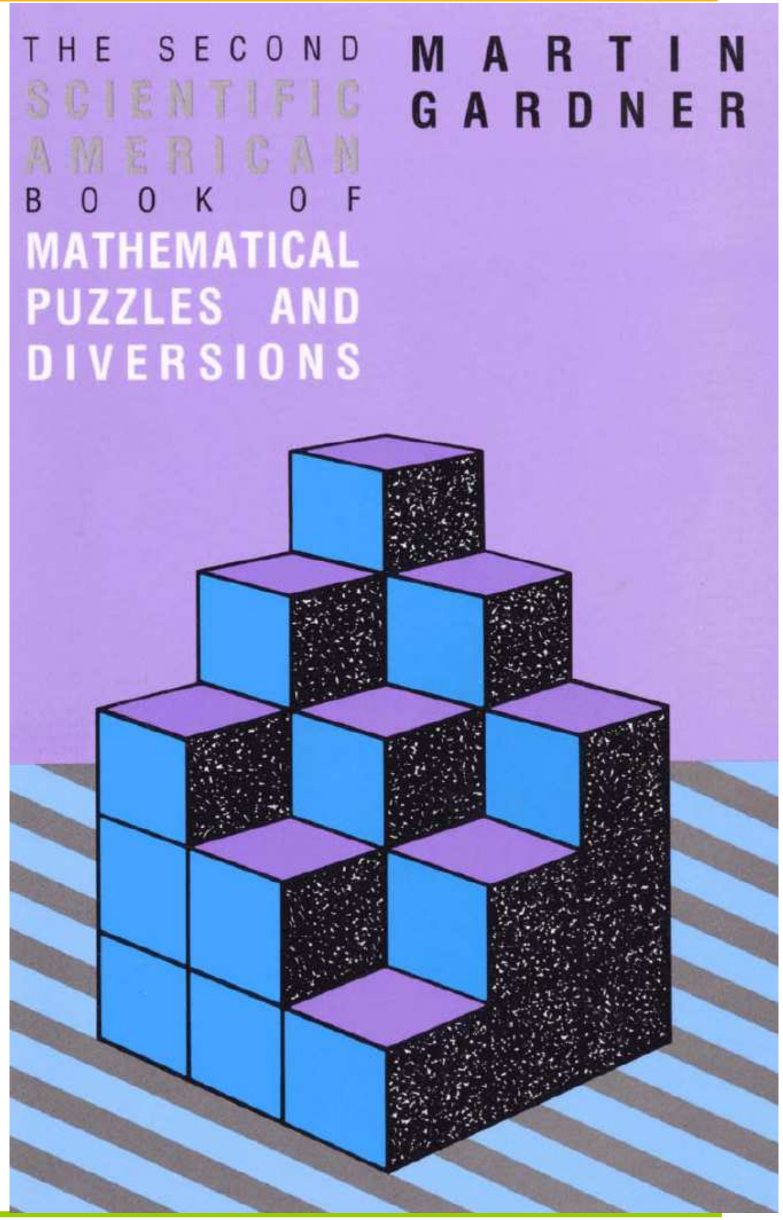
Japoniaiseries

Origami, Sangaku

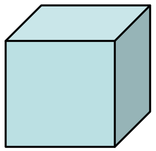
CHAPTER SIXTEEN

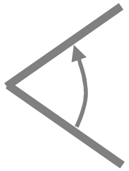
■

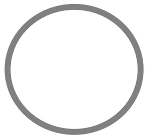
Origami



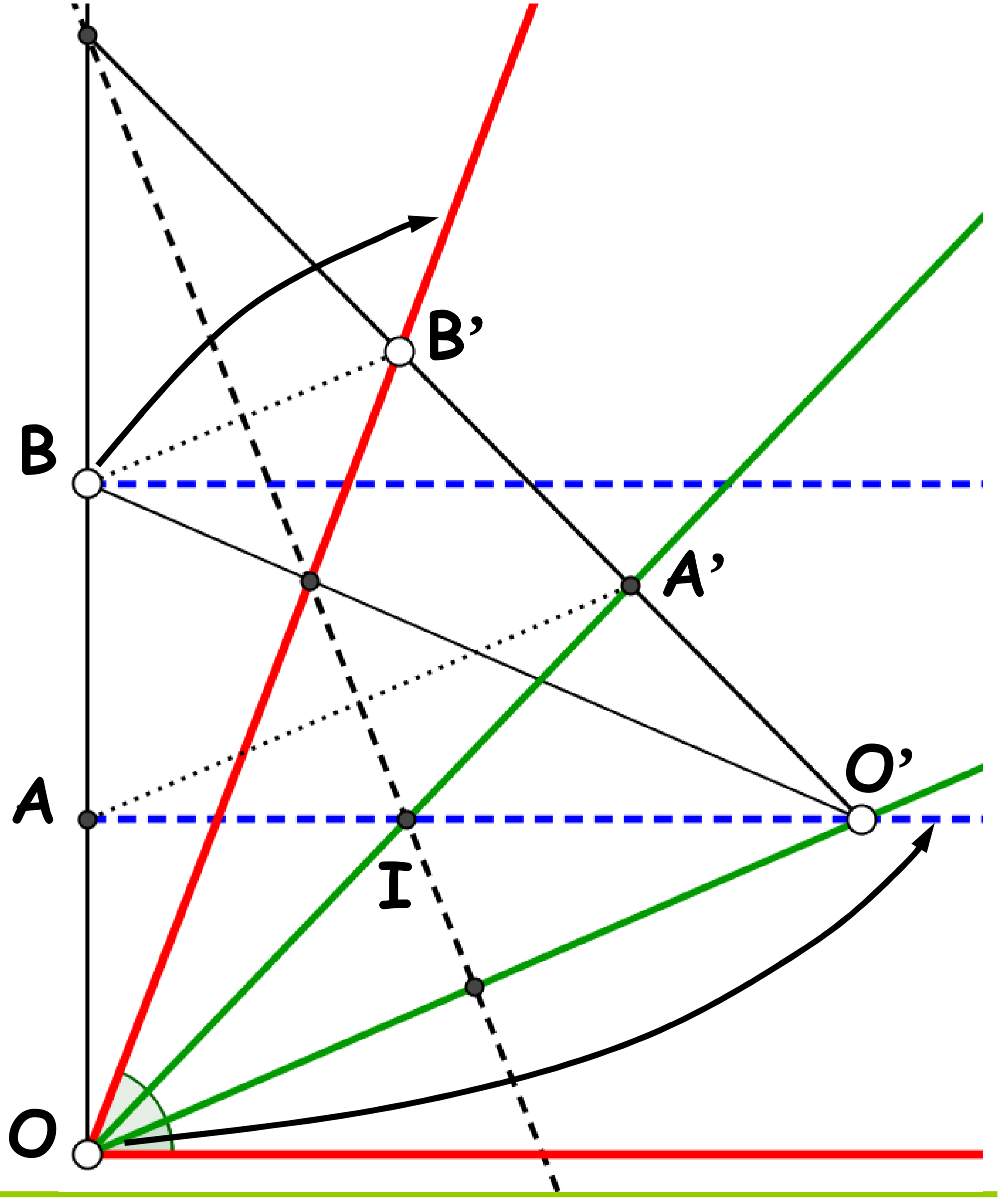
Ne pas se plier à la règle

2 - 

3 - 

4 - 

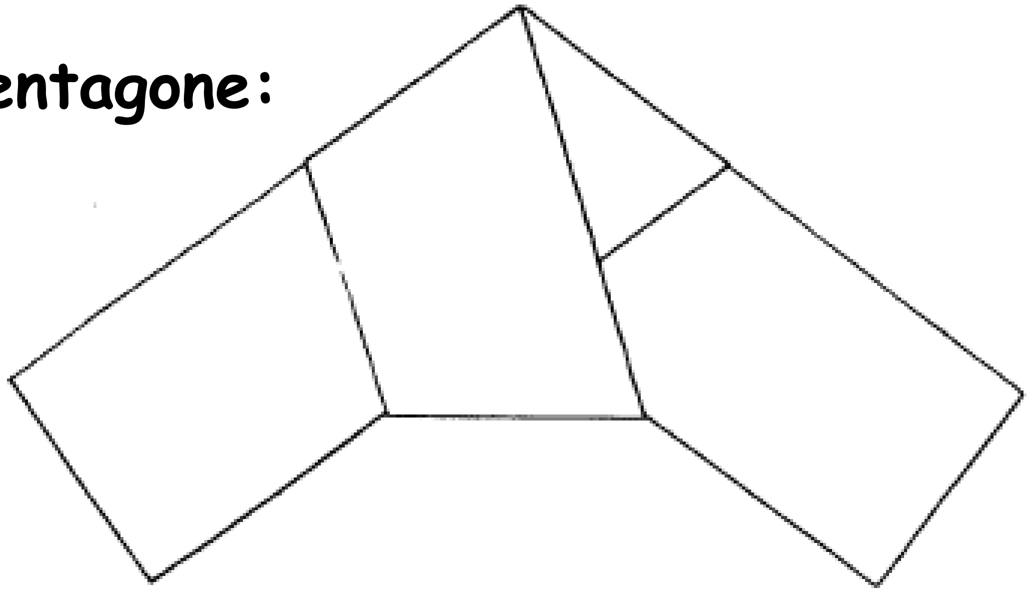
Trisection impossible
à la règle et au compas
(*P.-L. Wantzel* 1837)

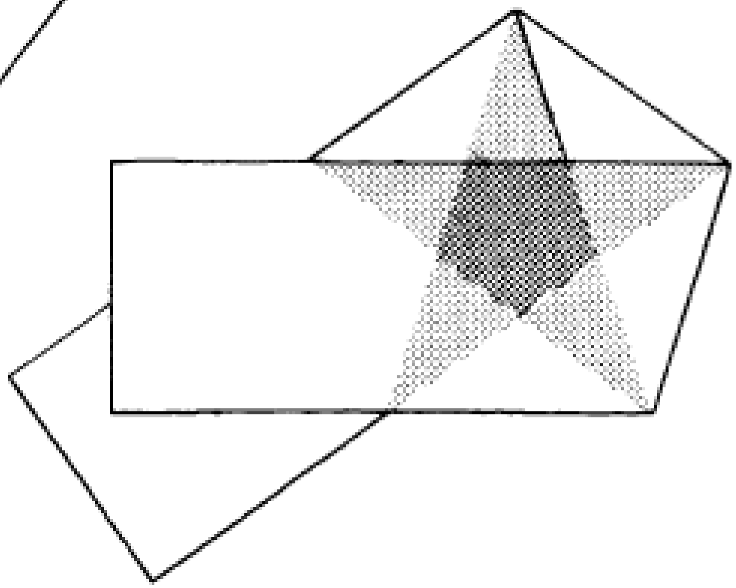


Pintes, agones

Polygones réguliers: 3, 4, 6, 8, ...

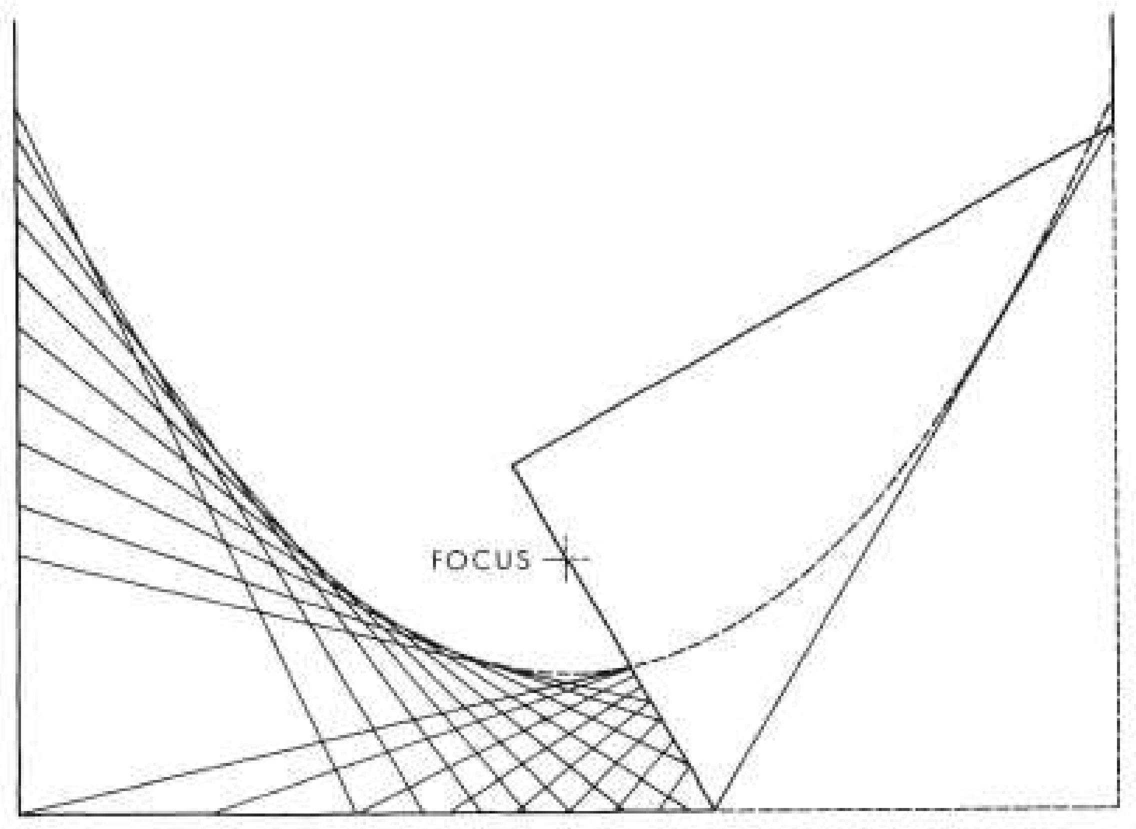
Pentagone:





***Urbano d'Aviso* (1682)**

Parabole : lieu des points à égale distance d'un point (*foyer*) et d'une droite (*directrice*).



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Lieu des points à égale distance
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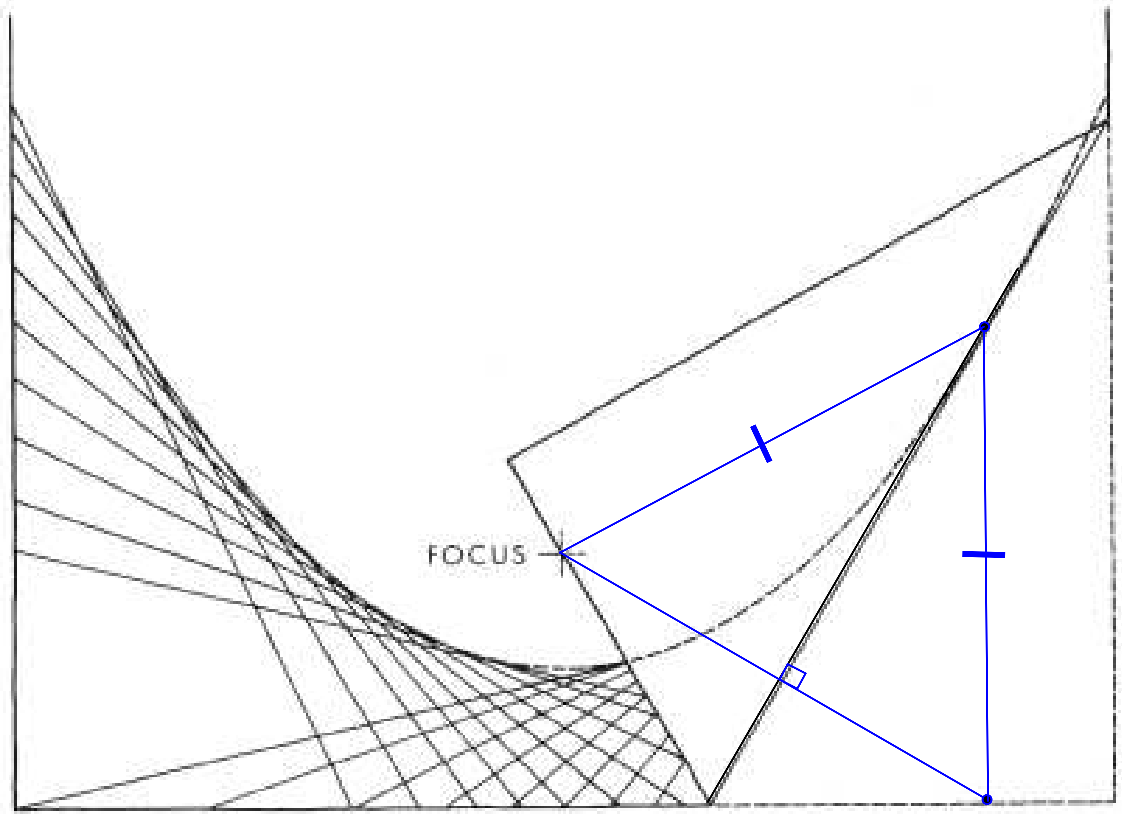


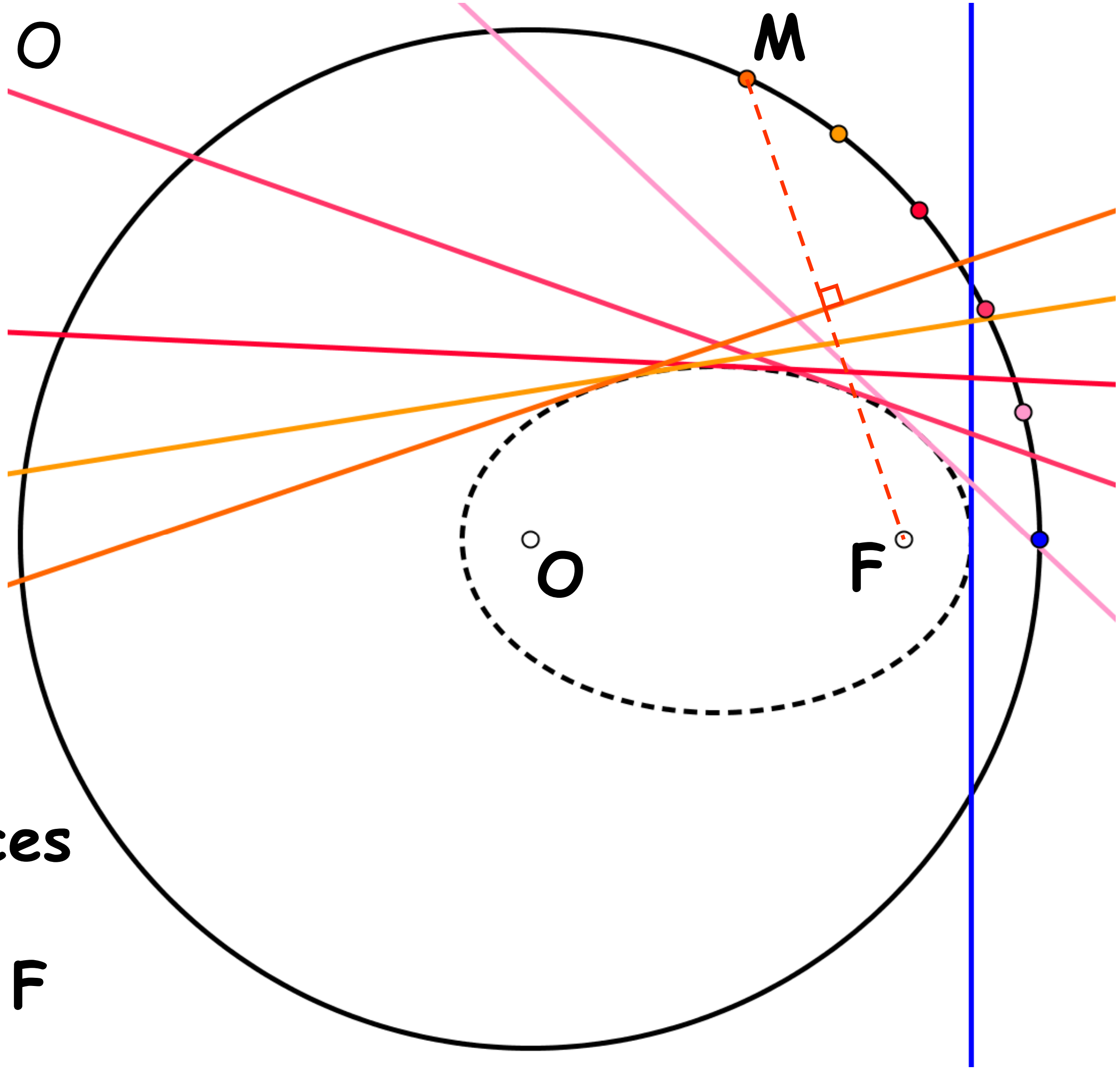
directrice

Enveloppes sous plis

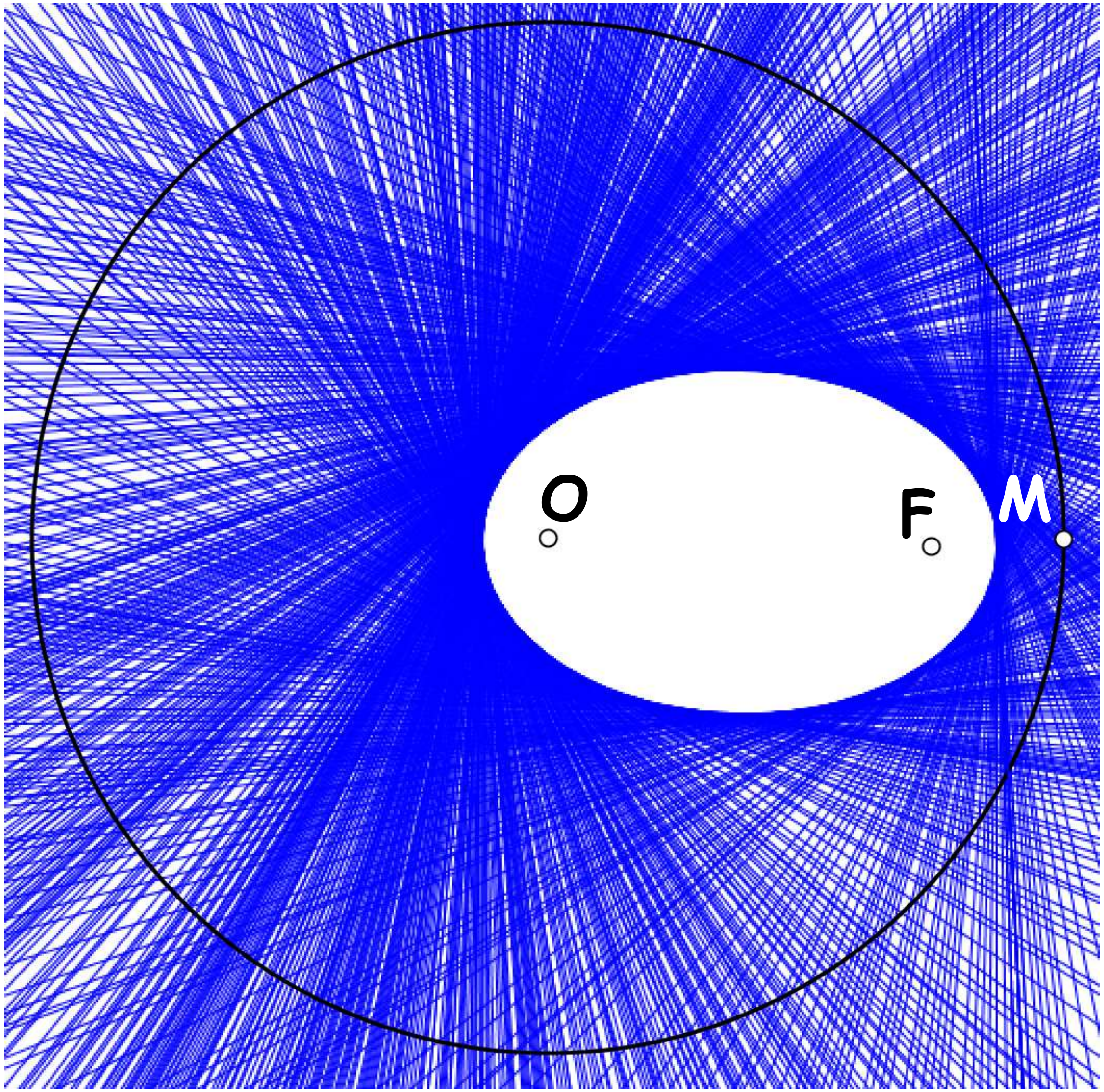
M point courant
du cercle de centre O

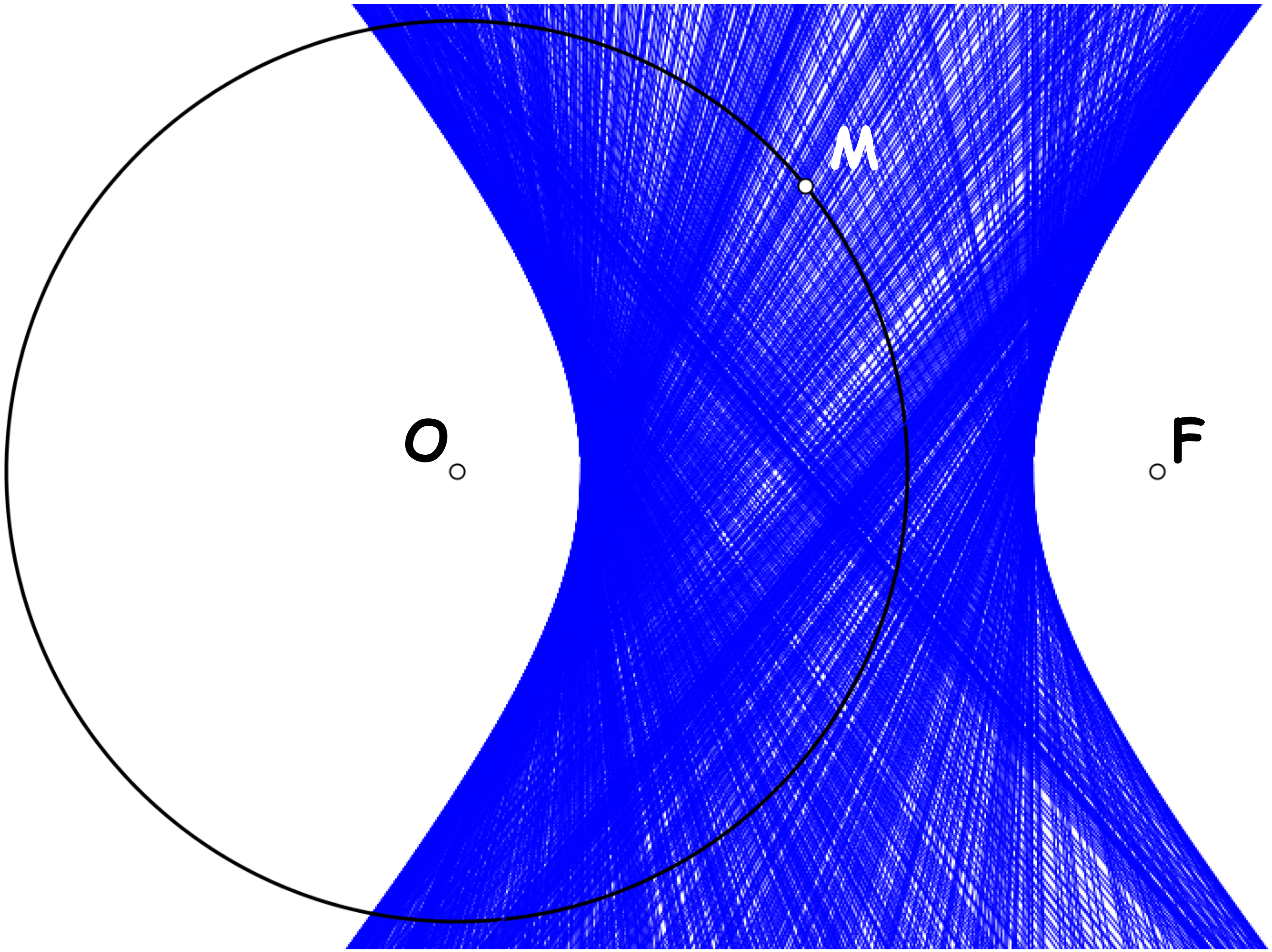
F point intérieur
du cercle

Médiatrice
du segment FM

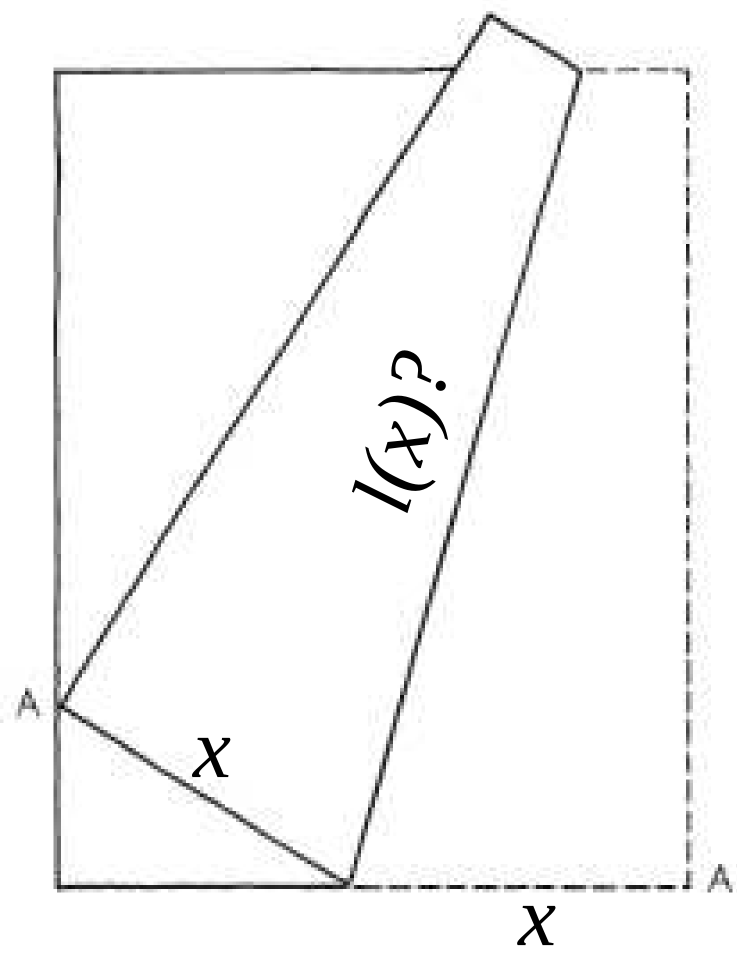


Enveloppe des médiatrices
=
Ellipse de foyers O et F

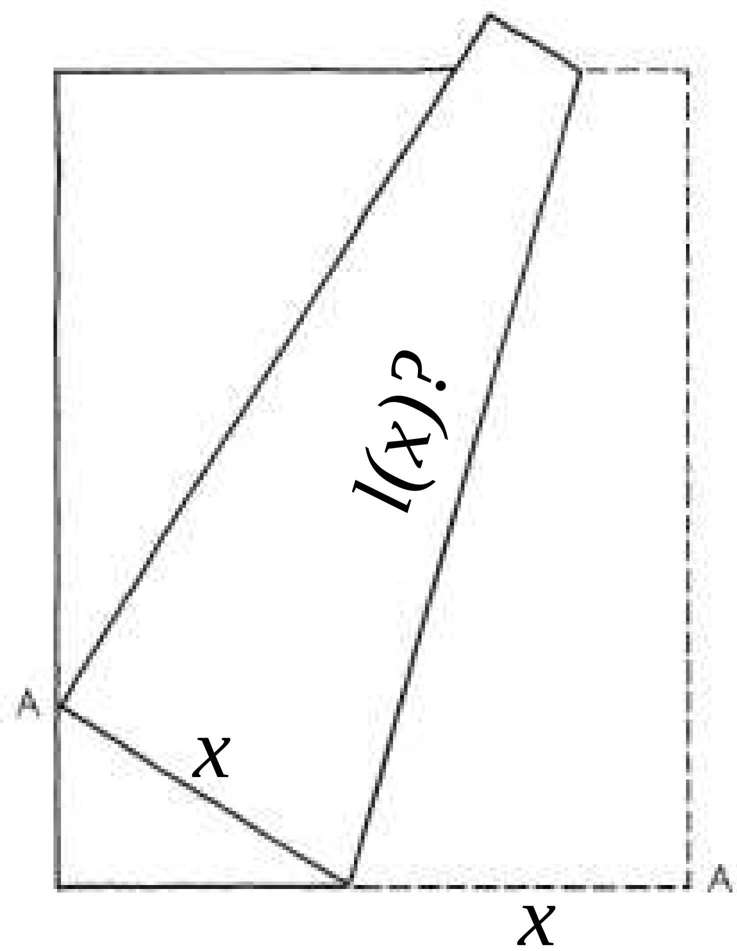

O
F
M

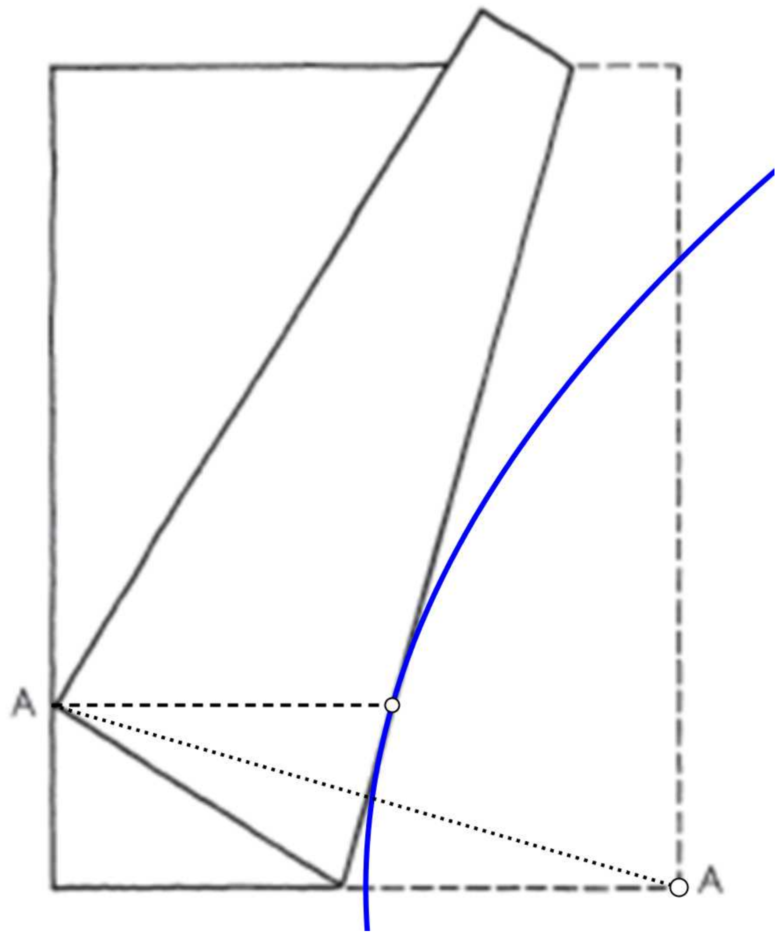

M
O
F

Problème de calcul élémentaire

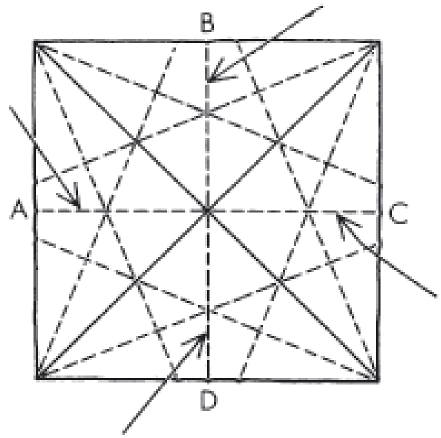


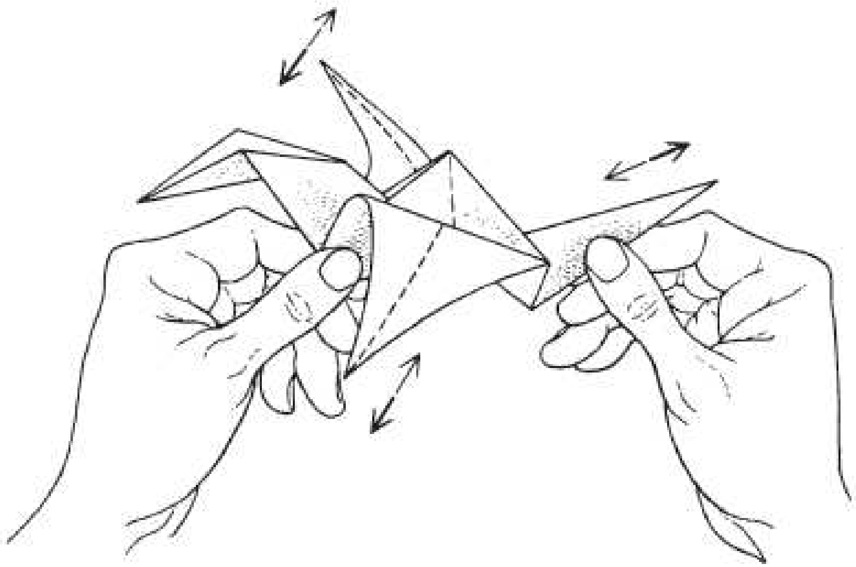
Problème plan



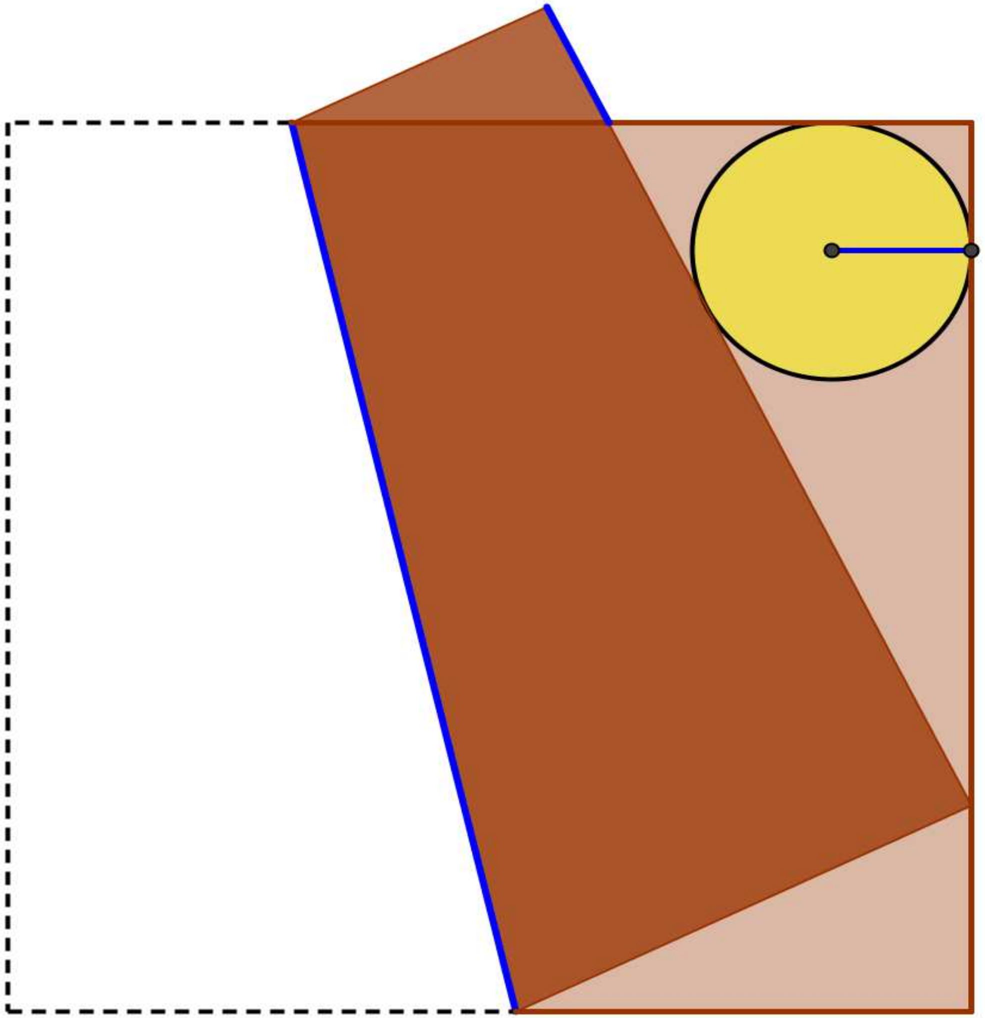


Solution plane





Plis dans l'enveloppe

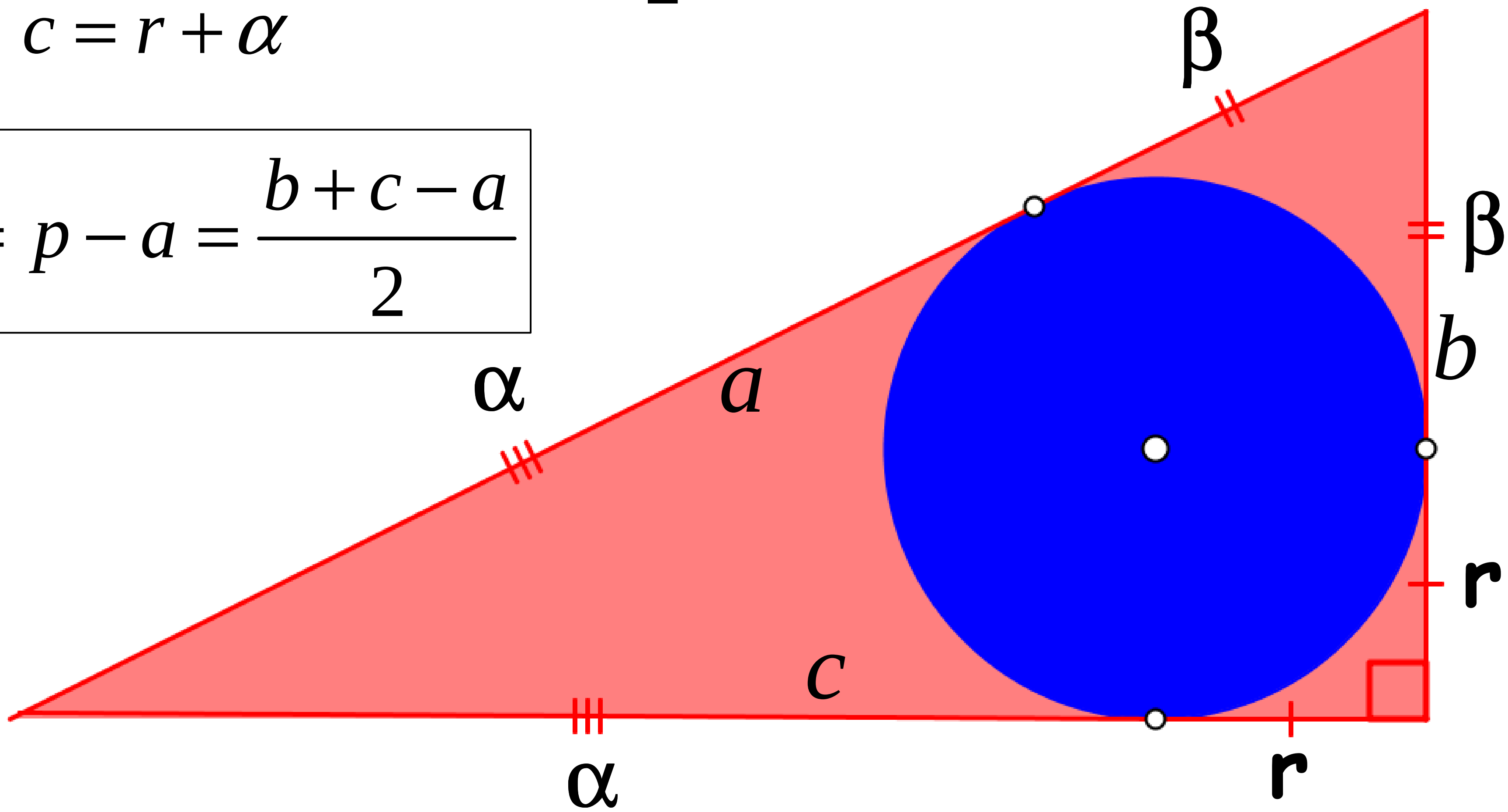


$$a = \alpha + \beta$$

$$b = \beta + r \qquad p = \frac{a+b+c}{2} = \alpha + \beta + r = a + r$$

$$c = r + \alpha$$

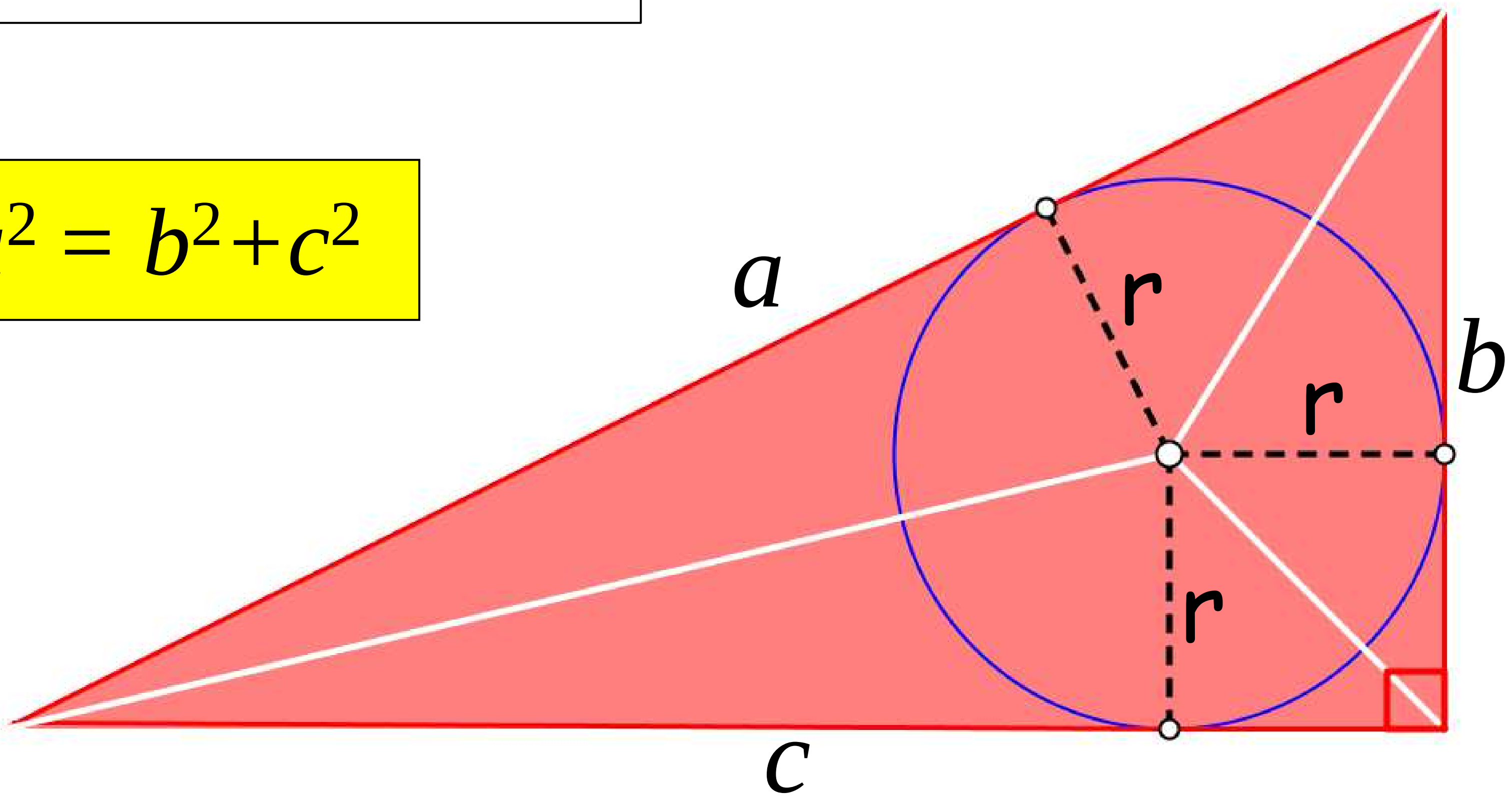
$$\boxed{r = p - a = \frac{b+c-a}{2}}$$



$$r \cdot (a+b+c) = bc$$

$$\boxed{r = \frac{b+c-a}{2} = \frac{bc}{b+c+a}} \Leftrightarrow (b+c)^2 - a^2 = 2bc$$

$$\boxed{a^2 = b^2 + c^2}$$



On n'est jamais assez fort pour ce calcul

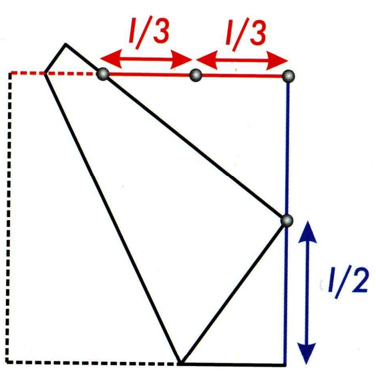


Figure 1

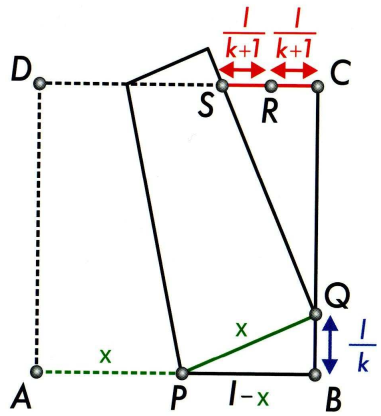


Figure 2

En effet, soit l la longueur du côté du carré, et $x = AP = PQ$, d'après le théorème de Pythagore :

$$x^2 = \frac{l^2}{k^2} + (l-x)^2, \text{ soit } x = \frac{l}{2}\,\frac{1+k^2}{k^2}.$$

Puisque l'angle $\widehat{SQP}$ est droit, les triangles PBQ et QCS sont semblables, donc :

$$\frac{CS}{CQ} = \frac{BQ}{BP} \Longrightarrow CS = CQ \times \frac{BQ}{BP}$$

Avec $BQ = \frac{l}{k}$, $CQ = l - \frac{l}{k}$ et $BP = l - x = l - \frac{l}{2}\,\frac{1+k^2}{k^2} = l\,\frac{k^2-1}{2k^2}$, on obtient :

$$CS = \frac{2l}{k+1},$$

ce qui termine la preuve.

Le sangaku devient maintenant immédiat. En effet, rien n'impose à k d'être entier.

Puisque $\frac{PQ}{BQ} = \frac{QS}{CS}$, on a :

$$QS = \frac{2l}{k+1} \times \frac{l}{2}\,\frac{1+k^2}{k^2}\,\frac{k}{l} = l\,\frac{k^2+1}{k(k+1)}.$$

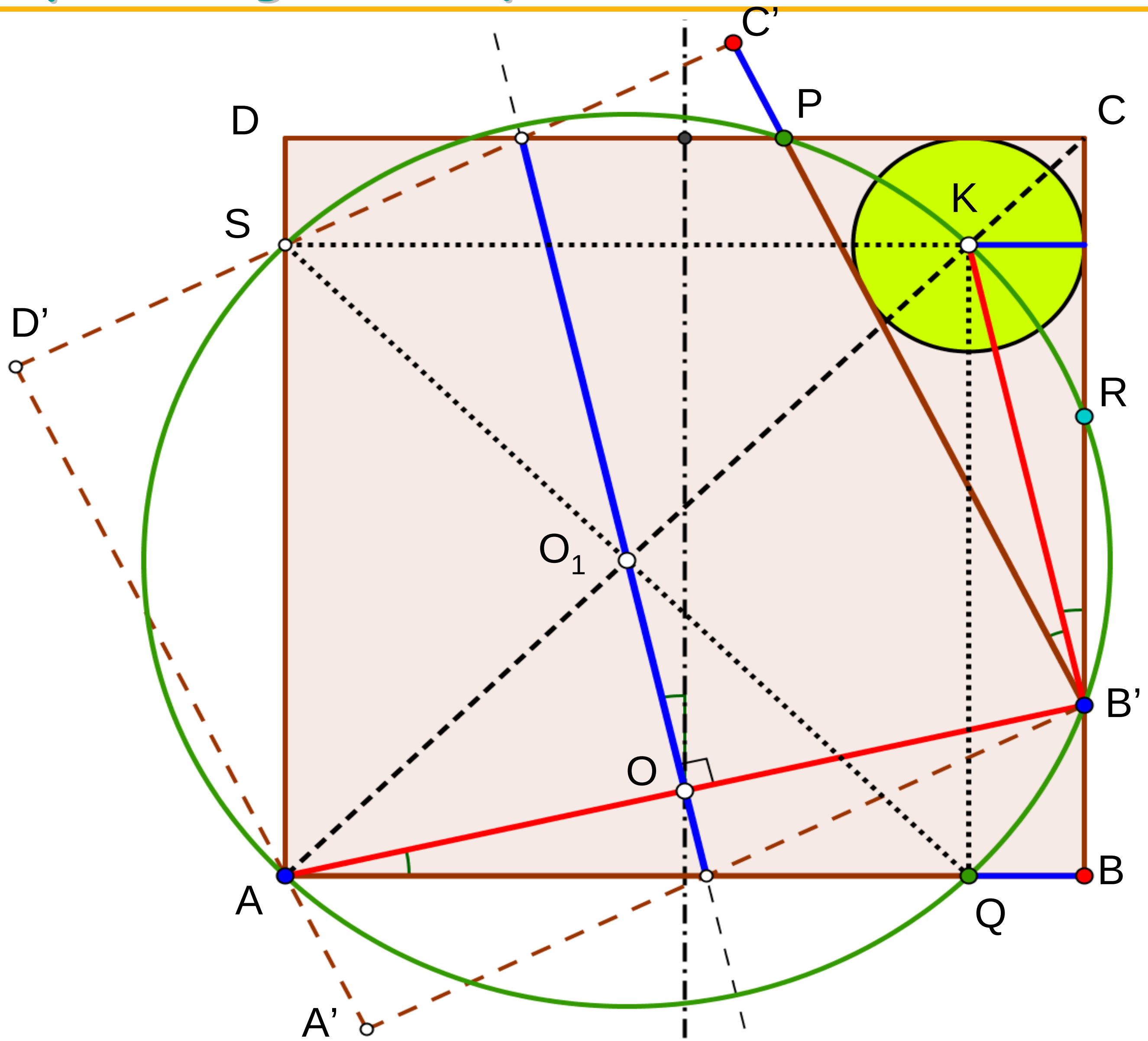
Le rayon du cercle inscrit au triangle rectangle CQS vaut alors :

$$r = \frac{CQ+CS-QS}{2} = \frac{l}{2}\left(1 - \frac{1}{k} + \frac{2}{k+1} - \frac{k^2+1}{k(k+1)}\right) = l\,\frac{k-1}{(k+1)\,k}$$

On vérifie que cette valeur est bien égale à $l - QS = l - l\,\frac{k^2+1}{k(k+1)}$ ce qui prouve que :

$$r = d$$

Propriétés géométriques



Origami pour Sangaku