

Les équations d'Euler (pour un fluide compressible)

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \quad \text{conservation de la masse} \\ \frac{\partial \rho V}{\partial t} + \nabla \cdot (\rho V \otimes V + P Id_3) = 0 \quad \text{conservation de la quantité} \\ \frac{\partial E}{\partial t} + \nabla \cdot ((E + P)V) = 0 \quad \text{conservation de l'énergie} \end{array} \right.$$

$$P = (\gamma - 1) \left(E - \frac{1}{2} \rho V^2 \right) = (\gamma - 1) \rho \varepsilon \quad \text{loi de comportement}$$

Si on considère les aspects viscosité (frottement sur les parois, ...) on doit résoudre les équations de Navier-Stokes

Le torseur des efforts aérodynamiques.

Les efforts (adimensionnés) exercés par un fluide parfait sur la structure s'expriment sous la forme :

$$\begin{aligned} f_{\text{aero}} &= \frac{1}{S_{\text{ref}}} \int_{\Gamma_s} K_P \mathbf{n} \, ds \\ M_{\text{aero}}^G &= \frac{1}{S_{\text{ref}} L_{\text{ref}}} \int_{\Gamma_s} \vec{GM} \wedge K_P \mathbf{n} \, ds \end{aligned} \quad (7)$$

où K_P est le coefficient de pression (valeur sans dimension) :

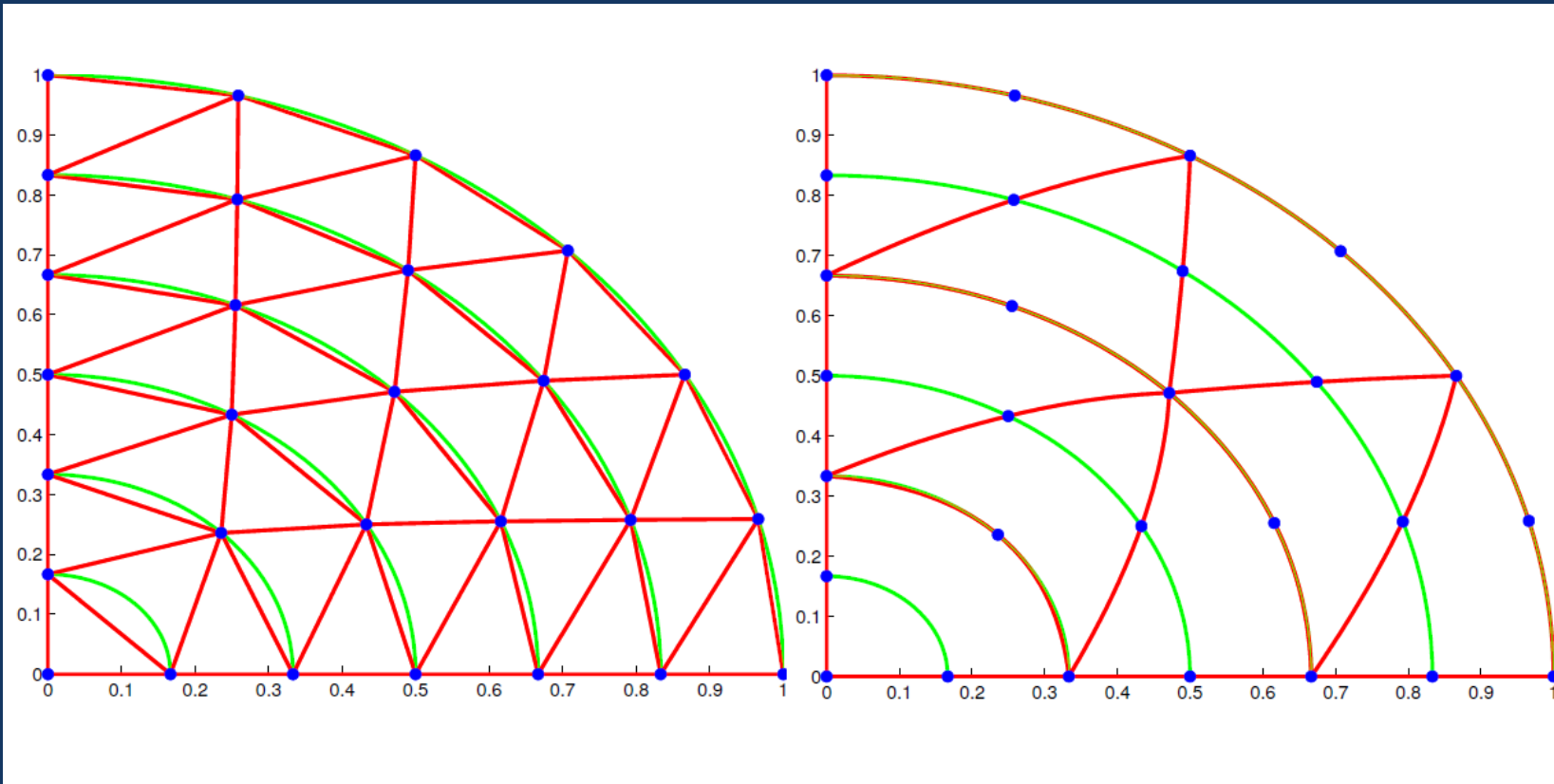
$$K_P = 2 \frac{P - P_{\infty}}{\rho_{\infty} v_{\infty}^2}$$

Les grandeurs L_{ref} et S_{ref} sont la longueur et la surface de référence respectivement, utiles pour l'adimensionnement.

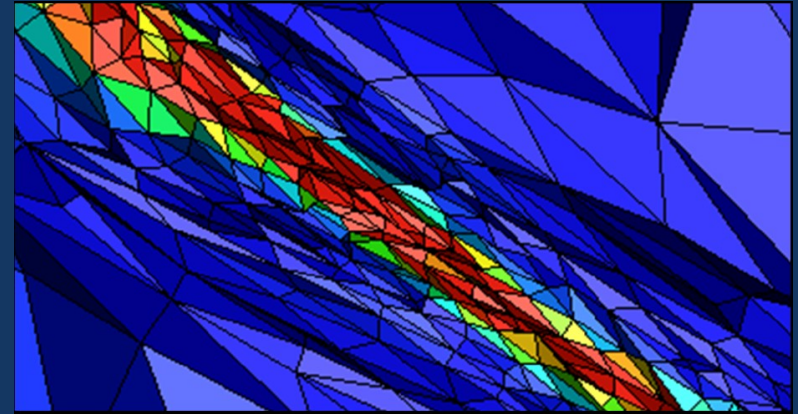
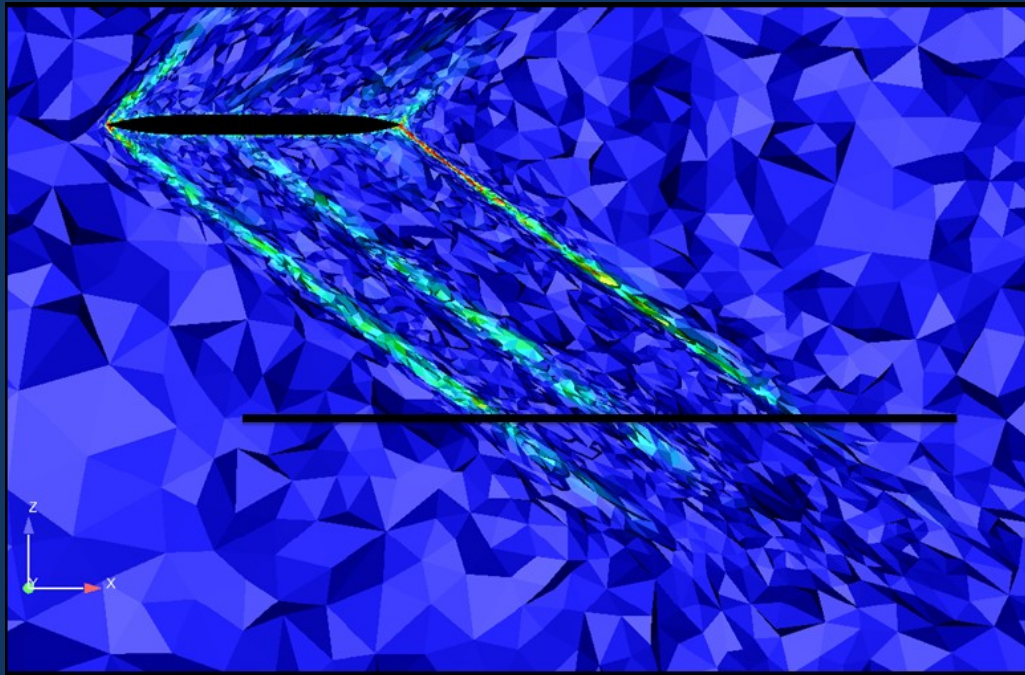
Coefficients de trainée C_x , de portance C_z , de moment de tangage C_m , ...



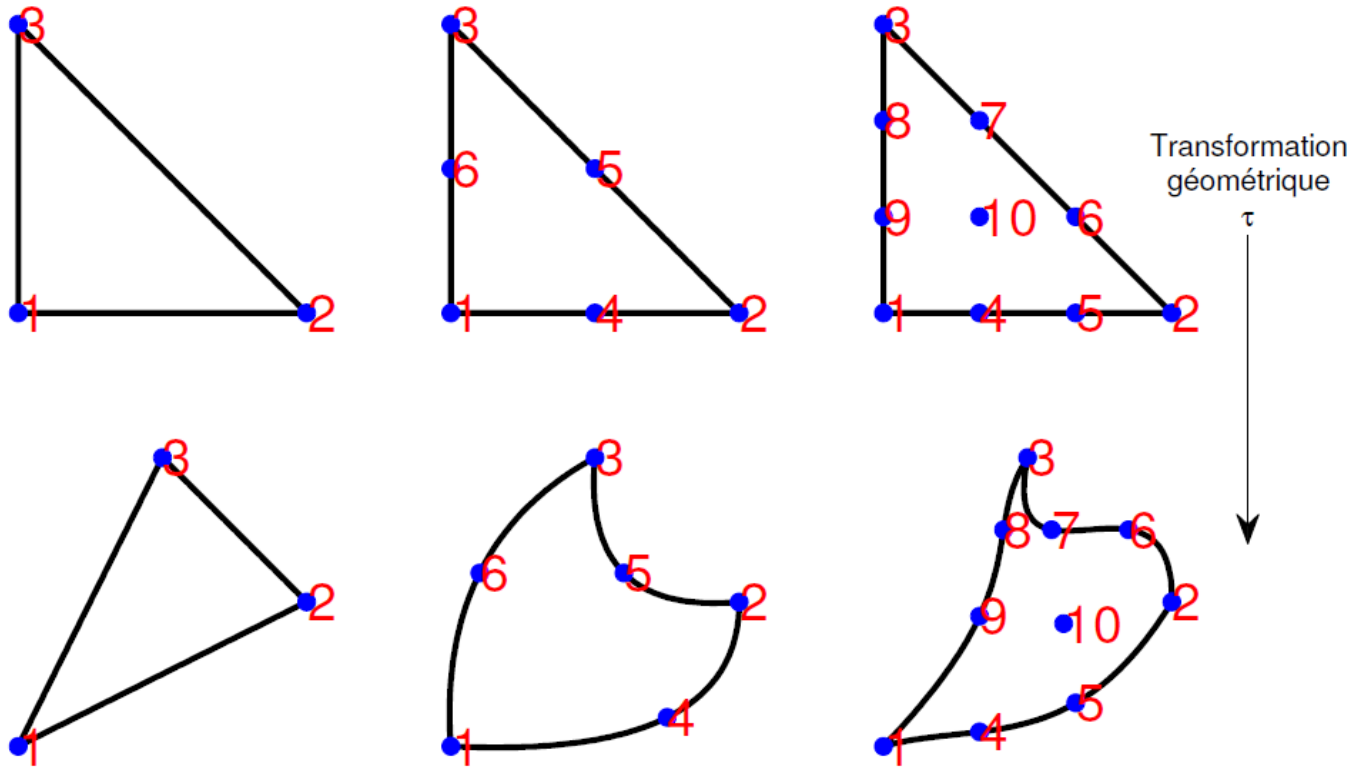
Discrétisation spatiale, Maillage, Éléments finis.



Discrétisation spatiale, Maillage, Éléments finis.



Élément de référence. Transformation isoparamétrique. Formules de quadrature



$$\int_K f(x) dx = \int_{\tilde{K}} f(\tau(\xi)) j(\xi) d\xi \simeq \sum_i w_i f(\tau(\xi_i)) j(\xi_i)$$

Formulation faible. Schémas numériques.

Résidu : $R(U) = 0 \Leftrightarrow \forall W \int WR(U) = 0$ Discrétisation : $U^h = \sum_j N_j U_j$

Galerkin	$\int_{\Omega} N_i R(U^h) = 0$
Streamline Upwind Petrov Galerkin	$\int_{\Omega} (N_i + \tau \lambda \nabla N_i) R(U^h) = 0$
Residual Distribution Scheme	$\beta_i(U^h) \int_{\Omega} R(U^h) = 0$
Général	$\int_{\Omega} W_i(U^h) R(U^h) = 0$

$$\sum_K \int_K W_i(U^h) R(U^h) = 0$$

Prise en compte des incertitudes (5/17)

Formulation for derivatives 1 and 2

E Non linear system for fluid (Euler or Navier-Stokes)

L Linear system for mesh deformation

W State, solution of E

X Volume coordinates

x Surface coordinates

J Observation

λ Aerodynamic parameters

ν Geometric parameters

ψ Fluid Adjoint, solution of $\psi^T \frac{\partial E}{\partial W} = \frac{\partial J}{\partial W}$

ϕ Mesh Adjoint, solution of $\phi^T \frac{\partial L}{\partial X} = \frac{\partial J}{\partial X} - \psi^T \frac{\partial E}{\partial X}$

Second derivative operator

$$D_{G_1, G_2}^2 M \cdot [V_1, V_2] = V_1^T \frac{\partial^2 M}{\partial G_1^2} V_1 + 2V_1^T \frac{\partial^2 M}{\partial G_1 \partial G_2} V_2 + V_2^T \frac{\partial^2 M}{\partial G_2^2} V_2$$

Aircraft



Aircraft Analysis



Prise en compte des incertitudes (6/17)

Derivatives 1 and 2

$$\frac{dJ}{d\lambda} = \frac{\partial J}{\partial \lambda} - \psi^T \frac{\partial E}{\partial \lambda}$$

$$\frac{d^2 J}{d\lambda^2} = \underbrace{D_{W,\lambda}^2 J \cdot \left[\frac{\partial W}{\partial \lambda}, 1 \right]}_{\text{Explicit}} - \underbrace{\psi^T D_{W,\lambda}^2 E \cdot \left[\frac{\partial W}{\partial \lambda}, 1 \right]}_{\text{Implicit CFD}}$$

$$\frac{dJ}{dv} = -\phi^T \frac{\partial L}{\partial x} \frac{\partial x}{\partial v}$$

$$\frac{d^2 J}{dv^2} = \underbrace{D_{W,X}^2 J \cdot \left[\frac{\partial W}{\partial v}, \frac{\partial X}{\partial v} \right]}_{\text{Explicit}} - \underbrace{\psi^T D_{W,X}^2 E \cdot \left[\frac{\partial W}{\partial v}, \frac{\partial X}{\partial v} \right]}_{\text{Implicit CFD}} - \underbrace{\phi^T \frac{\partial L}{\partial x} \frac{\partial^2 x}{\partial v^2}}_{\text{Implicit deformation}}$$



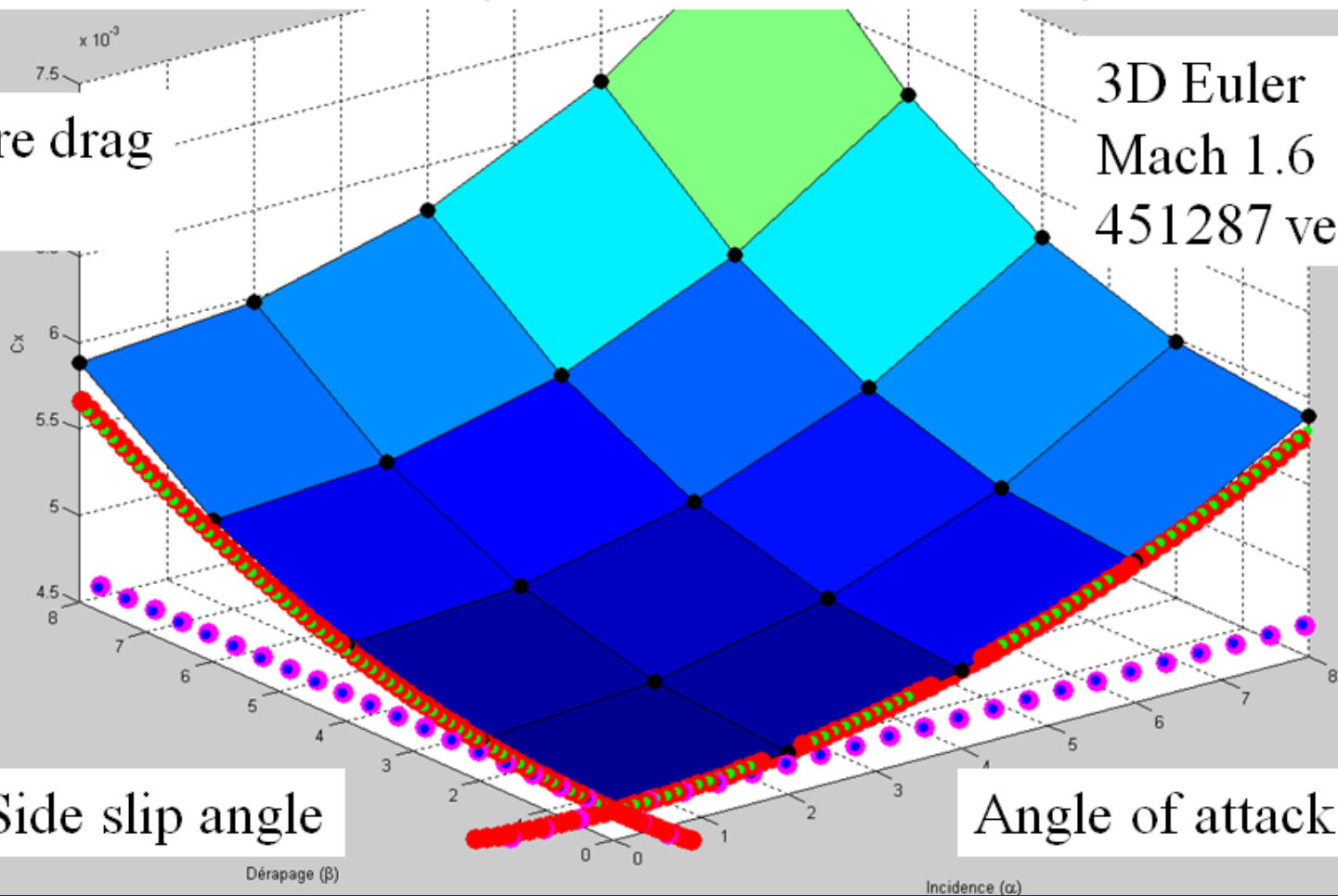
Prise en compte des incertitudes (7/17)

Validation visuelle dérivées - Tangentes, rayons de courbure

$$g(\alpha, \beta) = g(\alpha_0, \beta_0) + \frac{\partial g}{\partial \alpha} d\alpha + \frac{\partial g}{\partial \beta} d\beta + \frac{(d\alpha)^2}{2} \frac{\partial^2 g}{\partial \alpha^2} + \frac{(d\beta)^2}{2} \frac{\partial^2 g}{\partial \beta^2} + d\alpha \cdot d\beta \cdot \frac{\partial^2 g}{\partial \beta \partial \alpha}$$

Pressure drag
CDp

3D Euler
Mach 1.6
451287 vertices



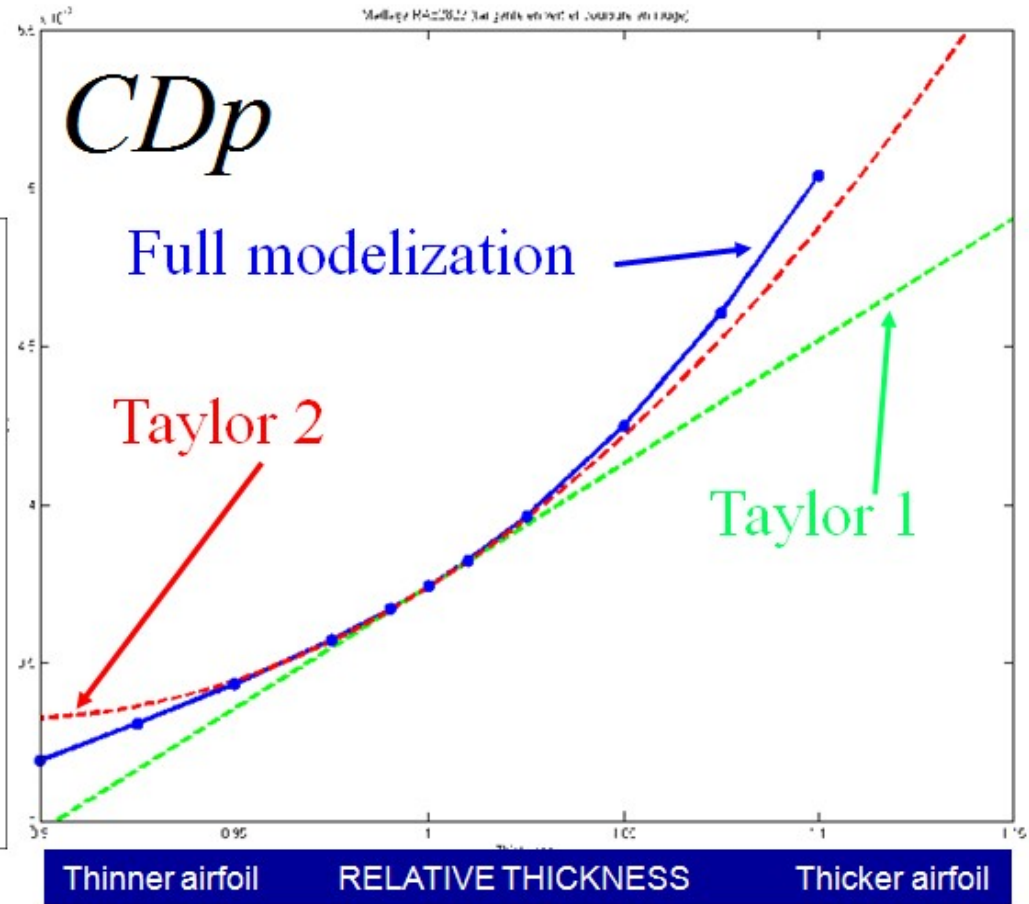
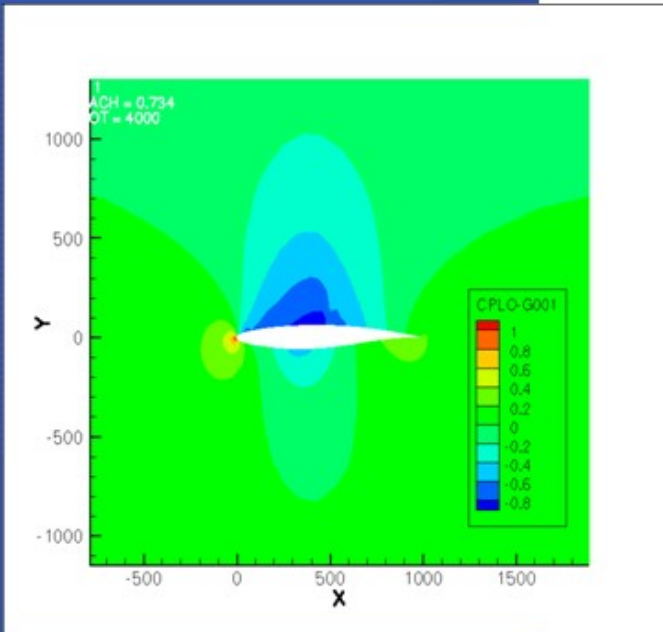
Side slip angle

Angle of attack



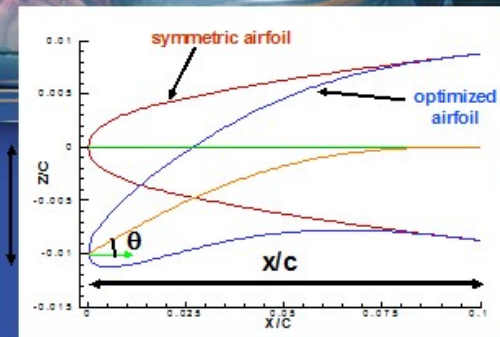
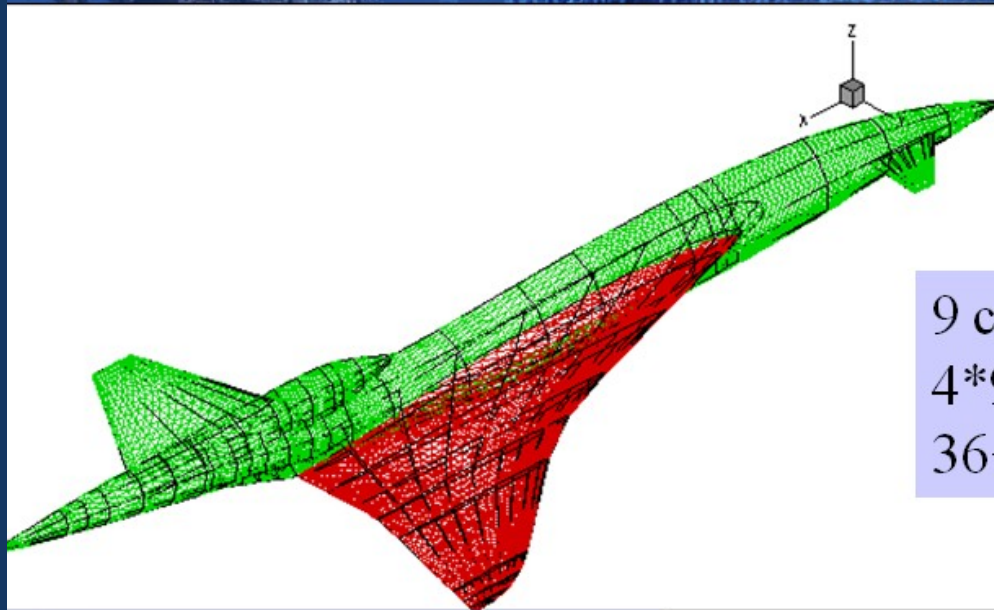
Derivatives 1 and 2 validation

aoa=1 deg



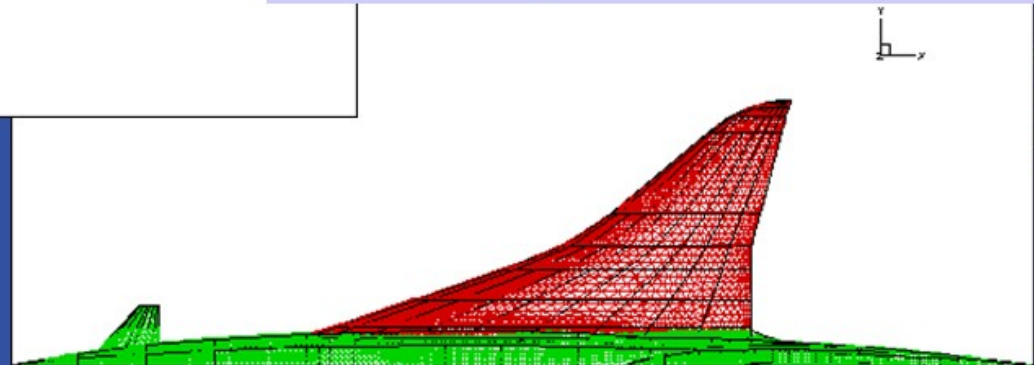
Mesh + CAD (symmetric wing)

3D view



9 control sections
4*9=36 design variables
36+1=37 optimization variables

Half geometry-mesh




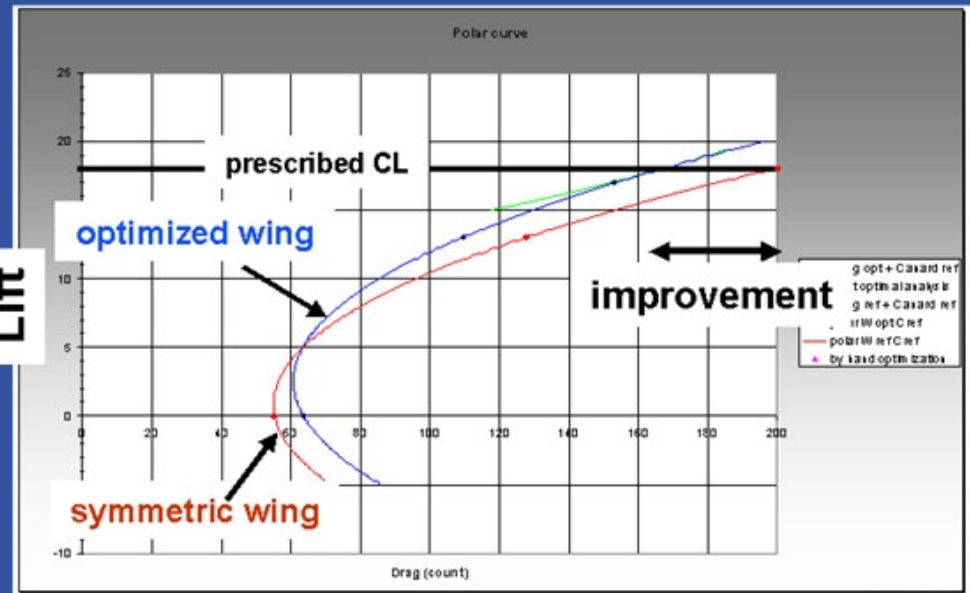
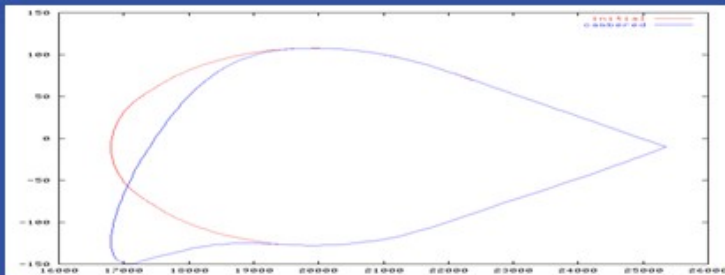
Top view



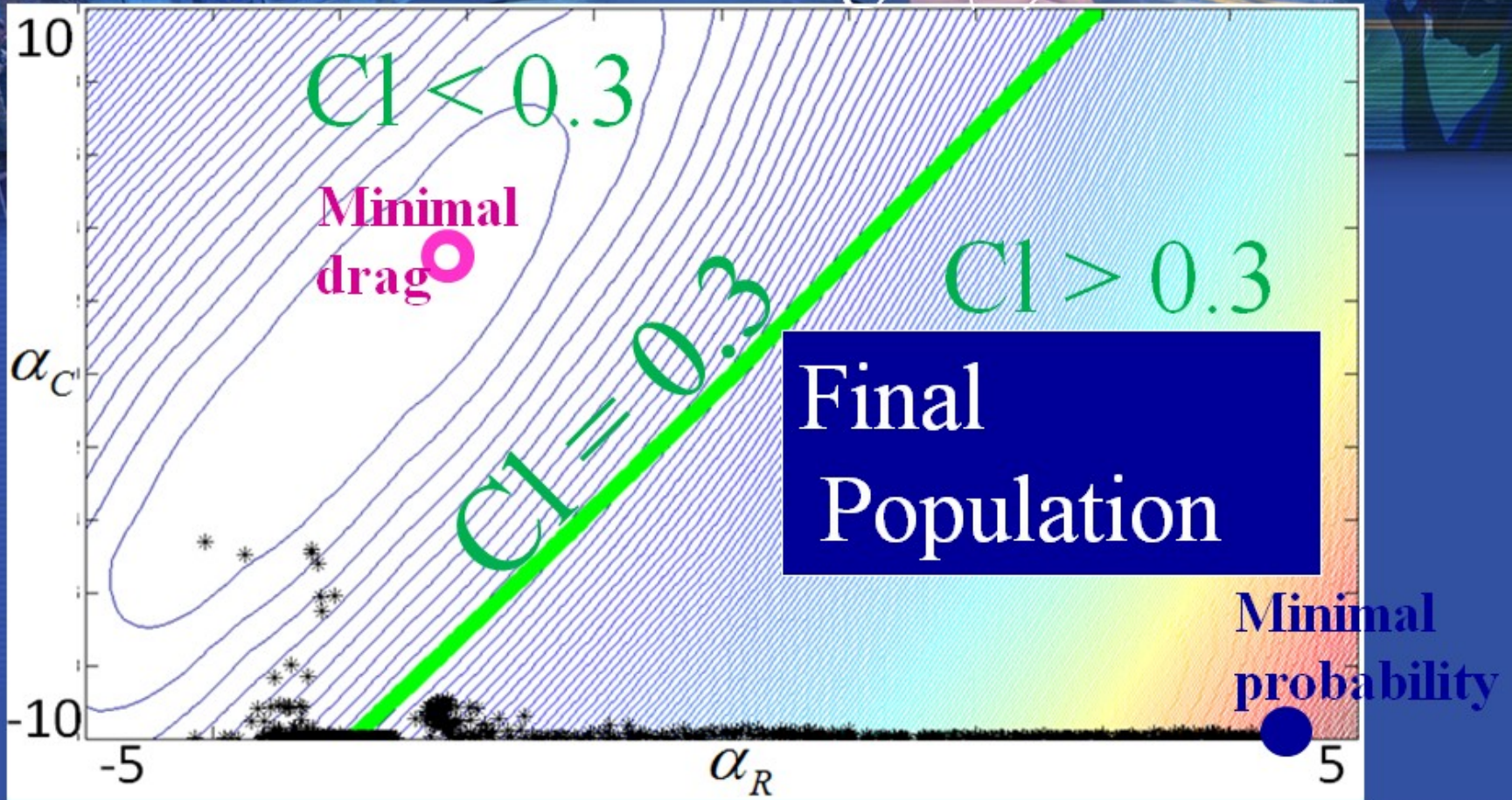
VA OÙ TON RÊVE TE PORTE

Conclusion for wing optimization

- Impact in term of time in the CFD design process (3 months  2-3 days)
- Impact in term of quality for the aerodynamic solution (18%)

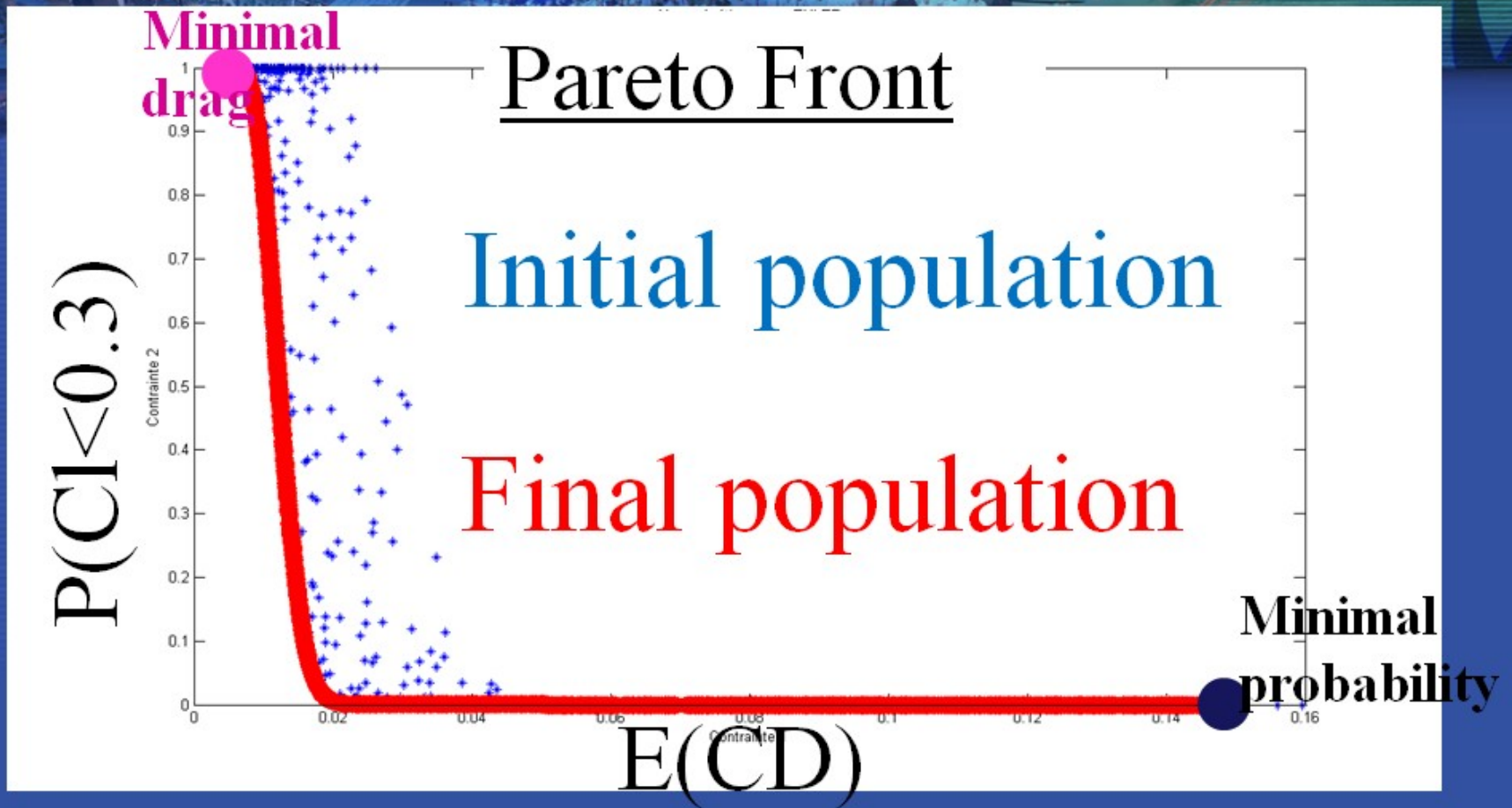


Robust Design (1)



ONERA M6 wing, 2 design parameters: twist and TE camber angles
Euler, RSM RBF like but with 1st and 2nd derivatives (original approach, Duchon extension)
MOGA, Robust design. Objectives: to control Drag and $P(CL < 0.3)$

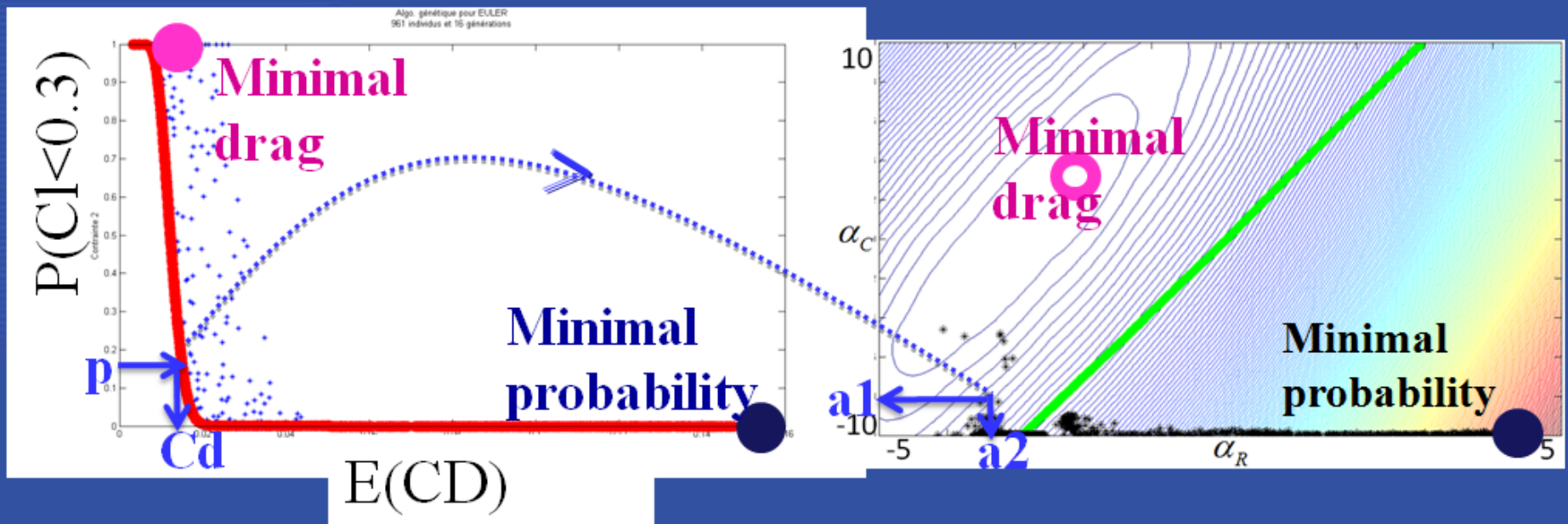
Robust Design (2)



Robust Design (3)

Decision: we accept a probability lift of $p = x\%$ with minimal drag

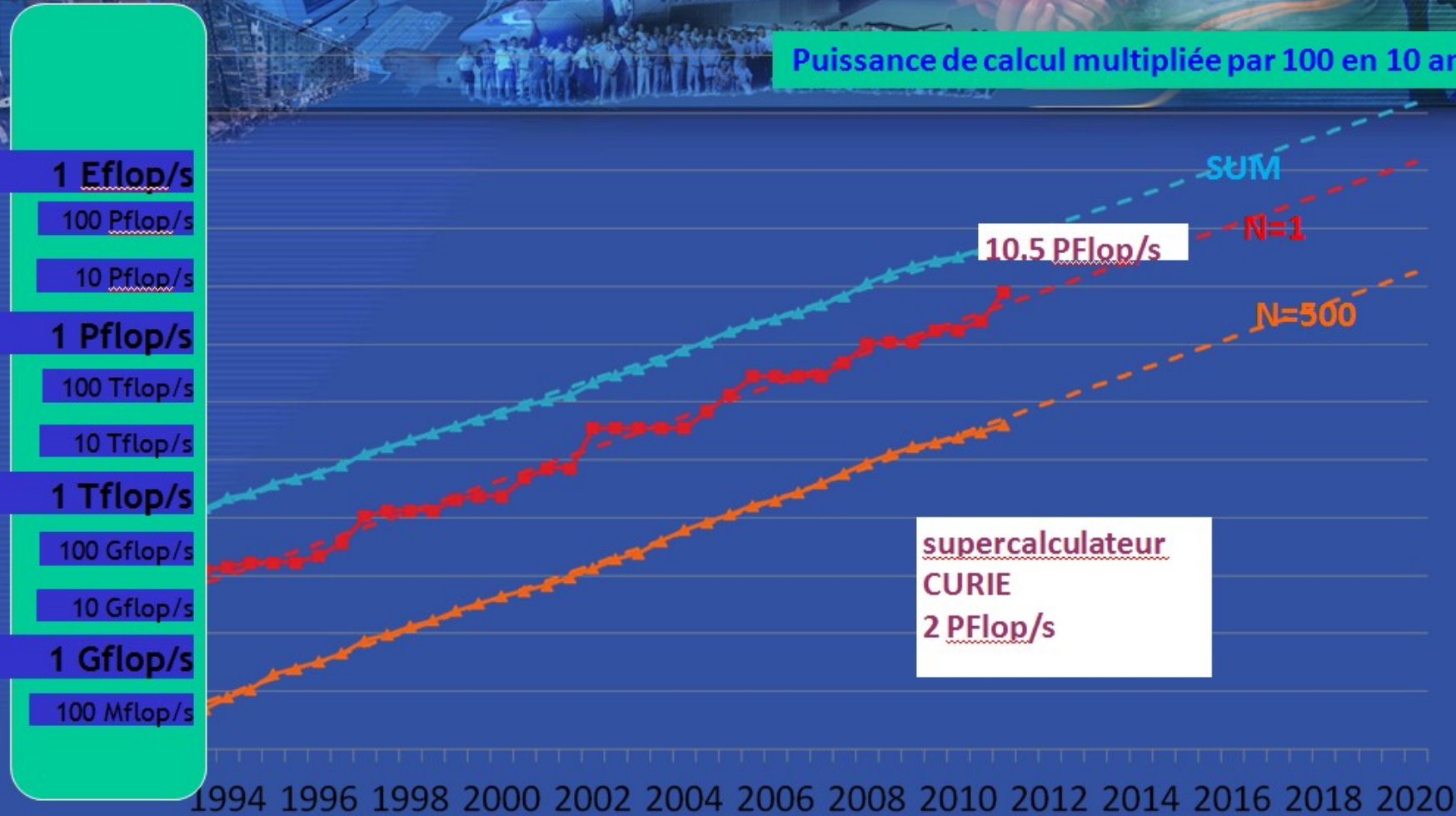
- Determination of drag mean CD (Pareto front)
- Determination of nominal values of geometrical parameters $a1$ and $a2$ (camber and twist angles)



Tera 10^{12}
Peta 10^{15}

Performance Development

Puissance de calcul multipliée par 100 en 10 ans



Dassault Aviation suit la loi de Moore

MPI, OPENMP, acceleration GPU



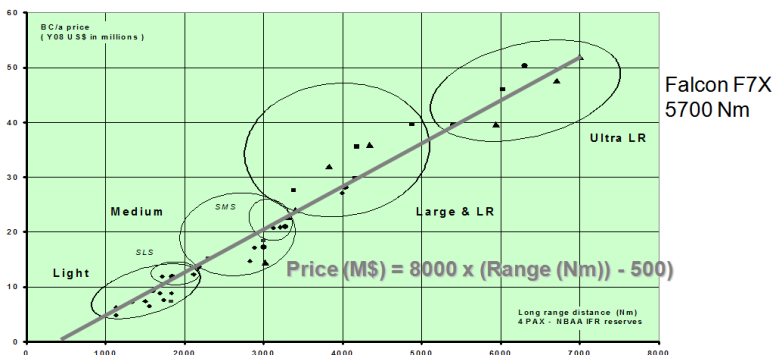
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Conclusion

- ✦ Le calcul scientifique a maintenant un impact fort sur la conception des avions
- ✦ Les codes de calcul sont des différentiateurs pour les avionneurs
- ✦ Beaucoup de travail a été fait en mathématiques appliquées, mais il reste encore beaucoup à faire
- ✦ Les avancées attendues concernent à la fois la physique, les mathématiques, l'informatique et la capacité à faire travailler ensemble les ingénieurs des différents métiers (mécanique des fluides, mécanique des structures, acoustique, ...)

Price of Uncertainties



- Δ Range of 2 to 3% ~ around 1M\$/aircraft sailed
- Δ Consumption of 2% ~ fuel cost 0.5 to 1M\$ during aircraft lifetime