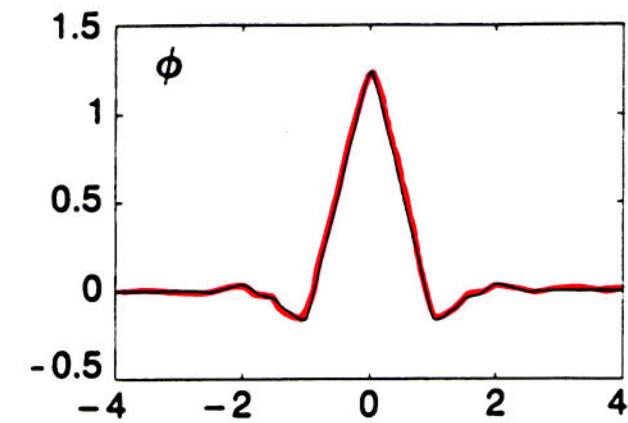


Coifman Wavelets
 $L = 3N$

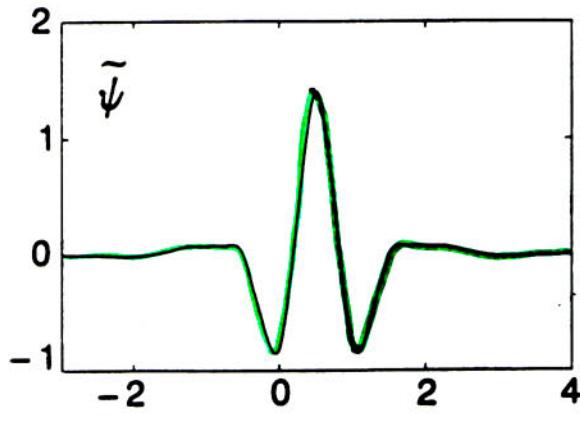
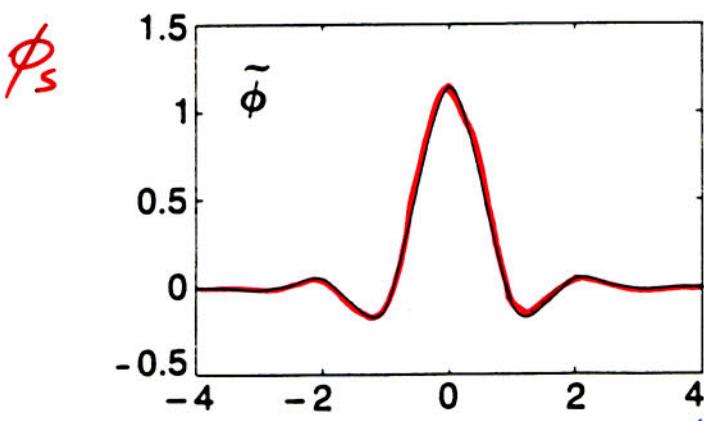
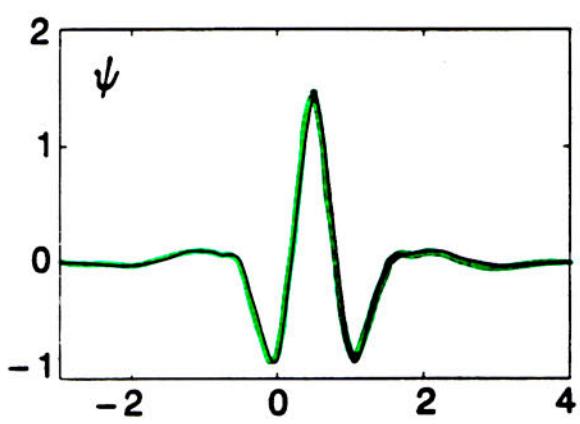
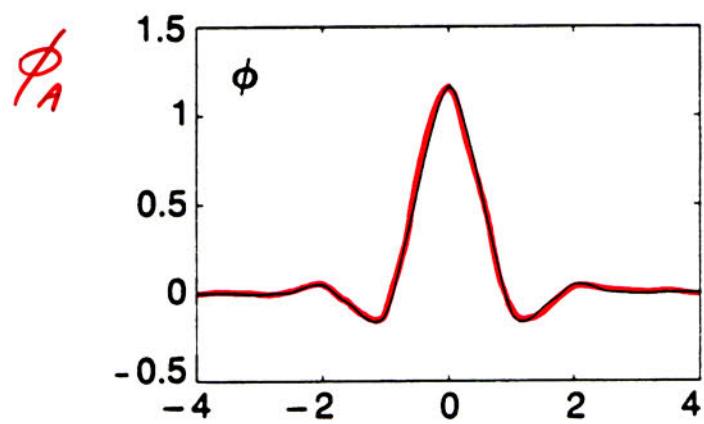


Biorthogonal
 Spline Wavelets

$$N_A = N_S = 4$$



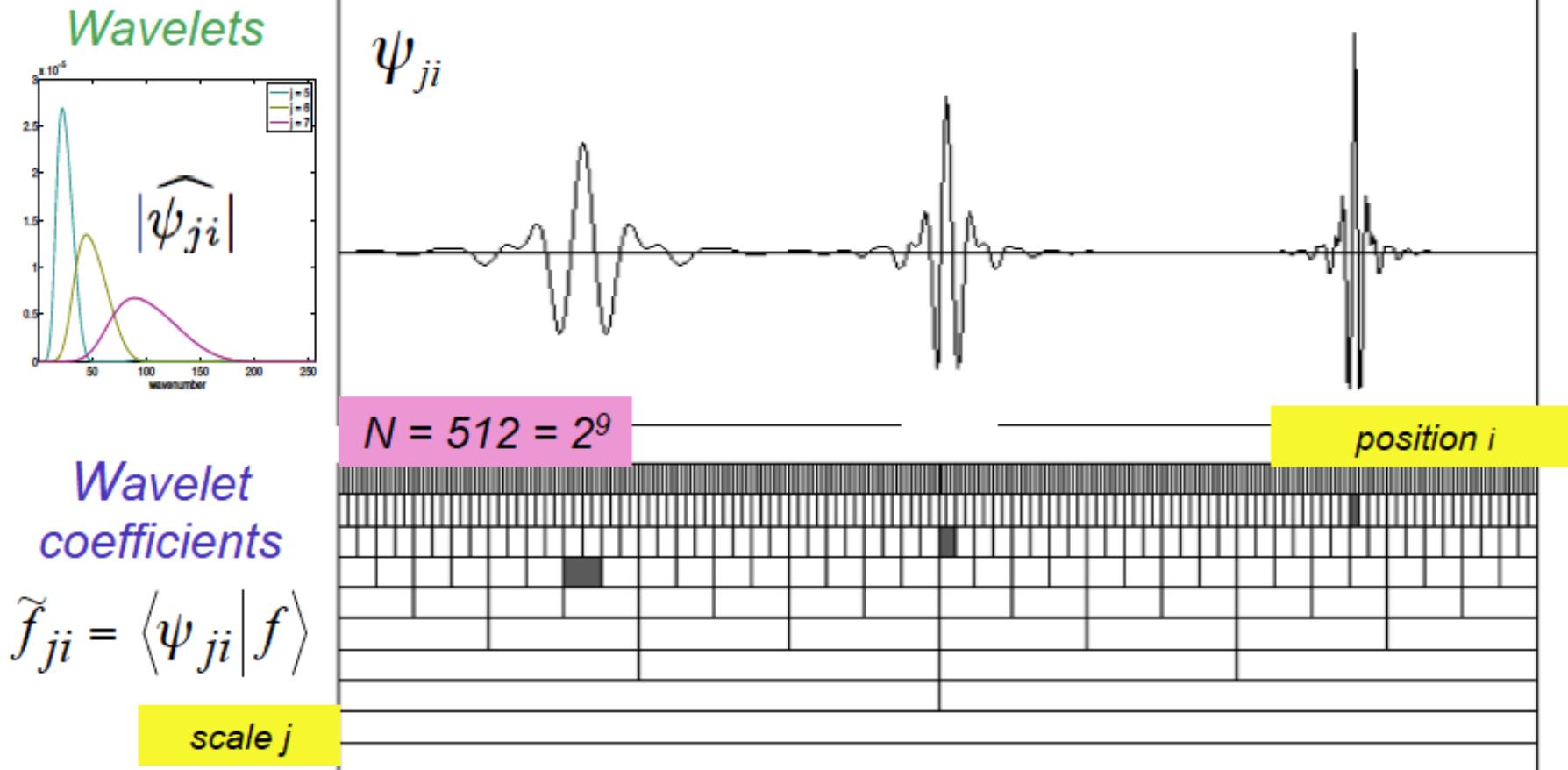
$L=4$



$L=10$

Biorthogonal Coifman Wavelets

Orthogonal wavelets representation

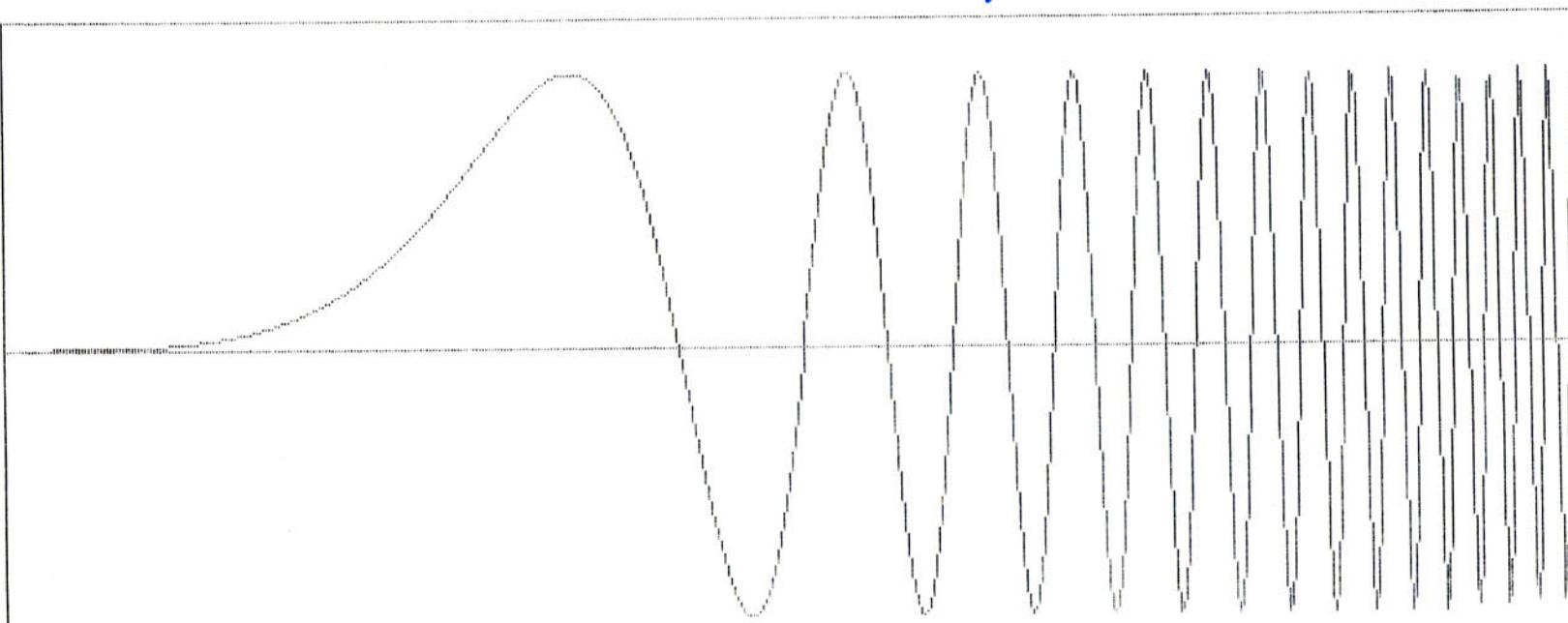


Mallat, 2008
A wavelet tour of
signal processing, 3rd edition,
Academic Press

filter ML. coefficients ?

Discrete Wavelet Transform $f = \sin t^2$

1.20E+00



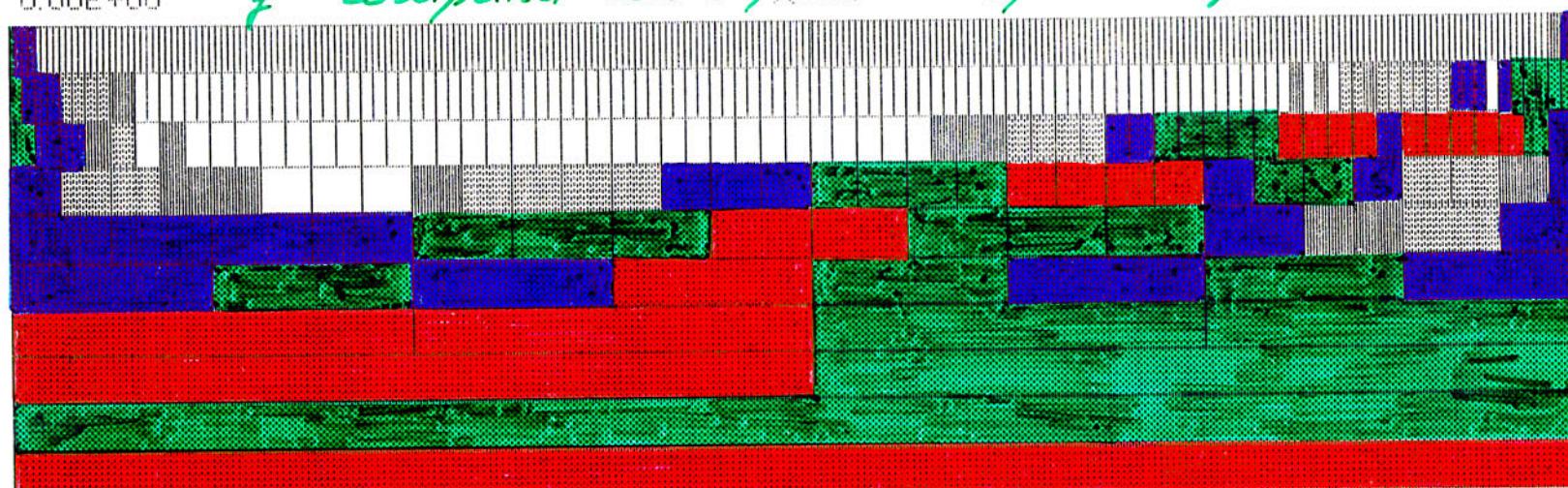
0.00E+00

\hat{f}^n computed with periodic splines degree 6

1.00E+00

$2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 16$
 $2^5 = 32$
 $2^6 = 64$
 $2^{11} = 2048$
 $2^7 = 128$
 $2^8 = 256$
 $2^9 = 512$

\hat{f}_g



-4.00E+00

LOGARITH.

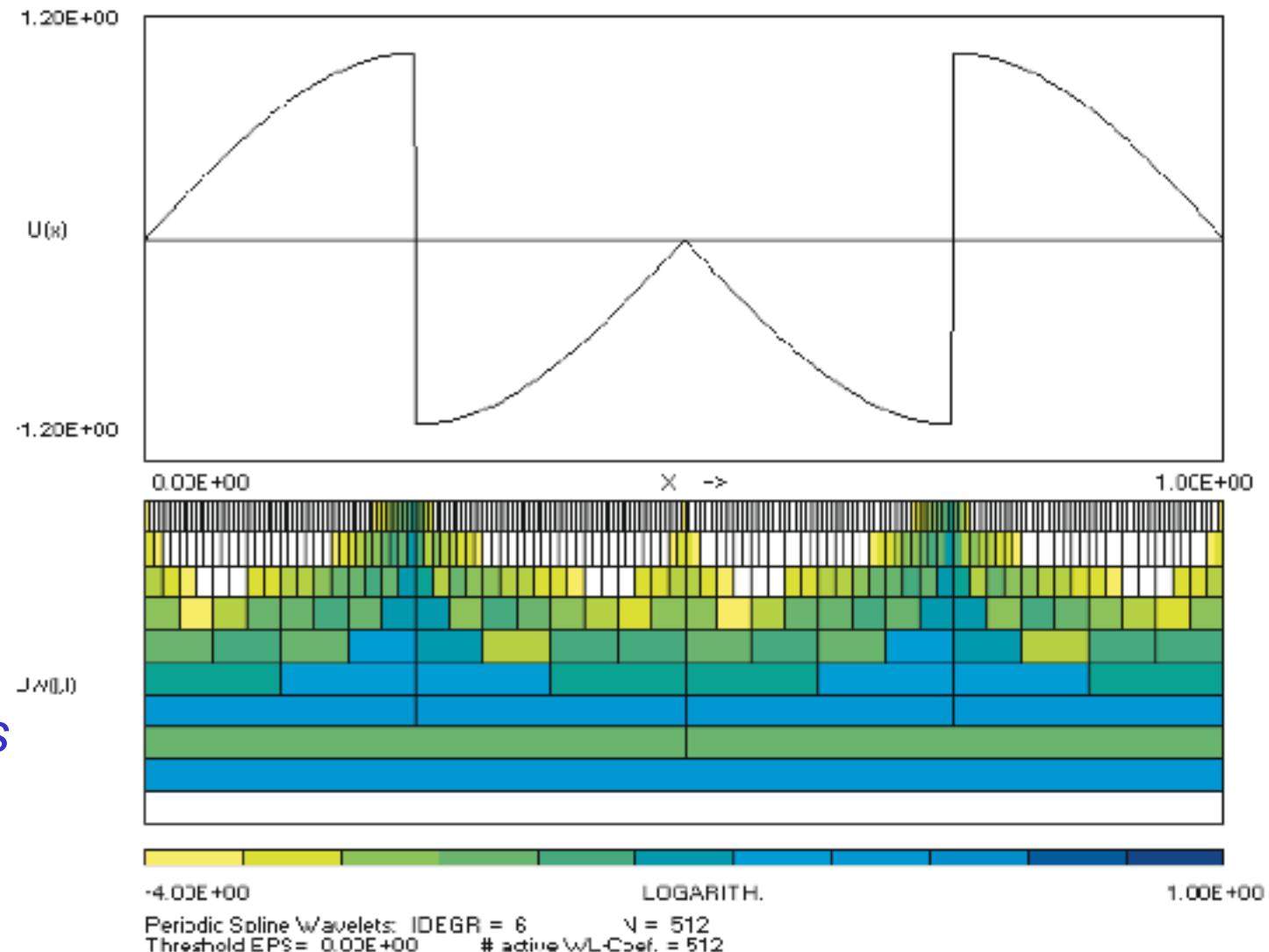
1.00E+00

Periodic Spline Wavelets: IDEGR = 6 N = 512
Threshold EPS = 0.00E+00 # active ML-Coeff. = 512

Academic example

*Function
sampled on
 $N = 512$
grid-points*

*$N = 512$
wavelet
coefficients*

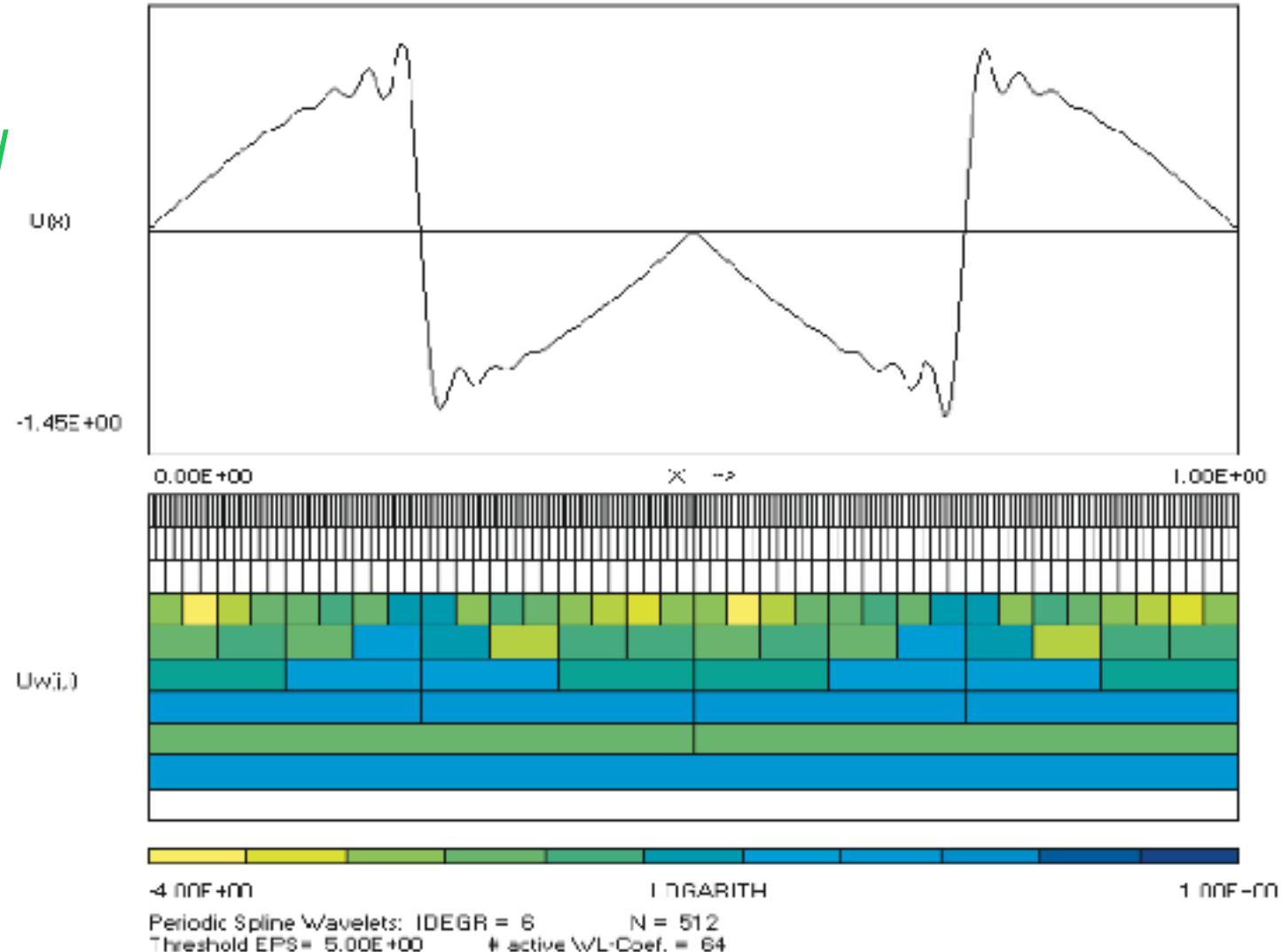


Linear approximation

*Function
reconstructed
from 64
wavelet
coefficients*

*64 wavelet
coefficients
such that*

$$j \leq 6$$

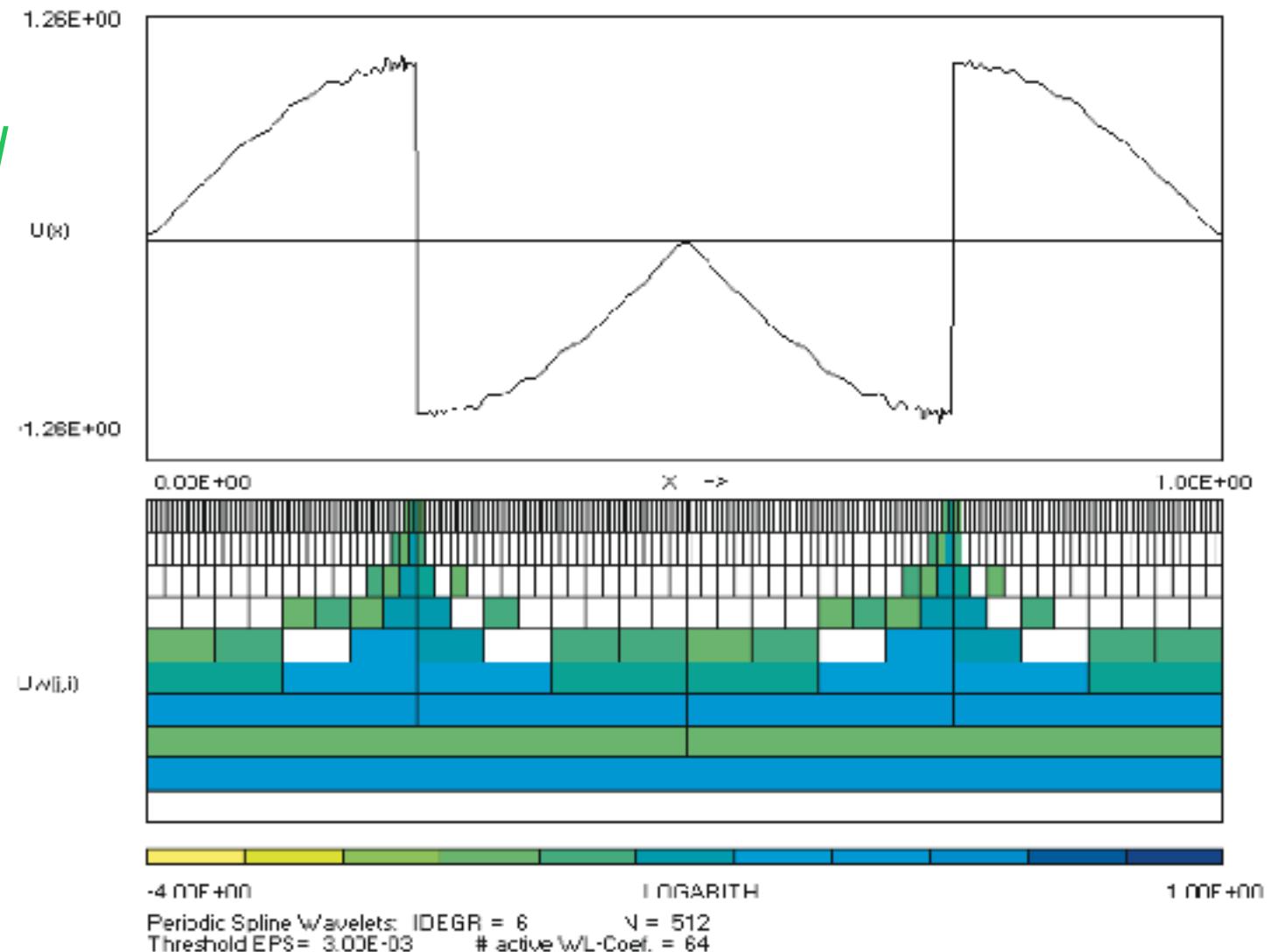


Nonlinear approximation

*Function
reconstructed
from 64
wavelet
coefficients*

*64 wavelet
coefficients
such that*

$$|\tilde{f}_{ji}| > \epsilon$$



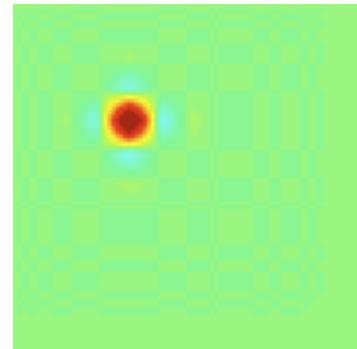
2D orthogonal wavelets

Scaling function *Wavelet*

$$\phi \sim \text{—} \begin{smallmatrix} & & \\ & \nearrow & \\ & & \searrow \\ & & \end{smallmatrix}$$

$$\phi(x)\phi(y)$$

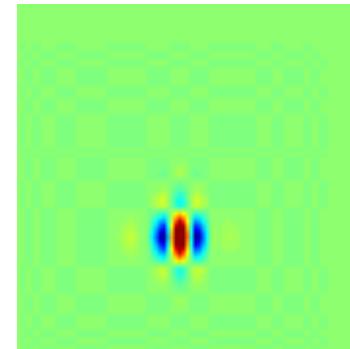
Coarse
approximation



$$\psi \sim \text{—} \begin{smallmatrix} & & \\ & \nearrow & \\ & & \searrow \\ & & \end{smallmatrix}$$

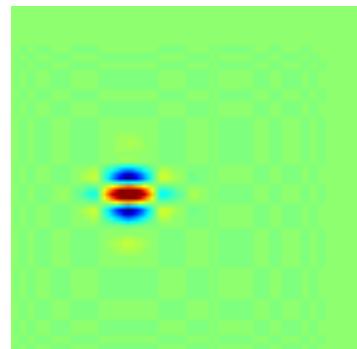
$$\psi(x)\phi(y)$$

Horizontal
details



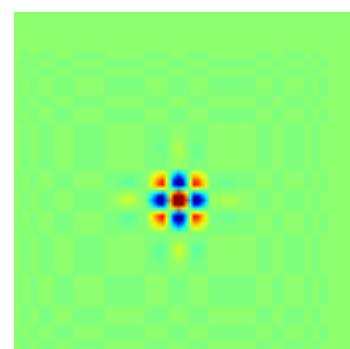
$$\phi(x)\psi(y)$$

Vertical
details



$$\psi(x)\psi(y)$$

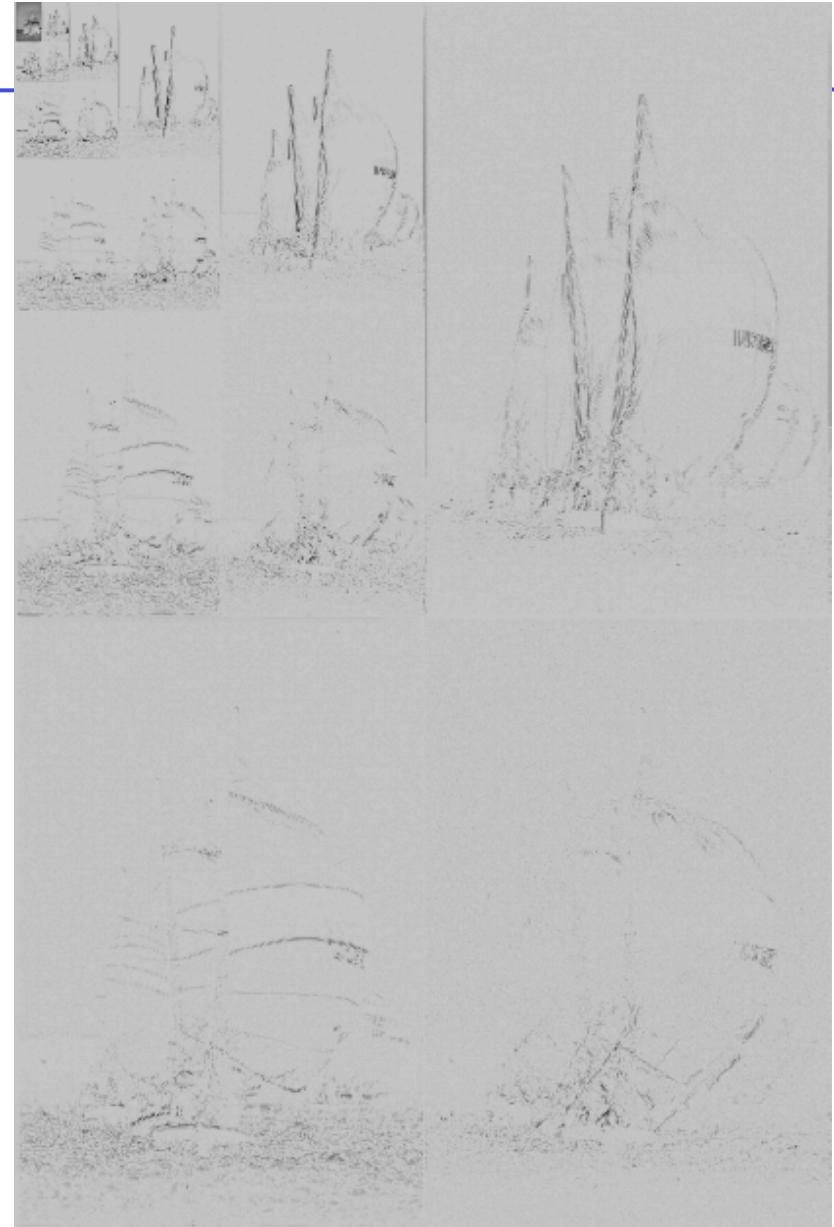
Diagonal
details



Wavelet decomposition



Image sampled on $N=(2^9)^2$ pixels



$N=512^2$ wavelet coefficients

Linear compression



Image reconstructed up to scale 2^4



$16^2 = 0.1\% N$ wavelet coefficients

Linear compression



Image reconstructed up to scale 2^5

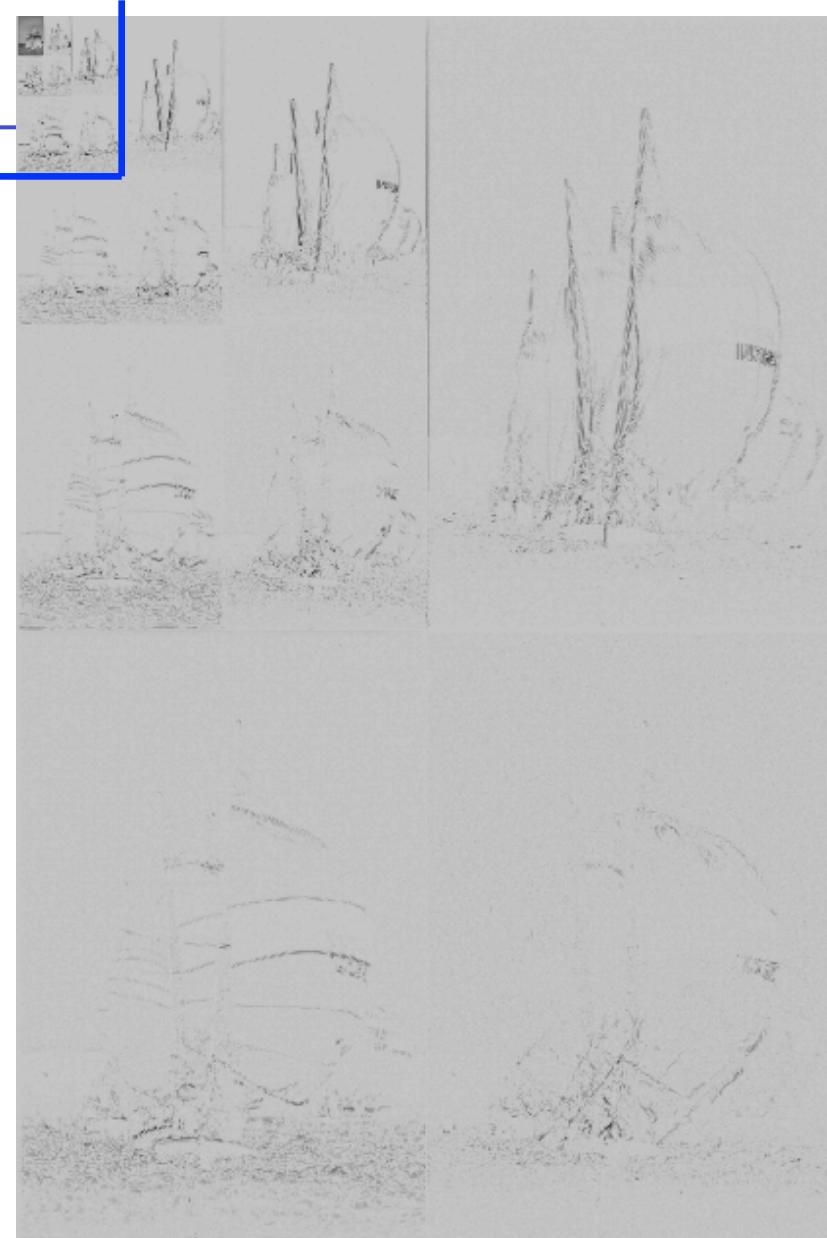


$32^2 = 0.4\% N$ wavelet coefficients

Linear compression



Image reconstructed up to scale 2^6

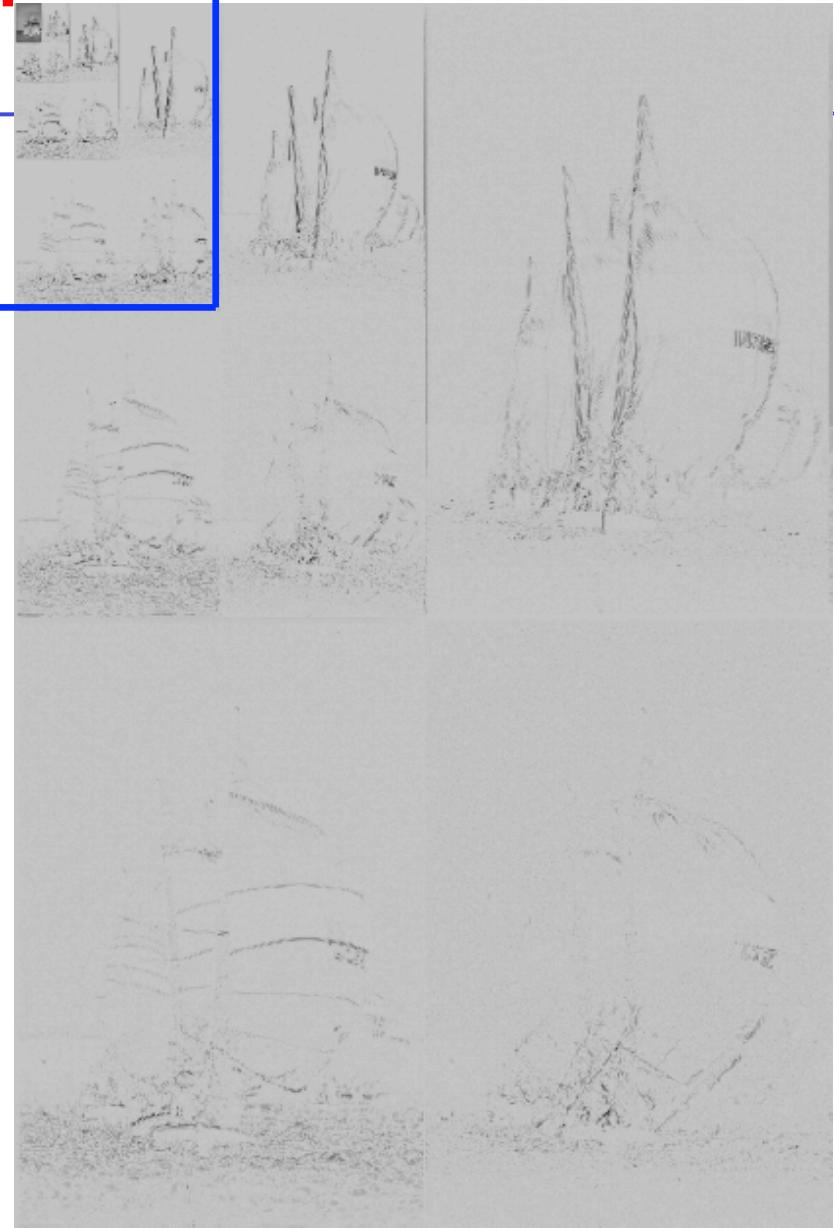


$64^2 = 1.6\% N$ wavelet coefficients

Linear compression



Image reconstructed up to scale 2^7

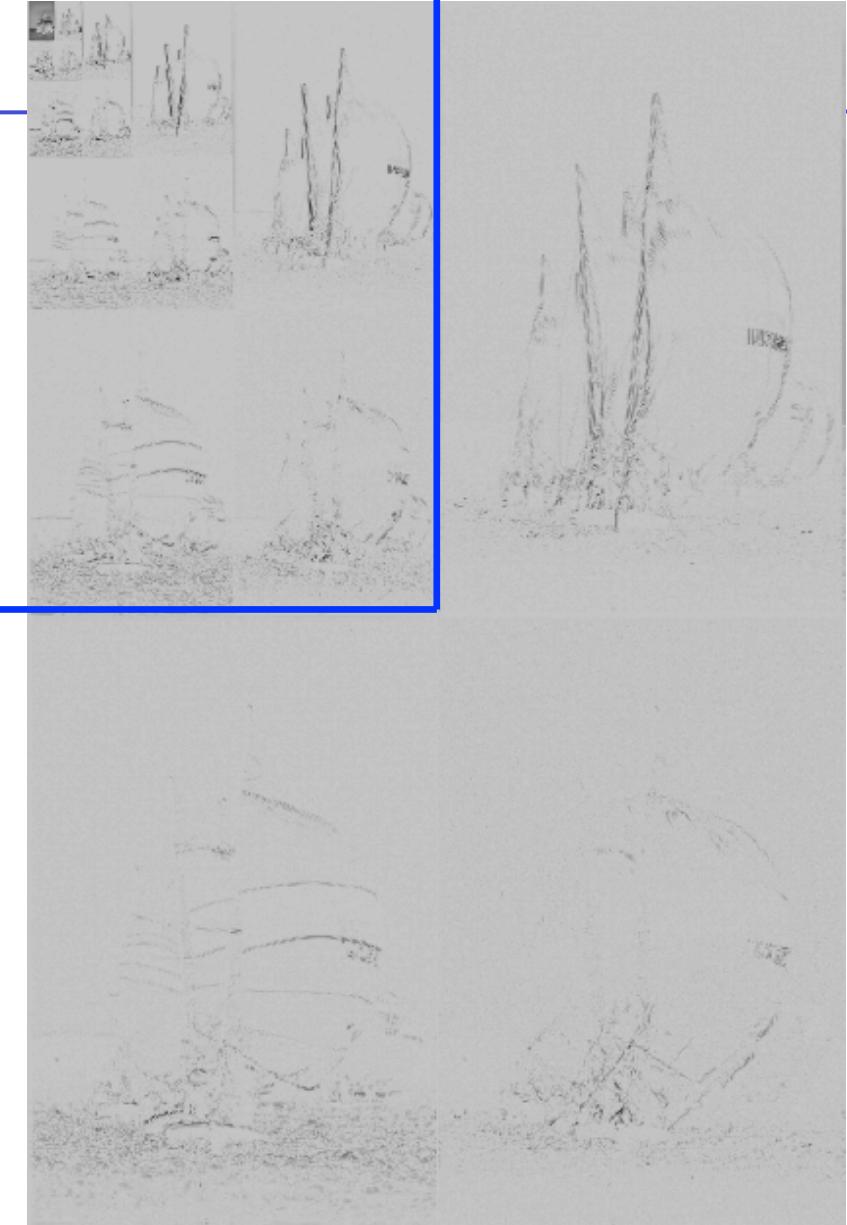


$128^2 = 6.2\% \text{ N}$ wavelet coefficients

Linear compression



Image reconstructed up to scale 2^8

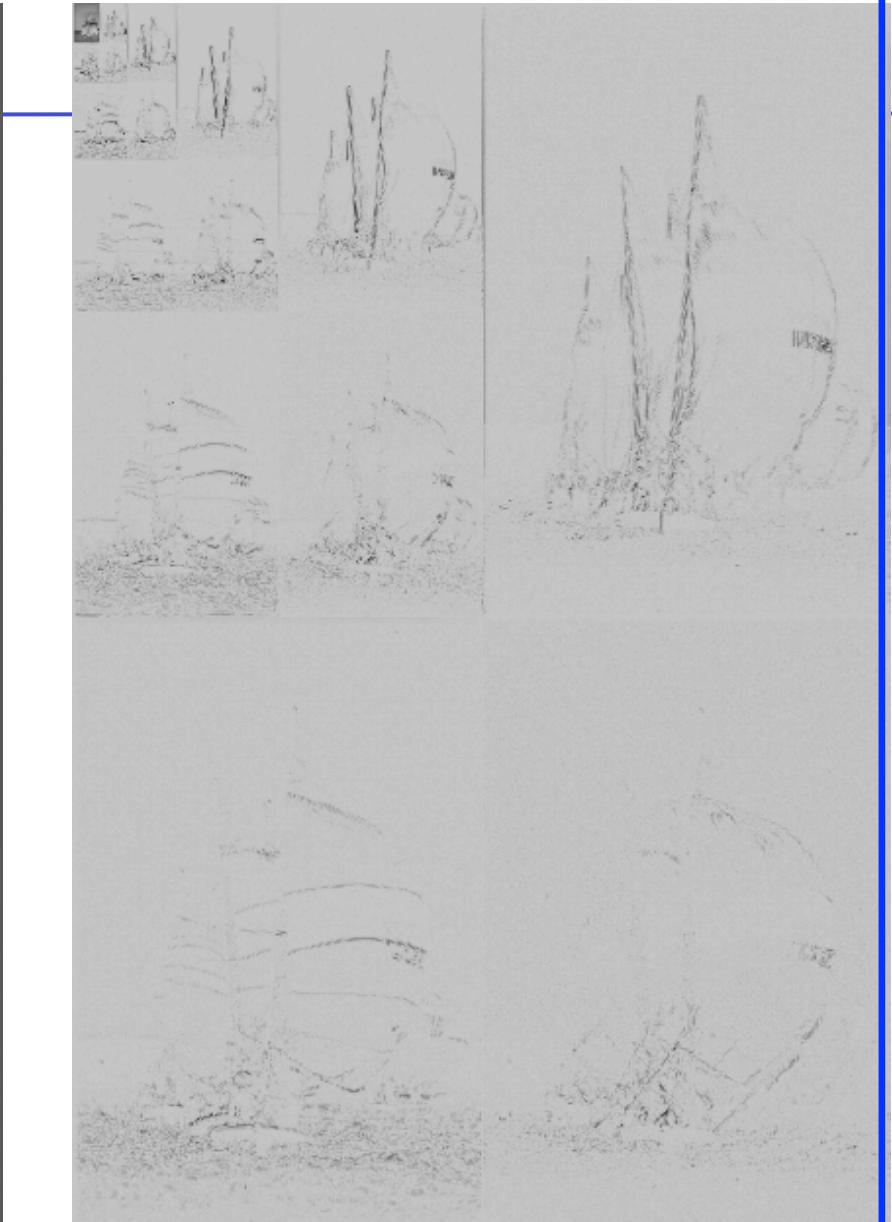


$256^2=25\%$ N wavelet coefficients

Linear compression



Image reconstructed up to scale 2^9

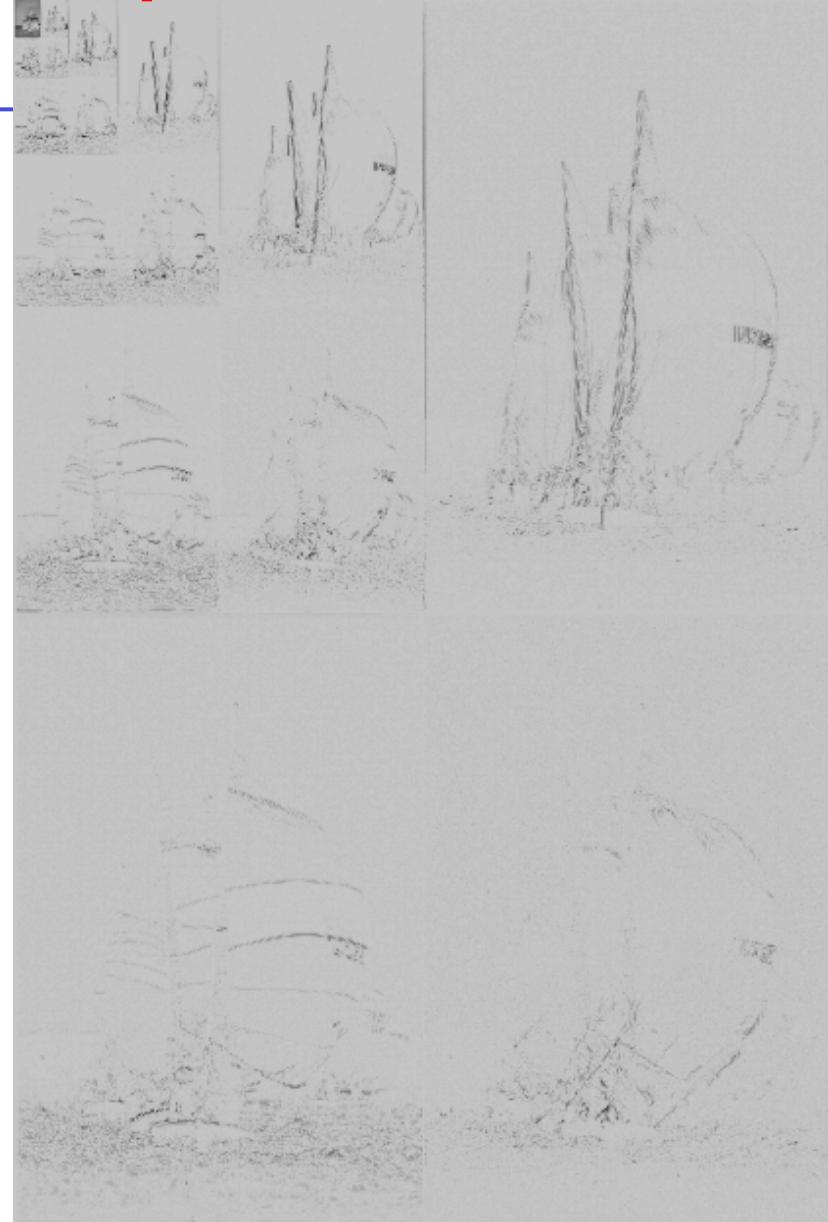


$512^2 = 100\% N$ wavelet coefficients

Nonlinear compression



Image sampled on $N=(2^9)^2$ pixels

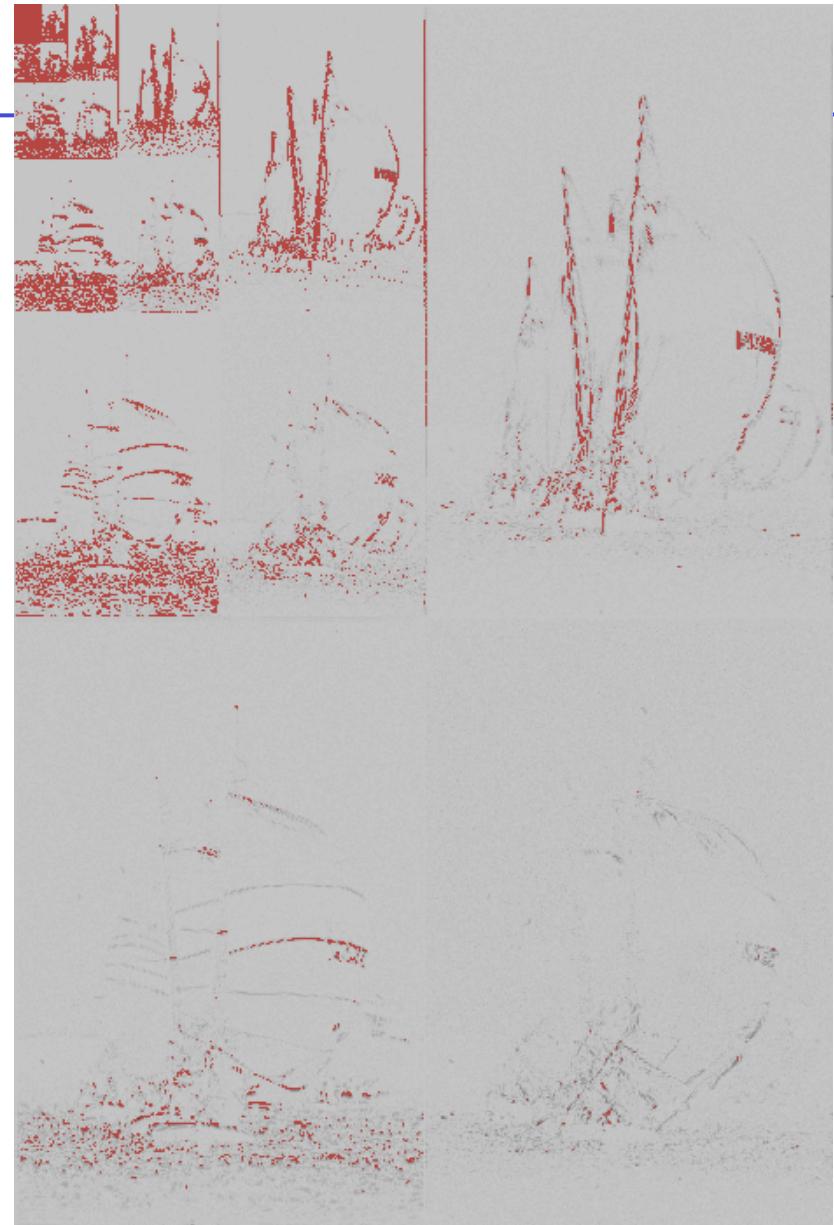


$N=512^2$ wavelet coefficients

Nonlinear compression



Image reconstructed from 3.3% N



3.3% N wavelet coefficients

Linear compression



Nonlinear compression

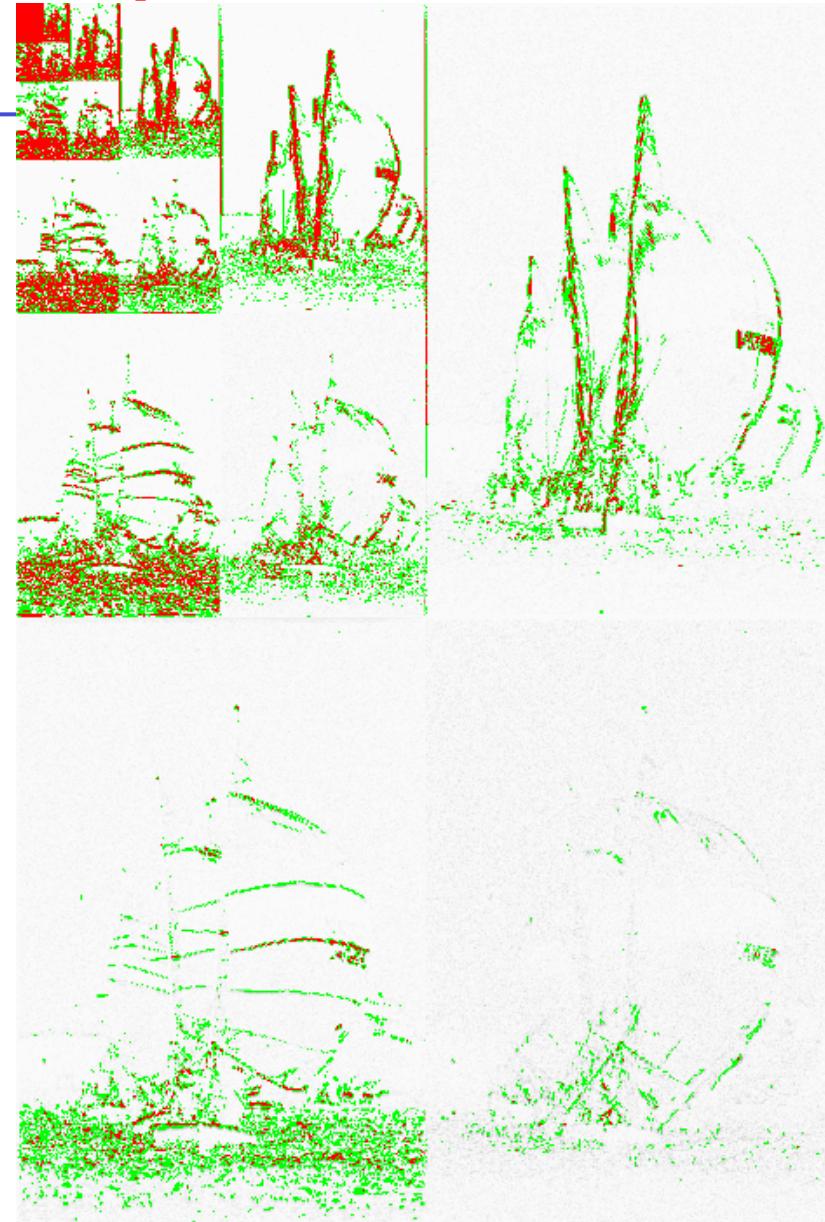


2% N wavelet coefficients

Nonlinear compression



Image reconstructed from 10% N



10% N wavelet coefficients

Wavelet denoising algorithm

Apophatic method :

- no hypothesis on the structures,
- only hypothesis on the noise,
- simplest hypothesis as our first choice.

Hypothesis on the noise :

$$f_n = f_d + n$$

n Gaussian white noise,
 $\langle f_n^2 \rangle$ variance of the noisy signal,
 N number of coefficients of f_n .

Wavelet decomposition :

$$\tilde{f}_{ji} = \langle f | \psi_{ji} \rangle \quad \begin{matrix} j \text{ scale,} \\ i \text{ position} \end{matrix}$$

Estimation of the threshold :

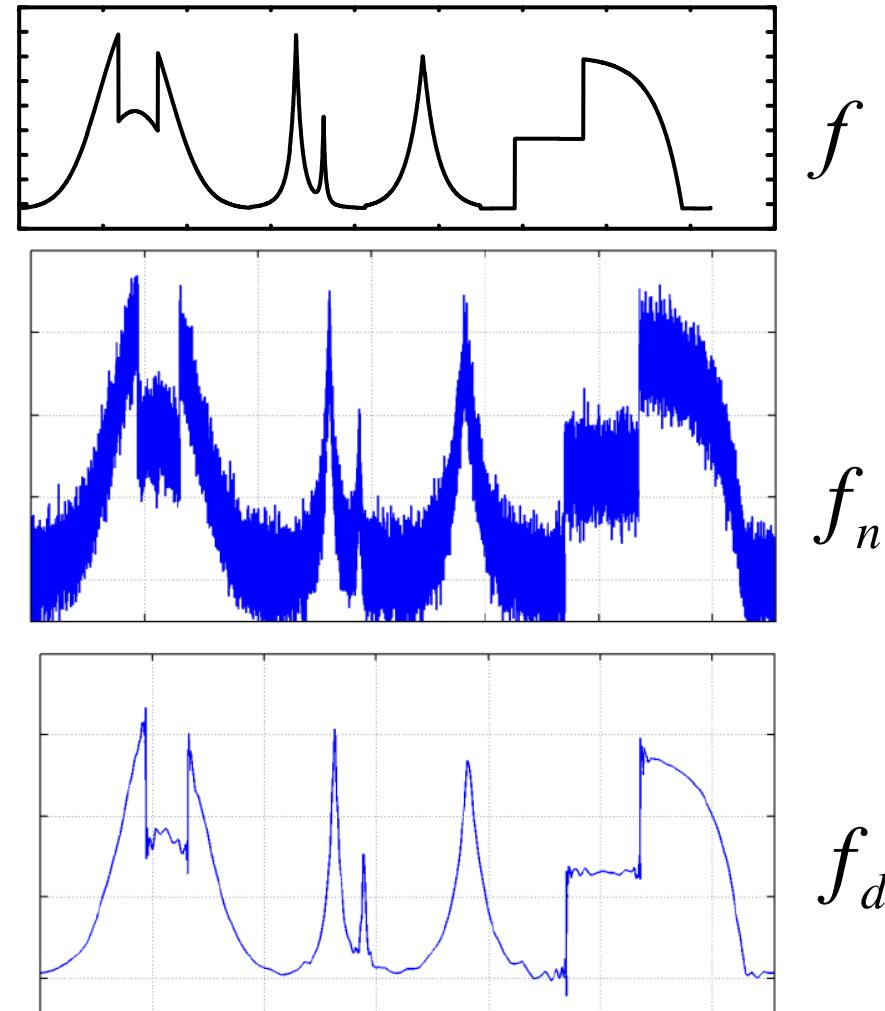
$$\varepsilon_n = \sqrt{2 \langle f_n^2 \rangle \ln(N)}$$

Wavelet reconstruction :

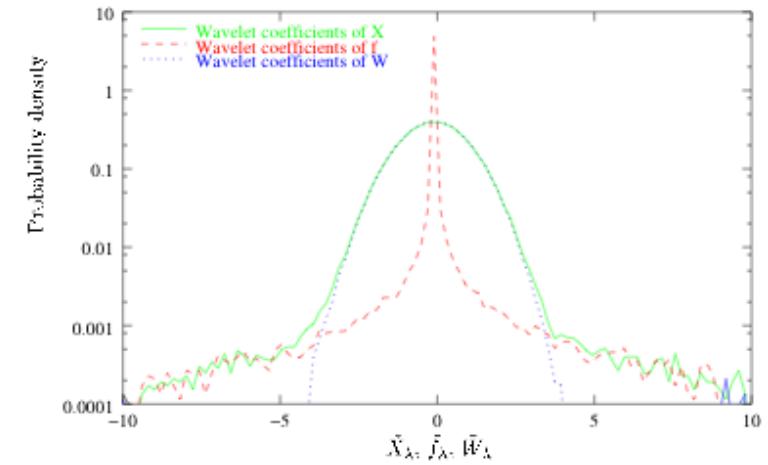
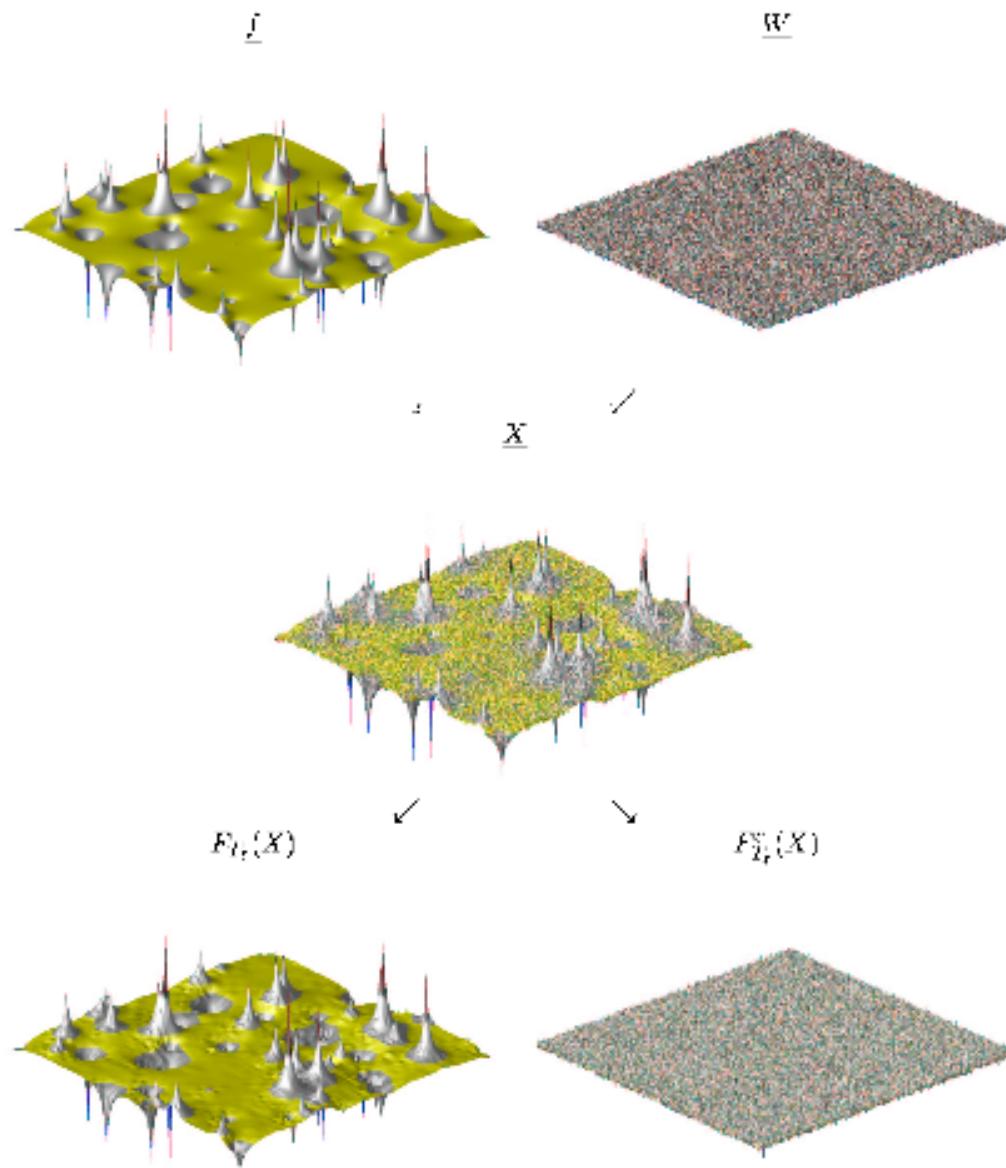
$$f_d = \sum_{ji: |\tilde{f}_{ji}| > \varepsilon_n} \tilde{f}_{ji} \psi_{ji}$$

Donoho, Johnstone, 1994
Biometrika, 81

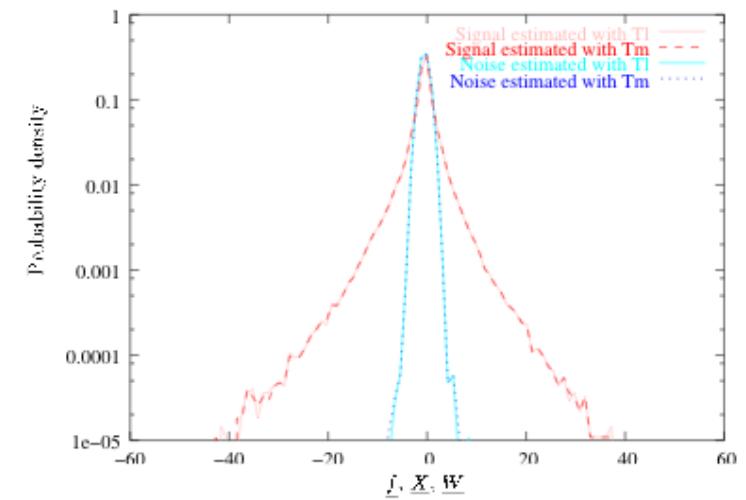
Azzalini, M. F., Schneider, 2005
ACHA, 18 (2)



2D academic example (SNR = 10 dB)



PDF of wavelet coefficients



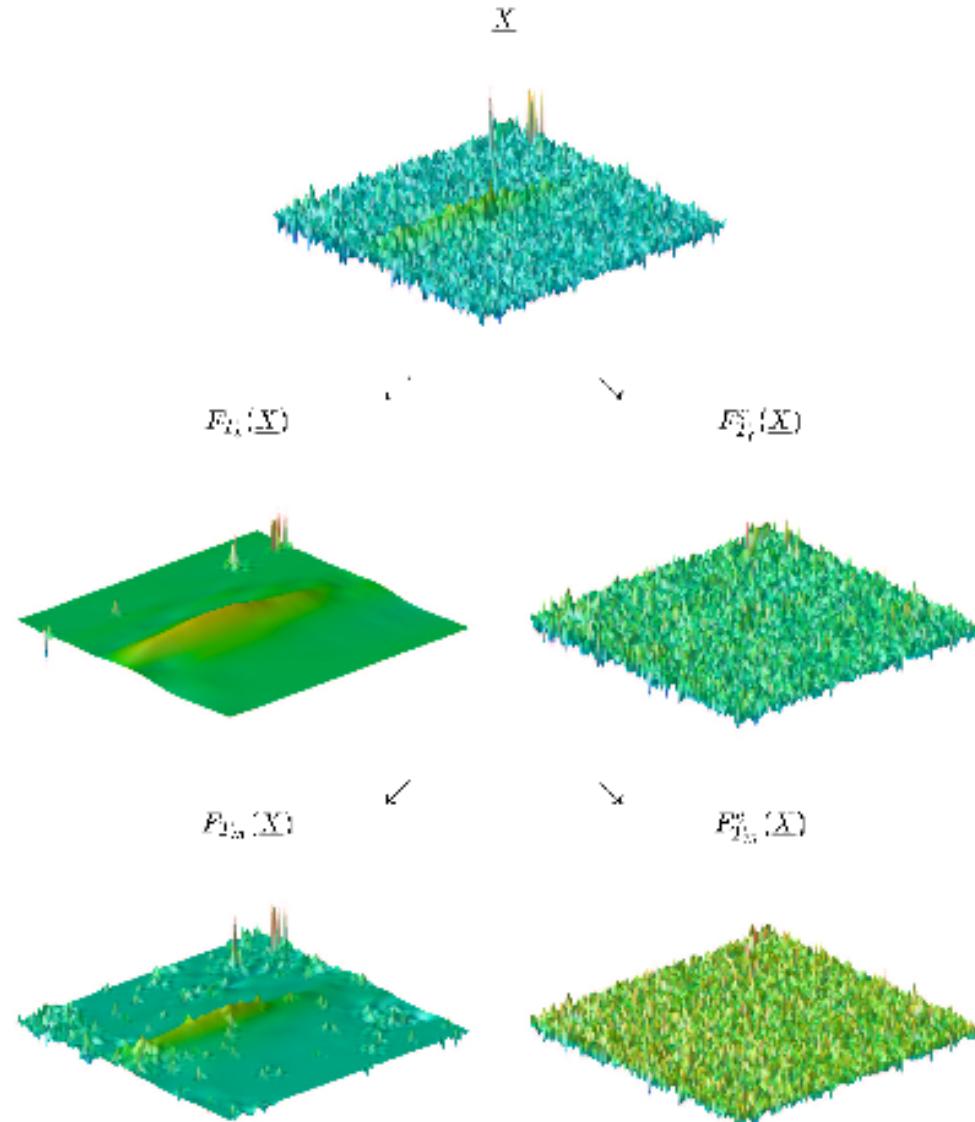
PDF of denoised signal and noise

Extraction of Bose-Einstein condensate

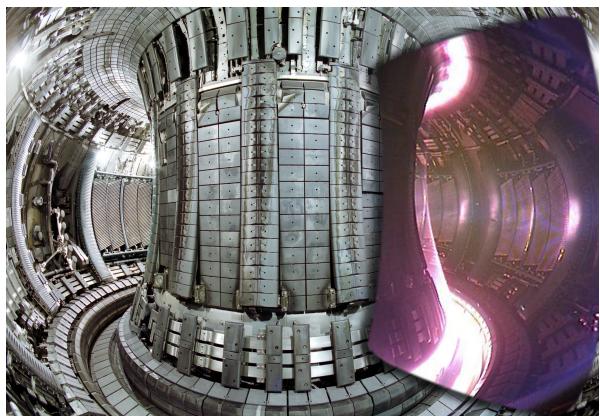
*Optical
density
of Lithium
atoms*

*Iterative
Wavelet
Denoising
(IWD)*

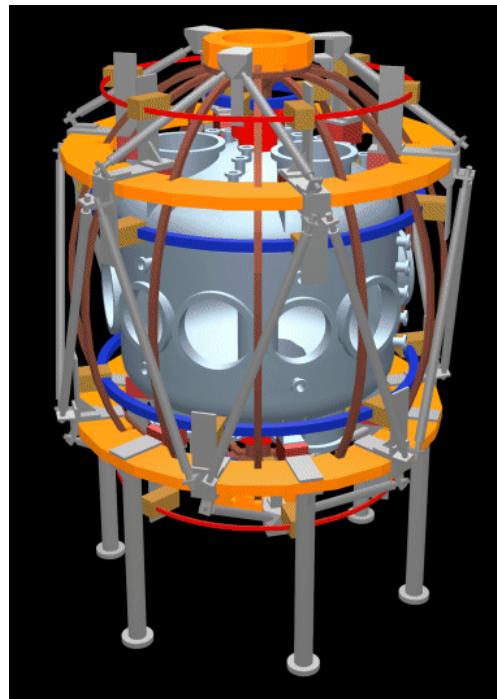
*Median
Absolute
Deviation
(MAD)*



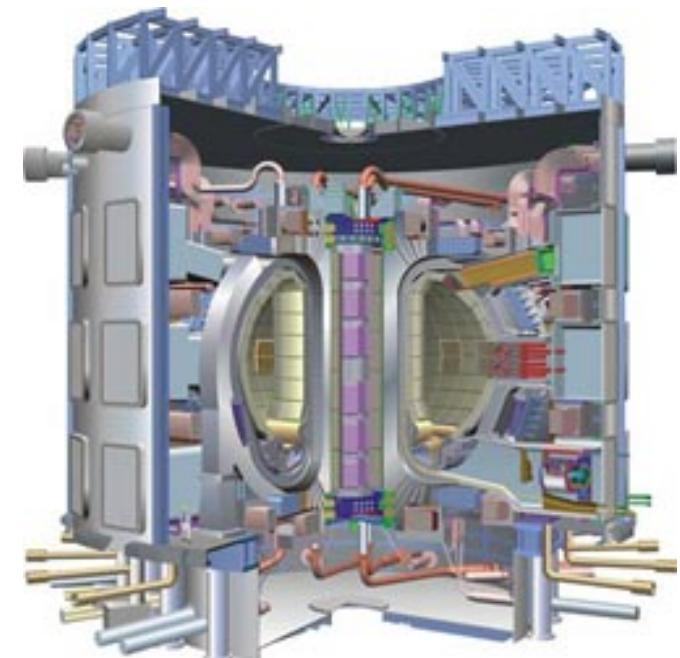
Tokamaks for controlled fusion



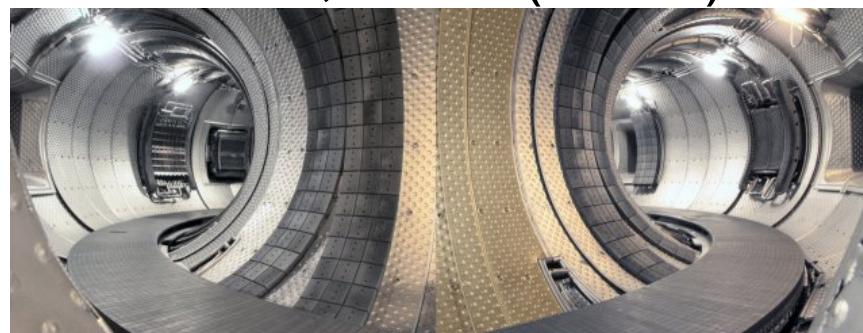
JET, Culham (UK)



ETE, INPE (Brazil)



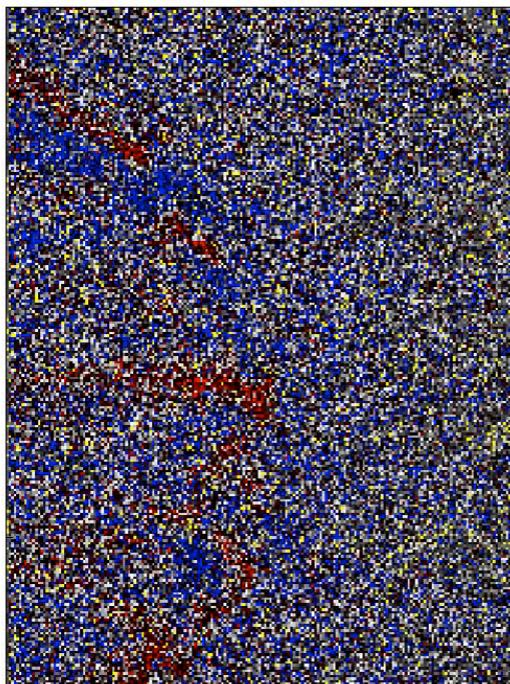
ITER (2015)



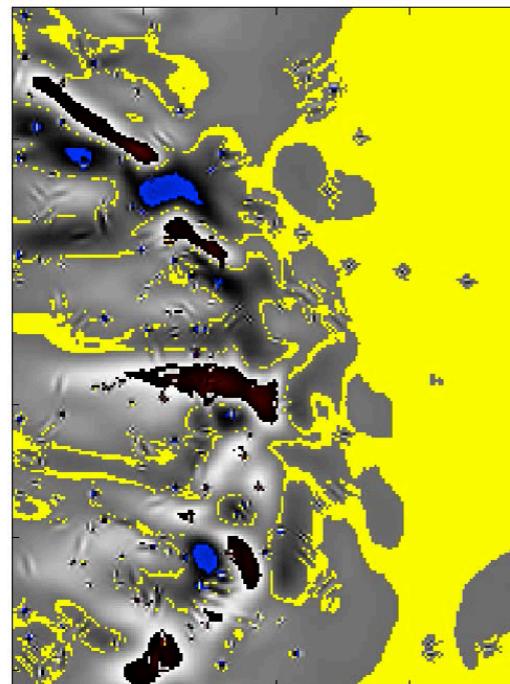
Tore-Supra, Cadarache (France)

Fast camera in Tore-Supra

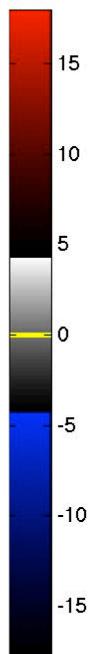
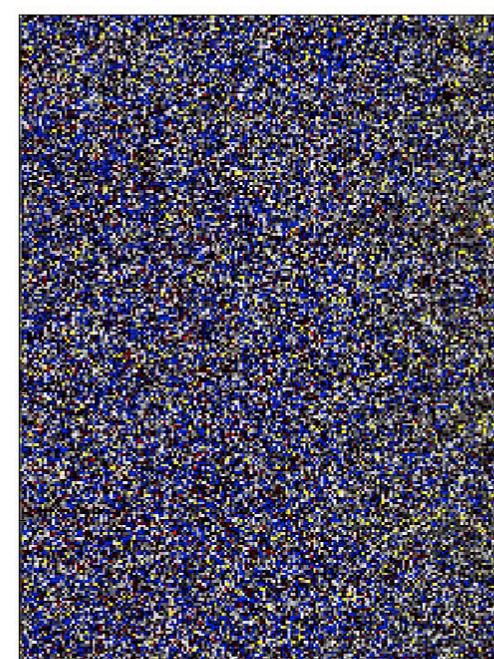
Total



Coherent

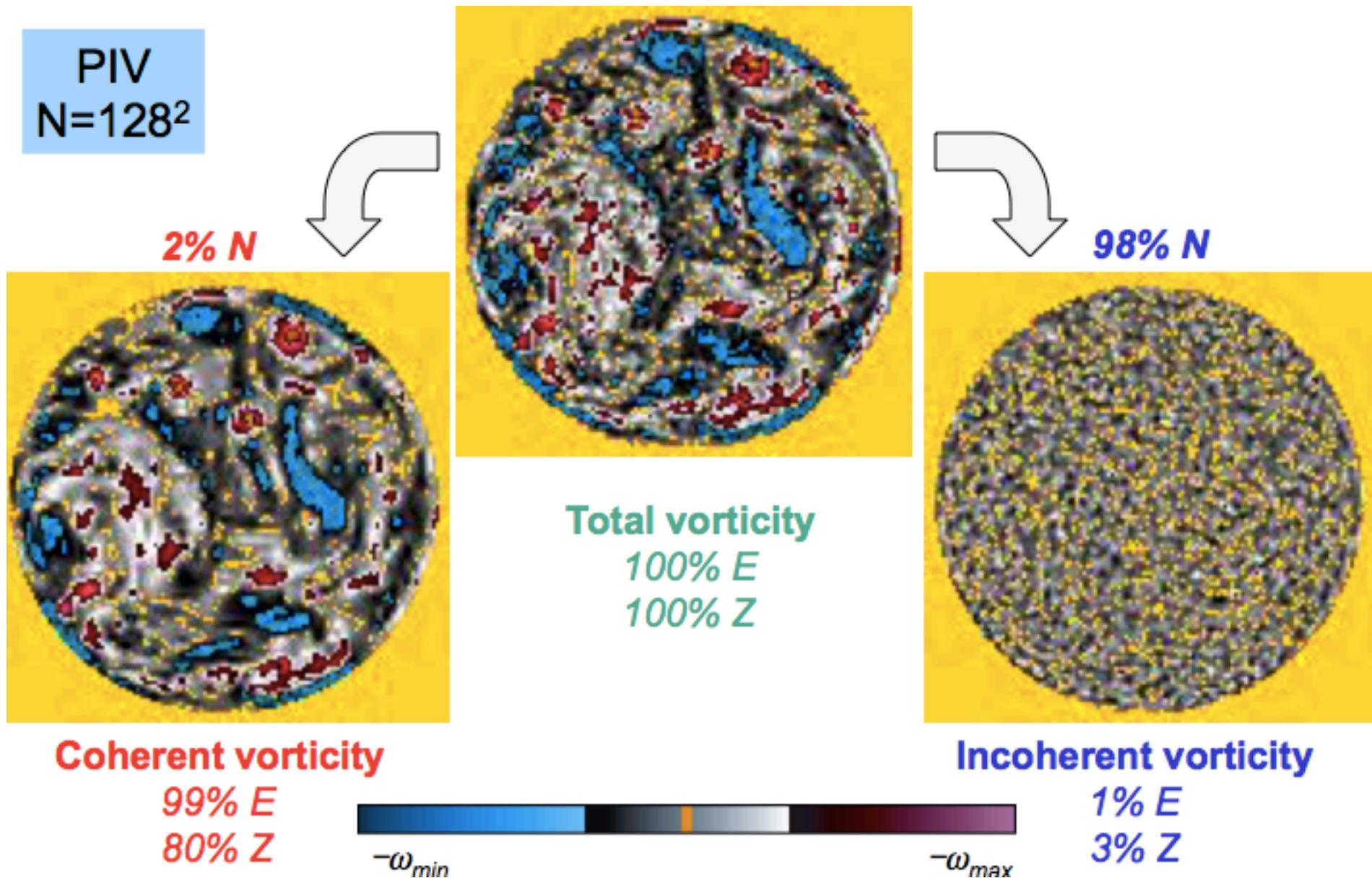


Incoherent



*Romain Nguyen van yen, Nicolas Fedorczak, Frédéric Brochard, Kai Schneider, Marie Farge and Pascale Monier-Garbet, 2012
Nuclear Fusion, IAEA (International Atomic Energy Agency), 52, 013005*

Extracting coherent structures in 2D

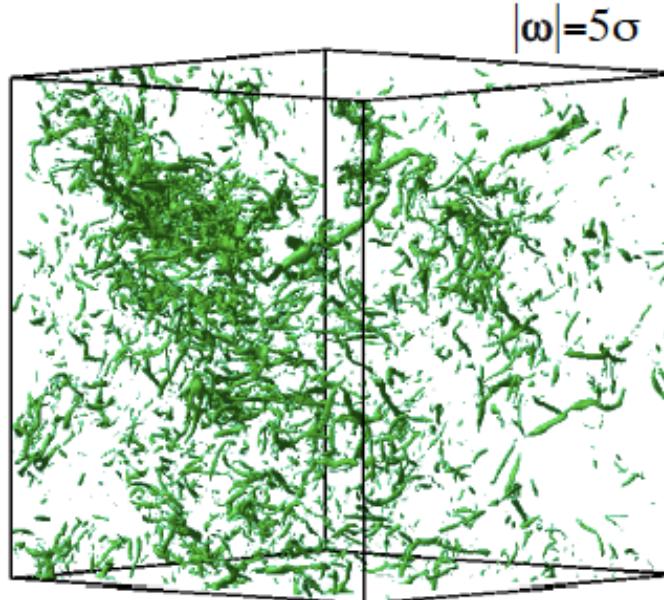


Extracting coherent structures in 3D

DNS
 $N=2048^3$

Coherent vorticity

2.6 % N coefficients
80% enstrophy
99% energy

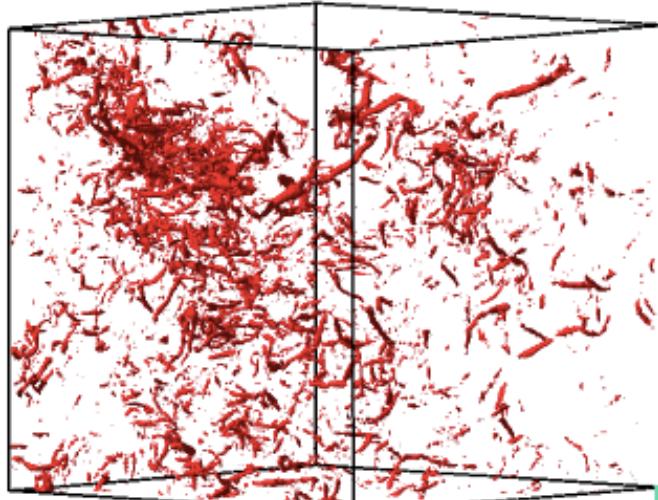


$$|\omega|=5\sigma$$

$$\text{with } \sigma=(2Z)^{1/2}$$

Incoherent vorticity

97.4 % N coefficients
20 % enstrophy
1% energy



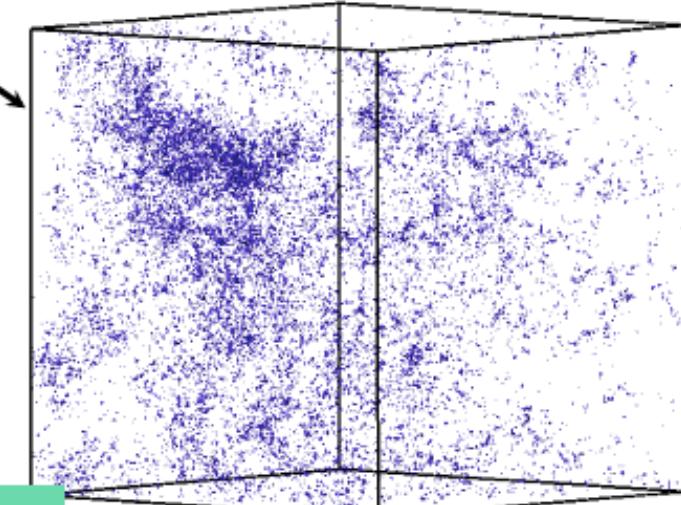
$$|\omega|=5\sigma$$

Total vorticity

$R_\lambda=732$
 $N=2048^3$
Visualization
at 256^3



Okamoto et al., 2007
Phys. Fluids, 19(11)



$$|\omega|=5/3\sigma$$

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DERMO-EXPERTISE



CLAUDIA SCHIFFER

LA CHIRURGIE ATTENDRA !

Le 1^{er} soin au boswelox™ qui décrispe les rides d'expression et rajeunit les traits.
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Environnement scientifique et technique de la formation



Laboratoire de météorologie dynamique - UMR 8539

RESPONSABLES

Marie FARGE
Directrice de recherche
UMR 8539
Kai SCHNEIDER
Professeur
UMR 7373

LIEU

GIF-SUR-YVETTE (91)

ORGANISATION

5 jours
De 7 à 10 stagiaires

Transformée de Fourier, transformées en ondelettes et paquets d'ondelettes : théorie et applications en traitement de signal et en traitement d'image

OBJECTIFS

- Apprendre les techniques classiques utilisant la transformée de Fourier et les nouvelles techniques développées à partir de la transformée en ondelettes
- Savoir appliquer ces techniques en traitement du signal et en traitement d'image
- Etre capable d'utiliser les différents programmes de transformées en ondelettes et en paquets d'ondelettes

PUBLIC

Techniciens, ingénieurs et chercheurs intéressés par le traitement de signal et le traitement d'image

PRÉREQUIS

Aucun

PROGRAMME

1er jour : transformée de Fourier

- Transformée de Fourier continue : propriétés, théorème de Parseval, convolution
- Principe d'incertitude et plan temps-fréquence
- Transformée de Fourier discrète. Théorème d'échantillonnage
- Transformée de Fourier à fenêtre

 Rechercher par mot-clé [Choisir un autre domaine](#)

OpenMP

Moteur essence turbocompressé : vers une commande à base de modèles

Débogage HPC

Les systèmes multi-agents : concepts et pratique NETLOGO

Fortran avancé

Analyse de la motricité humaine : cinétique, cinématique et électromyographie

Scripting pour Micromanager

Machine learning sous Python

Impression 3D par CAO

Hybride MPI / OpenMP

Geant4

Fortran 2003

Fortran base

MPI

Les saturations dans la boucle de commande : analyse et synthèse de contrôleurs